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Volume Two.

Form, Construction, and Working of the Engine: the Steam-turbine.

THE STEAM-ENGINE

AND

OTHER STEAM-MOTORS

A TEXT-BOOK FOR ENGINEERING COLLEGES AND A TREATISE FOR ENGINEERS

IN TWO VOLUMES

VOLUME ONE

THE THERMODYNAMICS AND THE MECHANICS OF THE ENGINE

BY

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PREFACE.

THE objects of this book are, to set forth clearly the fundamental principles of the steam-engine, to give a broad description of constructive practice, to explain fully the working of the machine in its several departments, and to show how to find its efficiency in performance. Manner of presentation and order in development of the subject have been strongly influenced by the fact that it is intended to be a text-book for the use of students: but with this has gone the purpose of making it a complete engineering treatise, in which the theory of the steam-machine shall be put into a shape convenient of practical application, and which, on the descriptive side, shall lay a solid foundation for the more special and detailed information which must be gained from experience and from the technical press.

As to the manner of presentation, it is assumed that the reader knows at least a few general facts about the engine, and has had a good preparation in mathematics, physics, and mechanics; but all physical and mechanical deductions are built up, in concise form, from basal principles, in order that the book may be, on this side, as nearly self-contained as possible. The more elementary subjects carry full explanations, many examples being given to impress and elucidate principles; but the advanced discussions are more concisely stated. Graphical methods are very generally used, as easier and clearer than algebraic.

The first volume is devoted to the Thermodynamics and the Mechanics of the engine, a sufficient foundation of practical knowledge being laid in the introductory chapter, which formulates general ideas and describes, in simple terms, the working of the engine. The fuller exposition of form and construction, at the beginning of volume two, is made to follow this development of fundamental principles, in order that the reasons underlying different types of design may be better understood.

In thermodynamics, the purpose continually kept in mind has

been to present clearly all that is essential to the theory of the steam-engine as a working apparatus. The primary theory of the heat-engine with a perfect gas is first built up, from the simplest statements to the establishment of the ideal thermodynamic process in the Carnot cycle. Then the properties of steam are fully described and illustrated, and its use as a medium exemplified, first in the Carnot cycle, then in the less perfect operations which approach more closely to actual conditions. Upon this follows a discussion of the secondary actions in the concrete machine, a development of methods for getting results from the indicator diagram (already introduced in Chapter I.), and a consideration of the means to be used for securing economy in the engine.

The working theory of the steam-jet apparatus is next taken up: the conditions of formation of the adiabatic steam-jet are reduced to simple quantitative expression, so that its energy and dimensions can be easily calculated; an attempt is made to set forth the mechanical action within the jet; the principles involved are applied to the flow of steam in pipes, the steam calorimeters, the injector, and the jet-blower; and a brief statement is made of the manner of action of the steam-turbine.

A development of the entropy-temperature analysis completes this division of the subject—which, besides its adaptation to teaching, is designed to serve as the basis of a course in experimental steam-engineering.

A full study of the mechanics of the main working-mechanism of the engine is given in Chapter VII., both as to force-action and as to motion. There is first a general view of the conditions to be investigated; next, a complete kinematic analysis, leading to the determination of the inertia-forces; third, a discussion of all the working-forces; last, a general solution of the problems of shaking-force and counterbalance.

Volume II. will cover the following subjects:

Form and construction of the engine;
Valve-gears and their action;
Governors and regulators;
Steam-action in the compound engine;
The steam-turbine;
Accessories.

The treatment of the turbine is intended to be sufficiently full

to meet the ordinary needs of the reader or student, laying a good foundation for the larger special works upon this subject.

This work is mainly descriptive and analytical, developing and illustrating principles and establishing methods for solving the various problems which arise in connection with the working of the engine. The writer has planned a subsequent treatise, on the Performance and Design of the Engine: this will take up the quantitative and synthetical side of the subject, giving the digested results of a large number of tests, now available in works of reference; setting forth methods of engine-testing; and developing, as far as practicable, a logical scheme of design, especially of the thermodynamic design of the engine. This is several times referred to, in the text, as Part II. (of a complete treatise).

It will at once appear that the ground here covered is more extensive than is desirable for many courses of study: but the arrangement of matter is such as to facilitate selection and omission. Without attempting to map out a course in detail, it may be well to remark that certain very special discussions, as § 6 (*k*), § 13 (*d*), and the greater part of § 27, are clearly indicated for omission. A good elementary class-room course would include, in Vol. I., §§ 1 to 17, or all of Chapters I., II., and III., with minor omissions, in Chapter IV., §§ 18, 19, 20 through (*h*), part of 21, 22, and some of 23—the rest of this chapter belonging to the experimental course and forming an introduction to Part II.; § 24, and as much of the rest of Chapter V. as time permits, with emphasis on § 29; in Chapter VII., §§ 32, 33 through (*e*), 34 (*b*), 35 as desired, 36 (*a*), 39, 40 (*a*). A strong course in the mechanics of the engine would use practically all of this chapter—such a course to be largely carried out through graphical work in the drafting-room.

Certain original features in this volume are, the formula for cylinder-condensation in § 21, the curves and tables for the steam-jet in Chapter V., and the full development of the radial analysis for shaking-force in § 40. In regard to the chapter on mechanics, the writer would acknowledge his indebtedness to Professor J. F. Klein, many of the methods there set forth being based upon those in "Notes on the Design of a High-speed Engine," or developed in connection with the long-continued use of that book.

R. C. H. H.

SOUTH BETHLEHEM, PA., December, 1904.

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THE STEAM ENGINE.

CHAPTER I.

A GENERAL VIEW OF THE SUBJECT.

§ 1. The Steam-power Plant.

(*a*) THE ELEMENTS OF THE STEAM-PLANT.—In Fig. 1 are shown in outline the essential elements or organs of a complete apparatus for the generation of power by means of heat derived from the combustion of fuel and utilized through the medium of steam. These organs are:

I. The Boiler (including the furnace as well as the boiler proper), where the fuel is burned and the steam generated.

II. The Engine, in which the expansive force of the steam is applied to the doing of useful work.

III. The Condenser, which receives the used or exhaust steam and abstracts its heat, bringing it back to the initial state of water.

Frequently the steam is discharged into the atmosphere and there condensed.

IV. The Feed-pump, which returns the condensed steam or an equivalent amount of water to the boiler, completing the cycle of operations.

The boiler and engine are naturally considered the principal parts of the plant; while the condenser and feed-pump, together with auxiliaries such as the feed-water heater and the various regulating appliances, come under the head of accessories.

The list of names of parts, given under Fig. 1, is to be used in

connection with the following description of the working of the plant.

(*b*) THE FUNCTION OF COMBUSTION.—In the operation of the steam-generator two distinct sets of phenomena are involved,

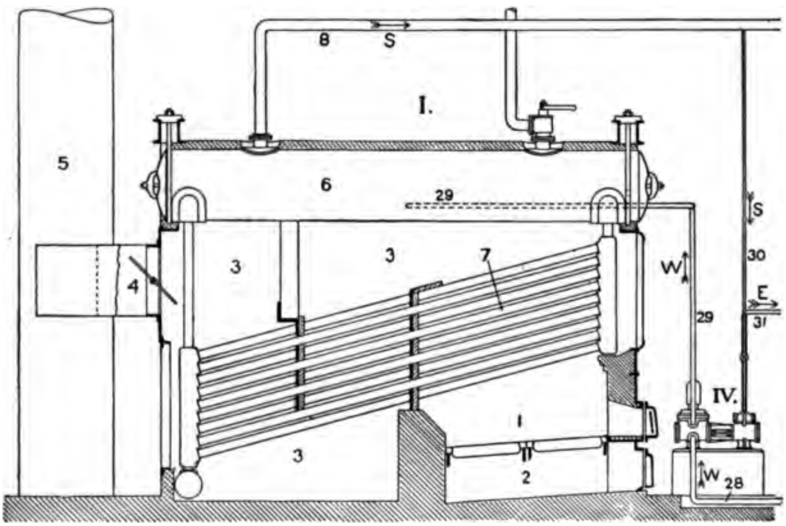


FIG. 1.—The Steam-plant. A. Boiler and Feed-pump.

I. The Boiler (Water-tube type).

1. Grate and Fire-space.
2. Ash-pit.
3. Hot-gas spaces.
4. Flue and Damper.
5. Chimney.
6. Boiler-shell, or Drum.
7. Tubes.
8. Steam-pipe.

II. The Engine (Corliss type).

9. Throttle-valve.
10. Cylinder.
11. Engine-frame.
12. Piston-rod.
13. Cross-head.
14. Connecting-rod.
15. Crank.
16. Fly-wheel.
17. Governor.
18. Exhaust to Condenser.
19. Exhaust to open air.

those of combustion and of heat-transfer and evaporation. The essential condition for combustion is that a sufficient supply of air be continually brought into contact with the fuel. To secure this, there must be first a suitable arrangement for holding the

bed of fuel, so formed that air can pass through it, with provision made for introducing fresh fuel and removing the solid waste-products; second, means for regulating the supply of air, both below and above the fire, and for producing and regulating the

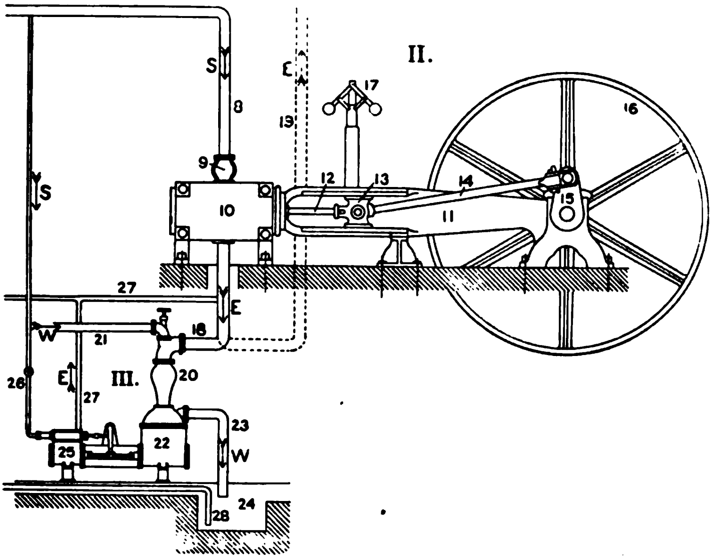


FIG. 1.—The Steam-plant. B. Engine and Condenser.

III. Condenser and Pump (Jet or mixing type).

20. Condensing-chamber.	24. Hot-well.
21. Cold-water supply.	25. Steam-cylinders.
22. Pump-cylinders.	26. Steam-pipe to pump.
23. Discharge-pipe.	27. Exhaust-pipe.

IV. The Feed-pump (Separate, steam-driven type).

28. Suction-pipe.	30. Steam-pipe.
29. Feed-pipe.	31. Exhaust-pipe.

draft which draws or forces the air and the combustion-gases through the fire and along the passages through or around the boiler; lastly, a sufficient space above the fire, in which combustible gases from the solid fuel can be completely burned before they are cooled below the ignition-temperature by contact with the relatively cold surfaces of the boiler. In Fig. 1, these requirements are

met by the grate, ash-pit, and fire-space, with the fire-door and ash-pit door, both provided with air-grids; and by the chimney and damper. With special fuels, liquid or gaseous, other arrangements take the place of the grate used with solid fuel. And various forced-draft appliances are frequently used to assist, or partially to replace, the chimney.

(c) THE FUNCTION OF EVAPORATION.—In order that the boiler may freely absorb the heat generated by combustion, it must have a large area of heating-surface, so disposed that there will be a rapid flow of the hot gases over the outer side, and that the steam as formed will be able to escape freely and rapidly from the inner side. A small portion of this surface is exposed to radiant heat from the solid fuel and from incandescent flame: this direct heating-surface is far more effective in absorption than is that which receives heat only by contact and conduction from the hot gas. The current of gas is split up into narrow streams, and the body of water is likewise divided into small parts, so that there shall be only a slight depth of gas acting upon, and of water heated by, any particular portion of surface. Whether this intimate contact is secured by the water-tube arrangement of Fig. 1, or by the fire-tube arrangement of cylindrical boilers, is a matter of minor importance. Means for insuring a full circulation of the hot gases over the whole of the heating-surface are shown in Fig. 1; and the boiler is so formed as to permit free internal circulation, whereby a current of mixed water and steam-bubbles is continually rising through the front connecting-tubes into the drum, where there is ample surface for the separation of the steam from the water.

(d) THE BOILER A SEPARATE SUBJECT.—The above general considerations are here stated in full because they are fundamental to an understanding of the thermal performance of the boiler, as a member of the steam-plant. But the boiler is made in so great a variety of forms, and there are so many special matters involved in its design, construction, and management, that it properly forms a separate subject—together with all the appliances for handling and controlling steam, such as piping, valves, steam-traps, separators, etc.

(*e*) THE ENGINE.—A simple engine (differing somewhat in type from that in Fig. 1) is fully described in the next section, so that no description need be given here. In general, the engine may be treated as a thermodynamic ("heat-work") apparatus and as a machine: both these phases of the subject are to be fully developed in this treatise.

(*f*) CONDENSING THE EXHAUST STEAM.—The two ways of getting rid of the exhaust steam are shown in Fig. 1. The simplest is, of course, open exhaust to the air; but the efficiency of the engine can be increased by condensing the steam at a low temperature and in a consequent vacuum, using a pump to remove the water and maintain the vacuum. In the figure, the exhaust steam meets, in the chamber 20, a jet of cold water from the pipe 21, and is condensed by direct contact and mixing. The water from the condenser, moderately warm, is discharged into a tank called the hot-well.

The difference in working here described marks the distinction between condensing and non-condensing engines.

(*g*) THE FEED-PUMP draws from the hot-well an amount of water equal to the steam condensed and forces it into the boiler; the rest of the warm water goes to waste. In a well-designed plant there is one feature not shown in the drawing; that is, the exhaust from the pumps, instead of going to the main condenser, would go to a feed-water heater, where the greater part of its heat would be utilized in raising the temperature of the feed. And with open exhaust there would be a larger feed-heater, in which the water, drawn from a cold supply, could be heated up to the temperature of the exhaust steam.

§ 2. Description of an Engine.

(*a*) THE ENGINE SELECTED for description, and illustrated in Figs. 2 to 7, is a good example of the simple, high-speed, self-contained type, especially adapted to driving electric generators—a service which requires smooth running and a close regulation of speed. The general form of the engine is shown in Fig. 2; the various working parts and the internal construction, in the several

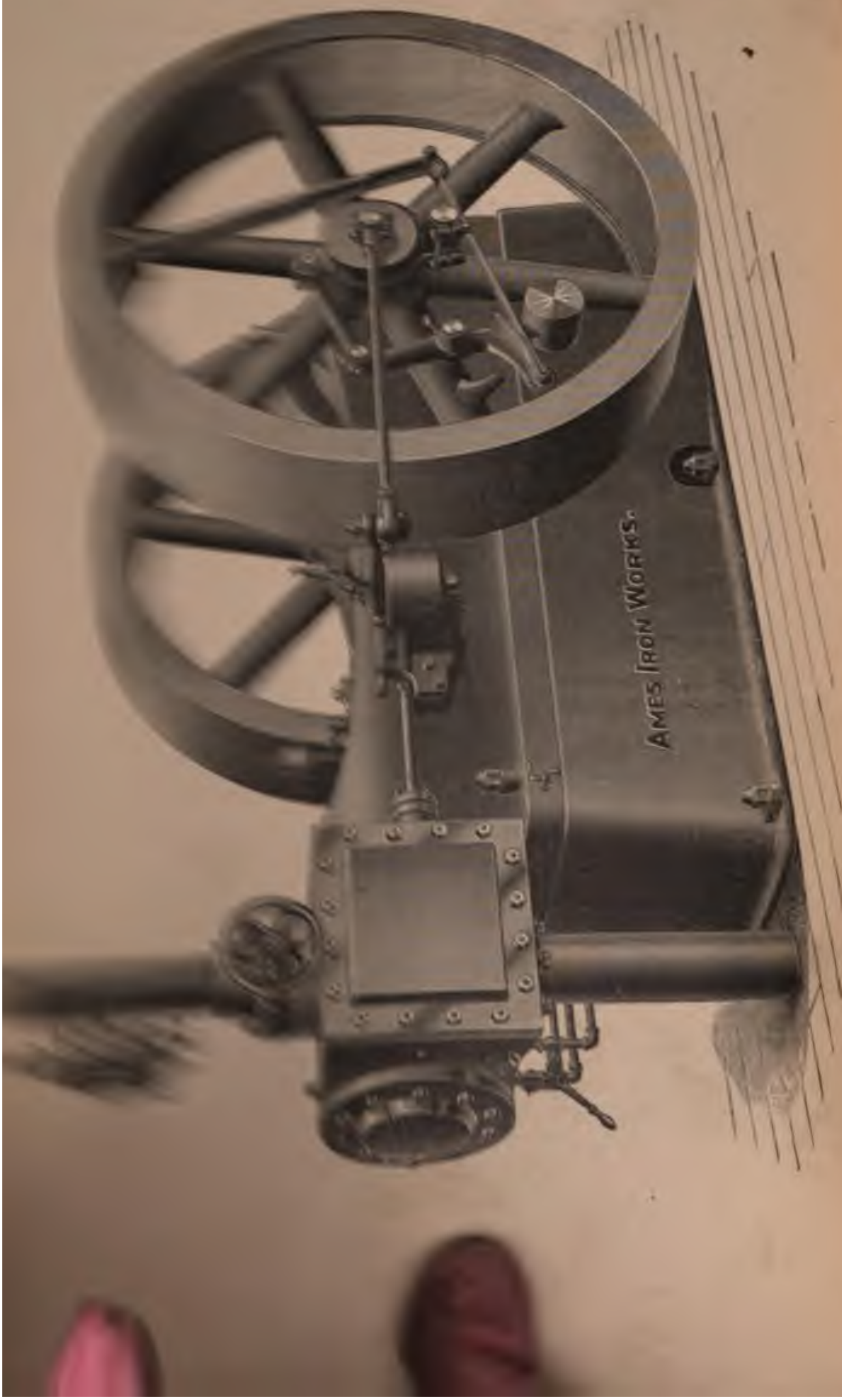


FIG. 2.—A High-speed Center-crank Engine.

sectional views. Leaving Fig. 2 for incidental reference, we will first take up the main working-mechanism of the engine, given in Fig. 3.

(b) THIS MECHANISM performs the function stated in § 1 (a), of applying the expansive force of the steam to the doing of useful work. The moving parts of the mechanism are three, the sliding piece made up of the piston, piston-rod and cross-head, the connecting-rod, and the rotating crank and shaft. Steam is admitted to the two ends of the cylinder alternately; and the steam-force acting on the piston is transmitted directly to the cross-head, and thence by the connecting-rod to the crank-pin, where it acts to turn the crank. The effect of this mechanism is, then, to change the reciprocating, straight-line motion of the piston into a continuous rotary motion of the shaft. In some types of engines, however, the motion of the piston is applied directly to the useful resistance—as in pumps, blowing-engines, and steam-hammers.

(c) THE PISTON—see also Fig. 6—is a thick disk, made hollow for lightness, but broad of face so as to have a liberal bearing-surface where it slides in the cylinder. It fits loosely, and the joint between cylinder-wall and piston is made steam-tight by the packing-rings, which fit into grooves cut in the piston and press outward by virtue of their own elasticity. The piston-rod is securely fastened into the piston and the cross-head: it passes out of the cylinder through a stuffing-box, which is filled with a packing material closely pressed into place around the rod, so as to prevent leakage of steam.

(d) THE RECIPROCATING PARTS.—Two sections of the cross-head are shown in Figs. 3 and 4, and a separate drawing of it will be given when the detailed description of the working parts is taken up. Besides having a hub into which the piston-rod screws and carrying the wrist-pin on which the connecting-rod oscillates, the cross-head is formed at the bottom to fit and slide upon the guide-surface on the engine-bed. The connecting-rod is of simple form, and is sufficiently shown in Fig. 3, except that the device for adjusting the wrist-pin bearing is not made clear; this also will be given later. The piston-slide and the connecting-rod together constitute the "reciprocating parts" of the engine.

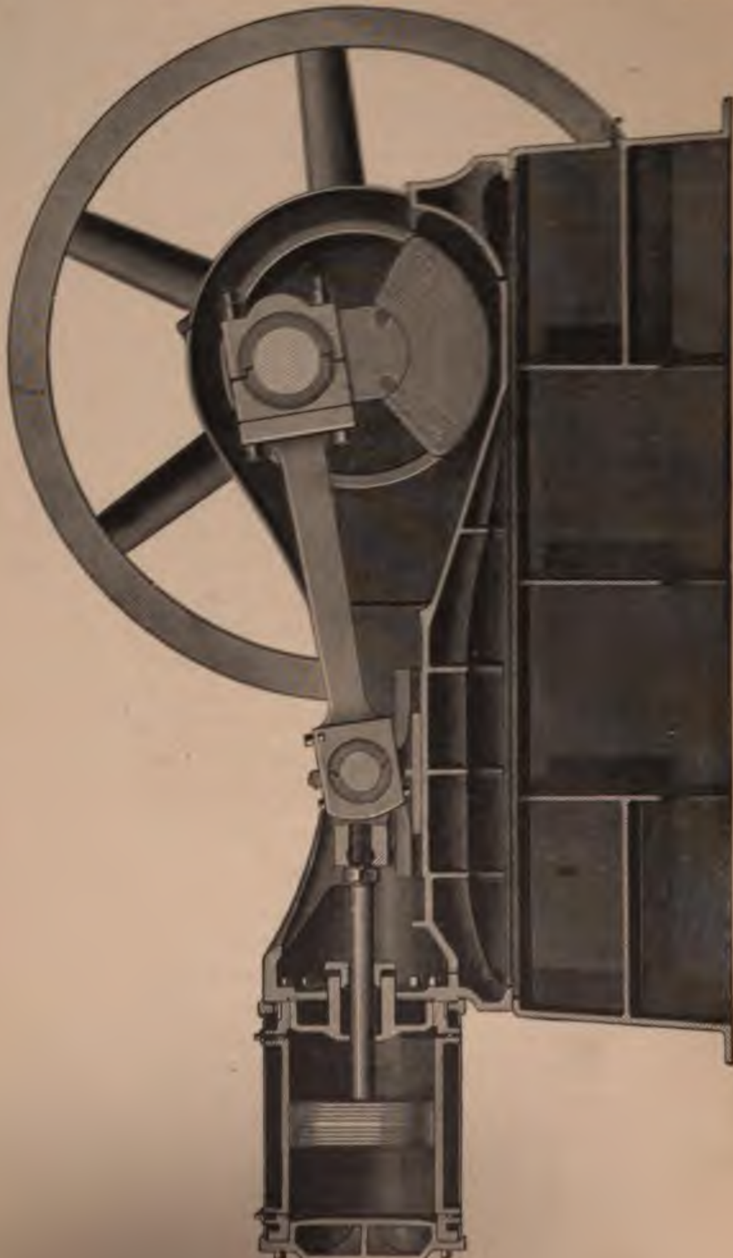


FIG. 3.—Longitudinal Section.

At high speeds, the inertia-forces due to their alternate rapid accelerations in opposite directions have a considerable effect upon the force-action in the engine.

(e) Shaft and Wheels.—The detail of the crank-shaft and wheels is well shown in Fig. 5. The shaft is one solid piece, and the reason for the name "center-crank" is at once apparent. The crank-disks are separate pieces, fastened to the shaft simply for

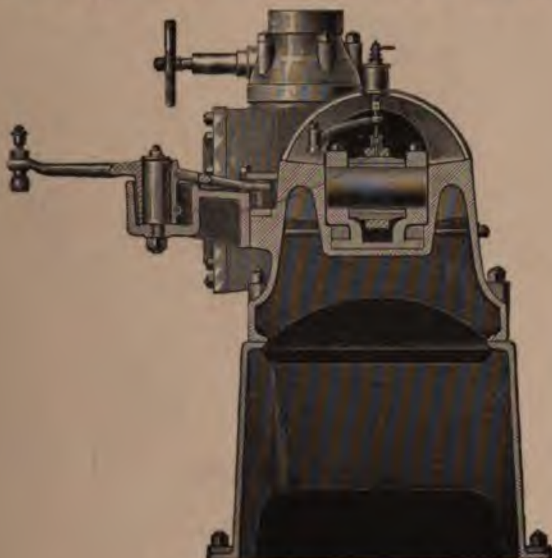


Fig. 4.—Cross-section through Frame.

the purpose of carrying the heavy fan-shaped weights opposite the crank-pin. The centrifugal force of these counterweights acts against the inertia of the reciprocating-parts, so as greatly to diminish the free force tending to shake the engine on its foundation. A distinctive feature of this type of engine is the very large bearing-surface of the crank-pin and of the shaft-journals. The heavy fly-wheels steady the running of the engine and also serve as belt-pulleys.

(f) The Frame or Bed.—As to the framework of the engine, we distinguish the engine-bed proper and the sub-base on which it rests, and which in turn is bolted to the foundation. This engine

is said to be "self-contained" because it is of such a form—all on one rigid base—that it might be picked up bodily, as by a travelling-crane, without displacing any of its parts. The design of a complex casting such as the bed of an engine of this class is very much a matter of skilled judgment. This particular design shows very graceful outlines and a judicious distribution of the metal.

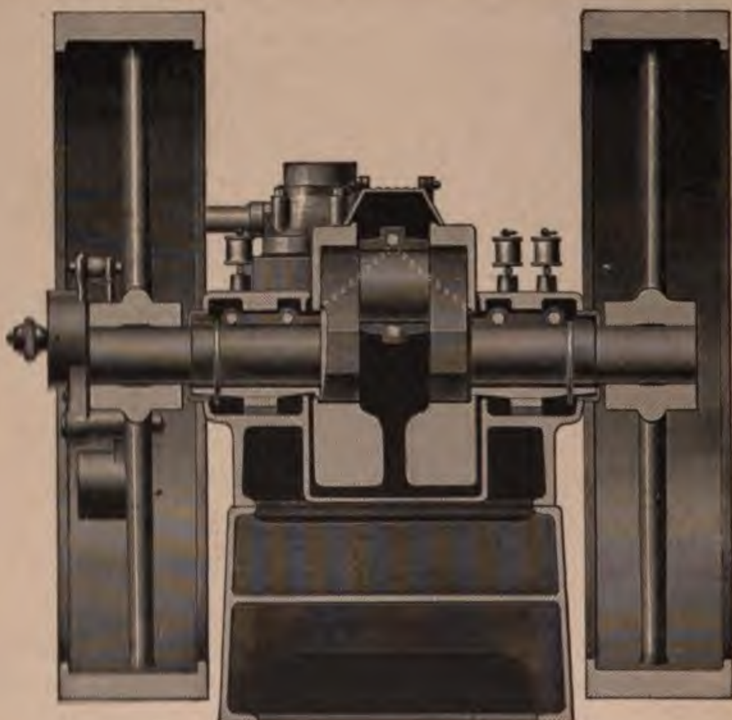


FIG. 5.—Cross-section through Main Bearings.

(g) THE CYLINDER AND VALVE-CHEST.—The sectional views given in Figs. 6 and 7 are especially intended to show the form of the valve and of the steam-passages. This particular slide-valve is of rather a complex form, though essentially the same as the plain valve used in Figs. 8 and 9; but its action in admitting steam to one end of the cylinder while permitting free exhaust from the

other end is clearly shown by the arrows in Fig. 6. This is what is called a balanced valve: it works between the plane surfaces of the valve-seat on the cylinder and of the heavy balance-plate, with a fit which, while permitting free movement, is yet so close that steam cannot leak between the surfaces. The balance-plate

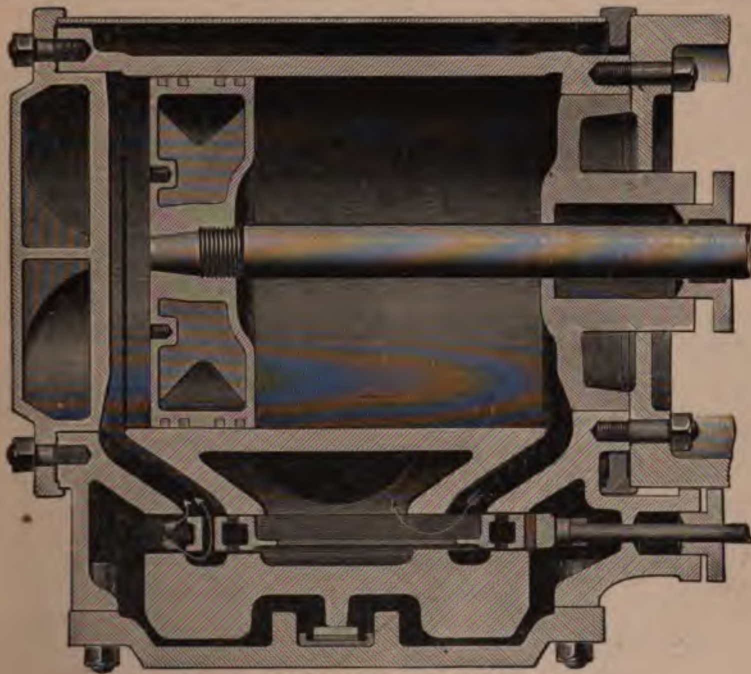


FIG. 6.—Horizontal Section of Cylinder.

is held out by distance-strips, at top and bottom, which are a few thousandths of an inch thicker than the valve. All the surfaces must be very truly finished; and the plate is made very heavy and stiff in order that it may not spring under the pressure of the steam on its back, and grip the valve. By the use of this device, the valve is relieved of the steam-pressure which forces a plain valve hard against the seat, causing great friction and rapid wear.

(*h*) THE STEAM-PASSAGES.—The live steam, coming from the steam-pipe through the throttle-valve, fills the valve-chest all

around the valve and the balance-plate. The passages through which the steam flows to and from the cylinder are called the steam-ports; the space in the middle, leading to the exhaust-pipe, is the exhaust-port. Evidently, it is necessary only to move the

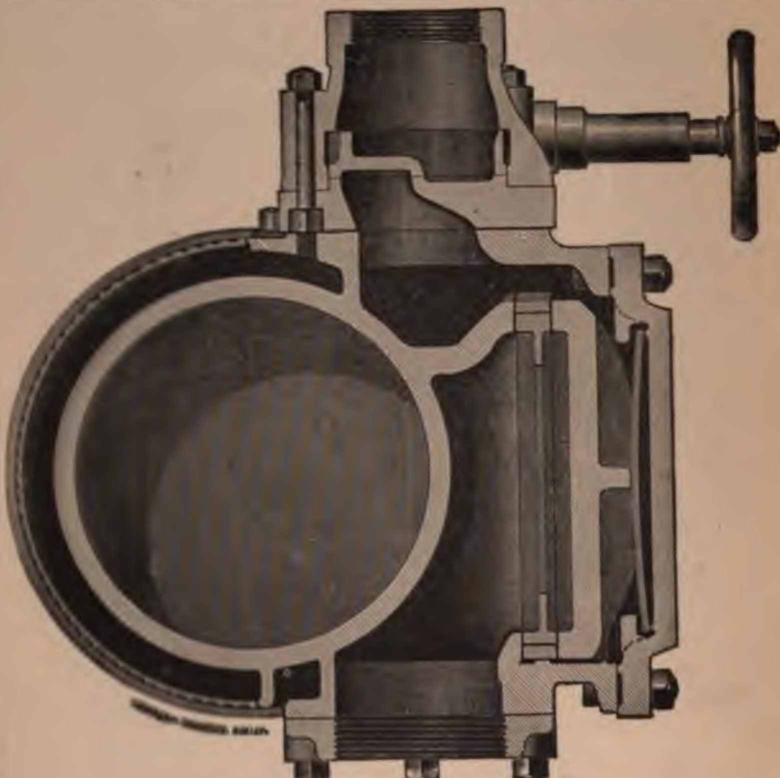


Fig. 7.—Cross-section of Cylinder.

valve back and forth in order to secure the desired end of alternate admission to and exhaust from the two ends of the cylinder. The working of the valve and the manner in which it controls the steam-distribution are fully described in the next section.

(c) The Valve-gear.—The mechanism for moving the valve is shown as a whole in Fig. 2, while details are given in Figs. 4 and 5. Starting with rotary motion, we have first the eccentric-pin, at a

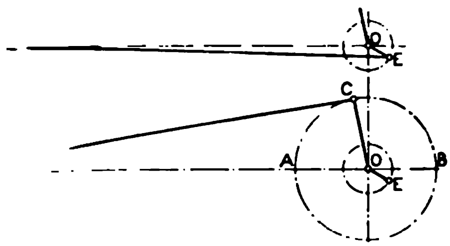
short distance from the axis of the shaft and travelling in a circle. The eccentric-rod runs from this pin to the outer end of the rocker-arm, transforming rotary into rectilinear motion: the rocker-arm, best shown in Fig. 4, serves to transfer this motion from a line outside the wheel to one near the side of the frame; and its inner end drives the valve-rod. This particular form of rocker-arm is peculiar in its action: since the eccentric-pin moves in a vertical plane, while the pin on the rocker-arm moves in a horizontal plane, it is necessary to use ball-and-socket joints instead of cylindrical bearings, at both ends of the eccentric-rod.

(*j*) THE GOVERNOR.—The speed of the engine is controlled by the governor, shown in Fig. 2. Briefly stated, the principle of its action is as follows: The centrifugal force of the weight on the governor-arm acts against the pull of the spring. If the load on the engine is increased or diminished, it will slow down or speed up until the change in centrifugal force causes the arm to move in or out far enough to change the working of the valve so as to accommodate the power of the engine to the new load. This change in valve-action is secured by moving the center of the eccentric, with reference to a line fixed on the plane of the wheel; and its chief effect is to vary the amount of steam admitted to the cylinder in each stroke, and thereby control the amount of work done per stroke. Being placed in the wheel or on the shaft, this is called a shaft-governor; and since the principal effect of its action is to change automatically the time in the stroke when the valve closes the steam-port, or "cuts off" steam, this engine is called an "automatic cut-off" engine.

§ 3. Valve Movement and Steam Action.

(*a*) THE MECHANISM SIMPLIFIED.—In order to find the relation between the movements of the two sliding-pieces in the engine, the piston and the valve, we use the method illustrated in Fig. 8. The two mechanisms, main and secondary, are represented by their skeleton outlines; and the second, the valve-gear, is simplified by bringing everything into one plane, replacing the rocker-arm by a simple slide-block. Also, for convenience, the section of the

mechanisms are turned
ertical plane, the crank-
a simple rigid figure
enter. For a particular
of the wrist-pin W on its
length WC and locate the
position of the eccentric-
be located. Of course,
terminations will be devel-



Valve and Piston.

Fig. 9 we now trace out the
dution or double-stroke of
extreme left-end position,
dead-center: the left port
steam has a chance to enter
the piston begins its stroke.
of the arrow right-hand or
valve move toward the right,
here the valve is at the
port has its fullest opening.
diminishing the port-opening,
closes the port or cuts off
now enclosed in the left end
continues to advance this steam
increases.
When the valve, still moving
shown by the dotted lines in H.,
releases the steam. A part of

the steam escapes while the piston is completing the forward stroke, the rest is expelled during the return stroke. In III., the dotted position shows fullest opening for exhaust, while the full-

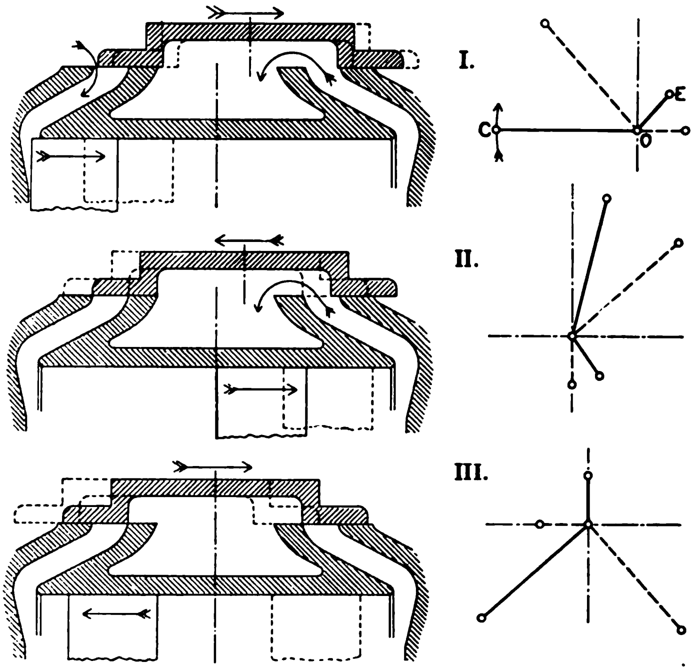


FIG. 9.—Valve Movement.

line figure shows where the piston is when the valve, returning from the left, stops the exhaust. For the rest of the stroke, the port is closed; and the steam caught in the cylinder is compressed into the clearance.

(*d*) THE STEAM DIAGRAM.—The performance of the steam, controlled by and resultant from this valve-action, is best shown by a diagram like Fig. 10. The base-line MN represents the stroke of the piston, and any ordinate, as HG, gives the pressure which exists in the cylinder when the piston is at a position in its stroke corresponding to that of G on MN. Diagrams of this sort are drawn automatically by an instrument called the Steam-engine

...dicator, which is illustrated in Fig. 11. This instrument com-...es the two functions of measuring the pressure, or getting the ...ngth of the ordinate HG, and of locating this ordinate on the ...agram. For the first, there is the piston 8 to receive the pressure ...om the engine-cylinder, and the spring to measure this pressure ... varying compression: together with the pencil-mechanism, ...ade up of pieces 13, 14, 15, and 16, which magnifies a small piston

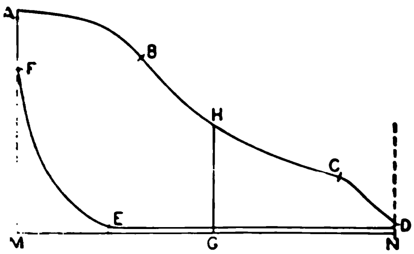


FIG. 10.—The Steam Diagram.

... to an indication of convenient size. The spring is ... so as to give to the pressure-ordinate a scale of a cer-... of pounds per square inch to the inch of pencil-rise. ... 23, draws the diagram on a slip of paper carried by ... This drum is driven, through a cord which pulls ... spring 31, by a special mechanism connected to the ... designed as to give a reduced copy of the motion ... piston. The atmosphere-line MN is drawn with ... the indicator and the pressure of the air acting ... sides of the piston: the ordinates of the diagram ... (or down, in a condensing engine) from this refer-

... OF THE STEAM.—Returning now to Fig. 10, and ... salient points of the working of the steam in the ... that the admission is made up of two parts, ... from F to A, and the filling of the cylinder back ... latter advances; this operation being carried ... AB, to cut-off at B. The clearance-space, ... while the pressure rises along FA, consists of ... between the cylinder-head and the piston when

the latter is at its extreme position, together with the volume of the steam-port. The pressure at A is nearly that of the steam in the boiler; but it drops off toward B because the gradual closing of the port chokes the entering current.

(*f*) The Steam-cycle Completed.—From B to C there is expansion of the steam enclosed in the cylinder, as already stated. The

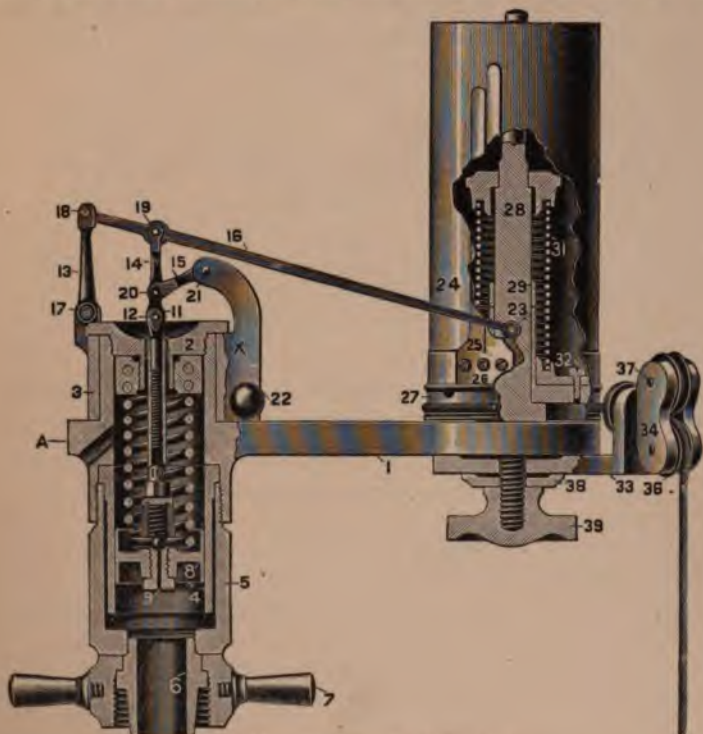


Fig. 11.—The Crosby Indicator.

two parts of the exhaust are shown by the drop of pressure from C to D, followed by expulsion at nearly constant pressure along DE. Compression from E to F, raising the pressure in the clearance part way up to that of the fresh steam, completes the cycle. In describing Fig. 9, we considered only the left or head end of the cylinder; there is, of course, a similar action in the other, the crank

end. It is to be noted that the diagram in Fig. 10 does not correspond, in the timing of its events, with the valve-action shown in Fig. 9.

(*g*) MEASUREMENT OF POWER OF ENGINE.—The steam diagram, besides showing how the valve is working and furnishing part of the data for a discussion of the thermodynamic behavior of the steam, also gives a measurement of the power developed by the engine. Since the abscissa represents distance moved by the piston, and the ordinate pressure acting, the area of the figure represents work.

(*h*) DIVISIONS OF THE SUBJECT.—The preceding general description of the steam-plant and of the engine forms a sufficient introduction to the detailed study of the several parts of the subject. This will follow the lines already laid out, taking up first the relations between work and heat, then the mechanics of the engine—the latter including the action of the forces in the machine and the working of the auxiliary mechanisms, the valve-gear and the governor. With this will be given a systematic description of the several parts of the apparatus, with representative examples of different types of design in each department.

§ 4. Classification of Steam-engines.

(*a*) SERVICE.—Steam-engines may be classified according to service, general form, mechanical features, and the manner of using the steam. As to service, we have first the great variety of stationary engines, used for driving machinery by means of the different kinds of power-transmission apparatus, or more directly applied to such work as turning electric generators, or hoisting-drums at mines, or metal-rolls in mills: an even more direct and intimate combination of prime-mover and load is found in steam-pumps, blowing-engines, and air-compressors. The locomotive type—including the traction-engine and the steam automobile—forms another class; and the marine engine a third.

(*b*) OTHER BASES OF CLASSIFICATION.—As to general form and mechanical features, the engine may be horizontal or vertical or of special shape; and may have one of several different styles of

framework. Besides the usual form of the main mechanism, there are several other types of construction, of which the oscillating engine is, perhaps, the most distinctive. The engine may be single or complex—in the latter case, having two or more complete engines combined in one, these elements being either simple or compound; thus in pumps and in locomotives duplex compound engines are quite common. The type of valve-gear and the kind of controlling apparatus likewise form grounds of distinction, as also the question whether the engine runs in only one direction, or reverses.

(*c*) Different Ways of Using the Steam.—In the matter of steam-working, engines may be either simple or multiple-expansion; that is, the steam may pass through only one cylinder, or through several cylinders, of increasing size, in succession. There is great variety in the arrangement of the cylinders of compound engines. The distinction between condensing and non-condensing engines has been already pointed out.

(*d*) Rotary Engines and Steam Turbines.—It is to the engine with the usual device of cylinder and piston for utilizing steam-force that the name steam-engine is commonly applied. Other forms of the steam machine, the rotary and turbine classes, are known by their distinctive names. Thermodynamically, they perform the same function, though with some modifications in the operation; mechanically, they very properly fall into entirely separate classes.

CHAPTER II.

ELEMENTARY THERMODYNAMICS OF THE HEAT-ENGINE.

§ 5. Heat and Work.

(*a*) THE FIRST PRINCIPLE OF THERMODYNAMICS is that heat-energy and mechanical work are mutually convertible, one into the other, in a definite quantitative ratio.

Of a certain amount of mechanical work or energy the whole may, with a suitable apparatus, be transformed into heat; but the complete reversal of this operation is not possible. That is, of a given amount of heat-energy, made available for instance by the combustion of fuel, it is inherently impossible to transform more than a certain fraction into mechanical work. The primary object of this discussion will be to determine the character and the limits of the thermodynamic process which will give the maximum proportion of heat-transformation.

An apparatus for converting heat into work is called a heat-engine; and in every heat-engine this conversion is effected through the action of an expansive fluid or working medium, which alternately expands and contracts as heat is given to or taken from it. Since air, which is practically a so-called "perfect gas," follows very simple, logical laws—far simpler than those governing the behavior of steam—it will be used as the medium in establishing the principles of working of the ideal heat-engine.

A concise statement of the physical facts and laws underlying the discussion will first be given—but without any description of the manner or of the history of their establishment.

(*b*) FACTS AS TO HEAT.—Heat is a form of energy. As existent in sensible form, in material bodies—the only form in which it is directly measurable—it is believed to consist in a vibratory motion

of the molecules of the body, and is then a form of kinetic energy. Intensity of heat-energy is measured by the temperature of the substance in which it is stored. The quantity of heat contained in a given body, above any certain reference state, depends upon the quantity of the substance, its unit-capacity for heat, and its temperature.

Temperature is so nearly a fundamental idea that it can hardly be defined in terms of anything simpler than itself. Since the first manifestation of a difference in temperature between two bodies—that on which depends our sense of heat—is a tendency of the heat to pass from the hotter to the cooler body, or from the more intense to the less intense state, temperature may be expressed as "the thermal state of a body, considered with reference to its power of communicating heat to other bodies." This makes it an entirely relative matter, until some standard reference condition has been chosen, as in the development of the thermometer.

(*c*) The Measurement of Heat.—It is not difficult to imagine a temperature-scale on which every degree shall represent the same change in heat-intensity. Actual thermometers only approximate this ideal heat-scale. Thus if a mercury thermometer were heated in comparison with an ideally correct instrument, its readings, which are based on the uniform expansion of mercury, would be found not quite to agree with those of the standard; this chiefly because the mercury does not really expand quite uniformly with rising temperature. The ideally correct thermometer is based on the expansion of a perfect gas.

The unit of temperature which we shall use is the Fahrenheit degree, and the unit of quantity for heat-measurement is the British Thermal Unit: this B.T.U. is the amount of heat required to raise by 1° F. the temperature of 1 lb. of water at about 40° F. That is, the heat-capacity of pure water at this temperature is taken as unity; and the heat-capacity of any other substance (including water at other temperatures), per pound and per degree F., expressed in terms of this unit, is called its specific heat. Instead of "British Thermal Unit" we shall use Heat Unit as a shorter name, with the abbreviation H.U.

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$$\ldots \qquad (2)$$

units,

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This symbol A, carrying the same meaning all through our thermodynamic discussions, and having the value $1 \div 778$ or .001285, is really the heat-equivalent of work.

When the statement is made in (a) that only a part of any given quantity of heat can be converted into work by a heat-engine, nothing is implied against the constancy and the reversibility of the ratio of conversion just given. Of the total heat supplied, a part is changed into work in the ratio of 1 to 778, the rest is rejected as heat: and this heat rejected is usually by far the larger part of the heat supplied.

§ 6. Heat-behavior of a Perfect Gas.

(a) A Perfect Gas is one which is so far from its point of liquefaction that it follows exactly the simple laws set forth below. Air, its component gases, and hydrogen are, for all practical purposes, perfect gases throughout the range of ordinary temperatures, even though certain very minute irregularities in their behavior have been found by extremely precise experimentation. The physical criterion of a perfect gas is, that its molecules shall have absolutely free motion, with no attractive forces exerted between or among them. The molecules are supposed to be in constant, rapid, rectilinear motion, rebounding as they impinge upon each other or upon the walls of a confining vessel; the distances between them being indefinitely large in comparison with their own dimensions. Then the effect of imparting heat is to increase the mean velocity of the molecules or to augment their kinetic energy.

(b) The Three Primary Laws for perfect gases, first discovered by experiment, but capable also of deduction from the theory as to the constitution of a gas which they have helped to develop, are as follows:

The Law for Constant Pressure.—If a portion of gas be heated and its pressure kept constant, the temperature and the volume will increase together at a constant relative rate.

The Law for Constant Volume.—If a confined body of gas be heated and its volume kept constant, the temperature and the pressure will increase together at a constant relative rate.

(*d*) WORK AND POWER.—For ... ...ned body of gas
be defined as the overcoming of r... ...es in volume and
to make the action subjective, w... ...e product of pres-
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opposite resistance, it does wo...
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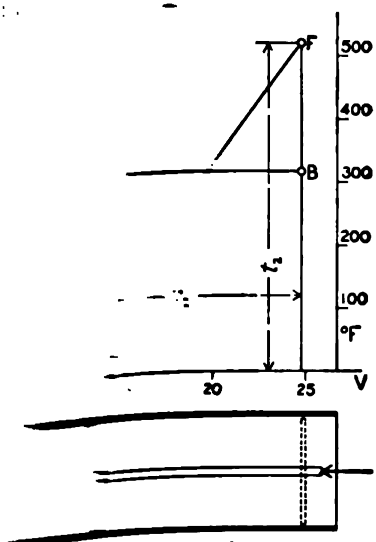

...Operations.

If we have ... The initial conditions, p_1
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... line AB, rise of pressure at
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...roportions, laid out to scale, are for one pound ... F. and at atmospheric pressure—is supposed ... a cylinder: in one case the piston is allowed ... a constant resistance, as the gas is heated; in ... piston is held fast.

RELATION BETWEEN TEMPERATURE AND VOLUME, ...t pressure, is further set forth in Fig. 13. If we

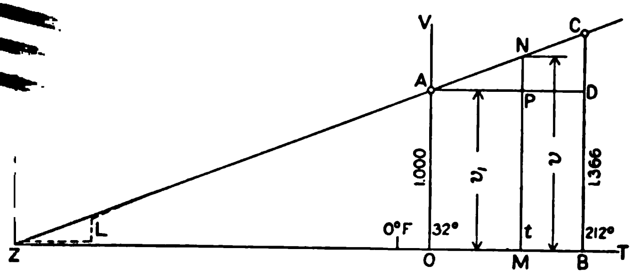

Fig. 13.—The Absolute Zero.

...t the volume at freezing-point be unity, then $v_1 = 1.000$ for 32° F.: and it has been found by experiment that the volume of the same weight of gas at 212° F. will be 1.366. These volumes being laid off at OA and BC, the first law is represented by the straight line AC; and the increment of volume per degree in terms of the volume at 32°, or the coefficient of expansion, is

$$a = 0.366 \div 180 = .002033. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now the volume v at some temperature t will be

$$MN = MP + PN,$$

or

$$v = v_1 + a(t-32)v_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where it appears that the increment of volume $(v - v_1)$ bears to the increment of pressure $(t-32)$ the constant ratio a or .002033.

Example 1.—The volume of 1 lb. of air, at standard atmospheric pressure and at 32° F., is 12.385 cu. ft.; what will its volume be at 400° F.?

Substituting in Eq. (5),

$$\begin{aligned} v &= 12.59(1 + .002033 \times 368) \\ &= 12.39 \times 1.748 = 21.66 \text{ cu. ft.} \end{aligned}$$

Again, at $-40°$ F. the volume will be,

$$v = 12.39[1 + .002033(-72)]$$
$$= 12.39 \times .854 = 10.58 \text{ cu. ft.}$$

(*e*) THE ABSOLUTE ZERO.—If now we produce the straight line CA until it cuts the temperature-axis at Z, we have there a new zero-point such that, if the temperature be measured from it, then the whole volume MN will bear to this temperature MZ the same ratio that the increment PN bears to OM, the rise of temperature above the first starting-point at O. The point Z is called the absolute zero, and from it absolute temperature is measured. The distance OZ is $1.000 \div .002033 = 492°$: that is, on the Fahrenheit scale the absolute zero is 492° below freezing or 460° below 0° F. Using t for common temperature and T for absolute, or for degrees AF., the relation between the two systems is

$$T = t + 460. \qquad (6)$$

Another argument for the absolute zero, based directly on the statement of the law for constant pressure, is, that if the gas continue to contract as heat is abstracted, with a constant decrement per degree, a point will at last be reached where, all the heat having been taken from the gas, its volume will be reduced to zero. Since the actual physical performance of the gas would be somewhat as indicated by the dotted line in Fig. 13, namely, liquefaction at L and then very slow contraction, it appears that the idea of the complete abstraction of heat at absolute zero is not rigorously established by this argument.

This deduction is then essentially the same as the more purely mathematical proof given above: in both cases we extend indefinitely the conditions existing at ordinary temperatures, and find a point which has a very important relation to the behavior of the gas over its range of "perfect" action.

(*f*) VOLUME AND ABSOLUTE TEMPERATURE.—Eq. (5) can be put into the form

$$v = v_1[1 + a(t - 32)].$$

The corresponding formula with absolute temperature is

$$v = v_1 aT. \qquad (7)$$

This can be derived geometrically from Fig. 13; for

$$MN : OA :: ZM : ZO,$$

or

$$v : v_1 :: T : T_1 :: aT : aT_1;$$

and $aT_1 = .002033 \times 492 = 1$. Again, by substitution, since $a = 1 \div 492$,

$$1 + a(t-32) = 1 + \frac{t-32}{492} = \frac{1}{492}(t+460) = aT.$$

Eq. (7) may further be put into the form

$$\frac{v}{T} = C \quad \text{or} \quad \frac{v}{T} = \frac{v_1}{T_1}, \quad \ldots \ldots \quad (8)$$

in which shape it is most convenient for finding the relation between volumes at any temperatures.

Example 2.—Derive from Eq. (7) an expression for the volume of 1 lb. of air under atmospheric pressure at any temperature.

We have only to substitute the particular value $v_1 = 12.39$ to get

$$v = .00203 \times 12.39T = .02518T. \quad \ldots \ldots \quad (9)$$

Example 3.—A receiver of 40 cu. ft. capacity is filled with air at 60° F. and a pressure p: how much must be allowed to escape in order that the pressure shall remain unchanged when the air is heated up to 200° F.?

$v_1 = 40$ cu. ft. $\qquad v_2 = ?$
$T_1 = 520°$. $\qquad T_2 = 660°$.

$$v_2 = 40 \times \frac{660}{520} = 40 \times 1.269 = 50.76 \text{ cu. ft.}$$

Of this, 40 cu. ft. remains in the receiver, and the fraction $\frac{10.76}{50.76} = .212$ of the original quantity escapes.

Example 4.—By what fraction of its volume will a body of gas, measuring v cu. ft. at $t°$ F., increase when heated 1° F. at constant pressure?

The volume at t being v, that at 32° would be

$$v_1 = \frac{v}{1 + a(t-32)}.$$

The increment per degree is then

$$av_1=\frac{a}{1+a(t-32)}v=\frac{1}{492}\times\frac{492}{460+t}v=\frac{1}{T}v.$$

Or, using absolute temperatures from the first,

$$a'v=av_1=a\frac{T_1}{T}v=\frac{1}{T}v. \qquad (10)$$

(*g*) THE LAW FOR CONSTANT VOLUME is expressed by formulas exactly similar to those for the first law: we have,

for (5) $$p=p_1[1+a(t-32)]; \qquad (11)$$

for (7) $$p=p_1aT. \qquad (12)$$

And if we wished to get pressure directly in pounds per square inch instead of in atmospheres, we should have, for the particular case where the gas just filled the confining vessel at 32° and one atmosphere pressure,

$$p=.002033\times14.7T=.02997T. \qquad (13)$$

(*h*) THE THIRD LAW is expressed by the equation

$$pv=C \quad\text{or}\quad pv=p_1v_1, \qquad (14)$$

and is graphically represented by the curve called the equilateral hyperbola, shown in Fig. 14, where a convenient construction for

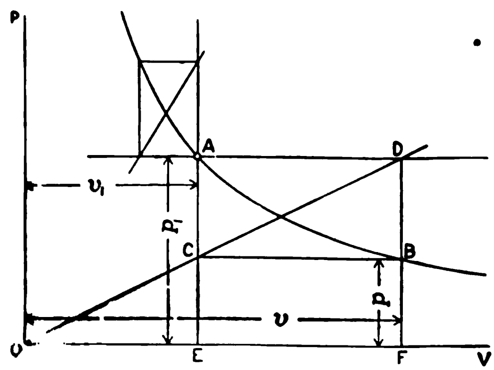

FIG. 14.—The Equilateral Hyperbola.

[illegible] the curve is given. Having the reference-axes OP and [illegible] A through which the curve is to pass, we draw

through A the lines AD and AE, parallel to OV and OP: then drawing from the origin O any radial line to cut these auxiliary axes at C and D, and completing the rectangle ADBC, we determine the point B of the curve. For, in the similar triangles DOF, COE,

$$DF \text{ or } AE : CE \text{ or } BF :: OF : OE$$

or

$$p_1 : p :: v : v_1,$$

which satisfies (14).

An operation which takes place at constant temperature is called an isothermal operation; and the curve drawn in Fig. 14 is the isothermal curve of a perfect gas.

(*i*) THE GENERAL LAW.—In these three laws is given the relation between any two of the three variables p, v, and t or T, when the other is kept constant. From the three expressions,

$$\frac{v}{T}=C \quad (8), \quad \frac{p}{T}=C \quad (12), \quad \text{and} \quad pv=C \quad (14),$$

it is self-suggestive that the general law must be

$$\frac{pv}{T}=C,^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

this, if for no other reason, because each of the particular laws can be directly derived from this general equation.

A more strictly logical deduction of this law is as follows:

Suppose that we have a unit-weight of gas under the conditions p_1, v_1, and t_1, represented as to pressure and volume by the point 1 in Fig. 15: and another equal weight, under the conditions p_2, v_2, and t_2 at 2. Now abstract heat from gas-body No. 1 at constant pressure, cooling it down to freezing-point or t_0: then the volume will become

$$v_{10}\dagger=\frac{v_1}{1+at_1} \quad \text{or} \quad v_{10}=v_1\frac{T_0}{T_1}.$$

* This symbol C is used simply to represent an undetermined constant, different in each of the several expressions.

† Read, "v sub one zero."

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Substituting in (15) a particular set of values, we get the constant C as follows:

For 1 lb. of air at 32° and atmos. pressure, $p_0=14.7$ lbs. per sq. in., $v_0=12.385$ cu. ft., $T_0=492°$; then

$$\frac{pv}{T}=\frac{p_0v_0}{T_0}=\frac{14.7\times 12.385}{492}=.37004,$$

or

$$pv=.37T. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

EXAMPLE 5.—If a receiver of 32 cu. ft. capacity is filled with air at 80 lbs. per sq. in. pressure and at 96° F., what weight of air does it contain?

For 1 lb. of this air the "specific" volume is, from (17),

$$v=\frac{.37\times 556}{80}=2.572 \text{ cu. ft.};$$

then the weight of air in the receiver is $32 \div 2.572 = 12.44$ lbs.

(j) THE GENERAL EQUATION $pv=CT$, having three variables, can be geometrically represented only by a figure of three dimensions. This is shown in Fig. 16: it is a curved surface whose sections by planes parallel to the pv-plane are equilateral hyperbolas; while planes parallel to either the vT- or the pT-plane cut the surface in straight lines. In the figure, ACOB (two straight lines) is the limiting isothermal curve at 0° AF., while $A_1C_1B_1$ and $A_2C_2B_2$ are the curves at 32° F. and 212° F. respectively. Then the figure $OO_1O_2D_2D_1$, on the vT-plane, is exactly similar to Fig. 13. Of course, a series of isothermals, all projected upon or drawn in the same plane, and each numbered with its degree, would be of more use than this "solid" figure, which is shown merely for illustrative purposes.

(k) THEORETICAL DERIVATION OF THE LAW FOR A PERFECT GAS.—From the theory as to the mechanical condition of the molecules of a perfect gas stated in (a), together with the experimentally established fact that the specific heat of such a gas is constant for all temperatures, the general law set forth in (i) can be logically derived.

The total heat-energy in one pound-weight of gas, which may

be thought of as put into the substance by heating it up from absolute zero, and in virtue of which the molecules have a kinetic energy proportional to the average velocity w, is

$$Q=cT=\frac{w^2}{2g}.$$

The pressure which the gas exerts upon the confining surface (or anywhere within its own substance) is due simply to molecular impact, which is supposed to take place under conditions of perfect elasticity. The intensity of this pressure is proportional to the product of the force of each impact by the number of impacts within a unit of area in a given time. Of course, these impacts are so numerous and individually so small in energy that their sensible effect is a truly "static" pressure. If now we let f be the force of the impact and n the impact-rate, then the proportionality just stated is expressed by

$$p \propto fn.$$

Think of the molecules as minute spherical bodies in rapid rectilinear motion: then the simplest cases of impact are when two molecules meet and rebound in the same straight line and when a molecule impinges squarely upon an element of the confining surface. At the instant when the moving mass has just been brought to rest, and before the reversed motion begins, the kinetic energy has all been absorbed in work of elastic compression. Of the two factors of this work, the force has varied uniformly from zero up to the maximum, so that the mean value is just one-half this maximum; and the compression distance is directly proportional to the maximum force; wherefore the product depends upon f^2: and since the kinetic energy to which this work is equal depends similarly upon w^2, we see that the force of the impact is proportional to the velocity of the moving body.

For a given number of molecules per unit of volume, moving with a certain velocity, the number of impacts per unit of area and of time is arrived at as follows:—The mean linear dimension of the space occupied varies as the cube-root of the volume, and the surface as the two-thirds power of the volume. The

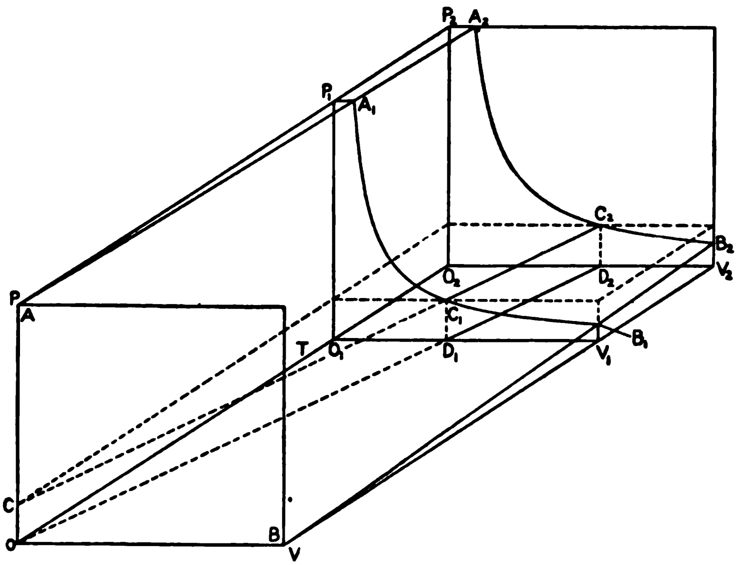

FIG. 16.—Geometrical Representation of the Law $pv=CT$.

rate of impact will vary directly as the velocity and inversely as the distance and as the surface; expressing these relations mathematically, we have

$$n \propto w, \quad n \propto \frac{1}{\sqrt[3]{v}}, \quad n \propto \frac{1}{\sqrt[3]{v^2}},$$

whence

$$n \propto \frac{w}{v}.$$

Also

$$f \propto w.$$

And since

$$p \propto fn \propto \frac{w^2}{v} \propto \frac{T}{v},$$

we reach the conclusion

$$pv \propto T, \quad \text{or} \quad pv=CT.$$

To be strictly correct, the kinetic energy of the molecules is proportional, not to the square of their mean velocity, but to the

mean of the squares of all the velocities. To prove that the absolute temperature is proportional to this mean square is a difficult feat of mathematical physics. But it is apparent that the impact-force and the kinetic energy of the molecule being similar functions of the individual velocity, the summation effects ought likewise to be similar.

§ 7. Simple Thermodynamic Operations with Gases.

(*a*) EFFECTS OF HEAT.—When heat is imparted to an expansive substance, with accompanying changes in pressure and volume, there are three ways in which the heat may be used or applied; these are:

First, in chan ing the temperature of the body: that is, the heat is directly stored in the body, in the form of increased thermal energy of the molecules, and without any change in the condition of the heat itself.

Second, in doing internal work in overcoming molecular forces or attractions. Since such work effects changes in the relative arrangements of the molecules, it is also called disgregation work. In perfect gases, where there are no molecular attractions, this work is zero. But where there is a change of state, as from liquid to vapor, disgregation work is of predominating magnitude. Heat used up in this manner changes from the state of active energy to that of passive, potential, or stored energy; ceasing to be sensible heat, but ready to reappear in that form whenever the conditions are suitable for its escape.

In both these methods of expenditure, the heat is to be thought of as doing internal work in or upon the body.

Third, in doing external work, whenever the volume changes against an external pressure equal to the internal stress or pressure in the substance,* by overcoming the resistance of the confining surface, of whatever form, through a certain distance. The heat thus used definitely ceases to exist as heat, being

* The full meaning of this proviso, and its practical bearing, will be brought out later.

transformed into mechanical work or energy: but is also capable of reconversion by a reversal of process.

We will now consider the thermal relations and quantities involved in the several simplest and most important operations with gases, using air as an example and giving the numerical values involved.

(b) EXPANSION AT CONSTANT PRESSURE.—When air is heated and expanded under constant pressure, the heat which must be

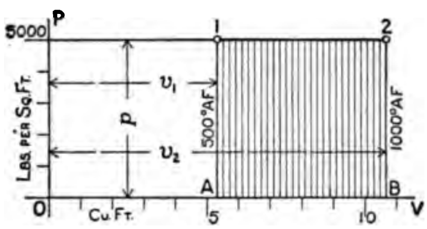

FIG. 17.—Expansion at Constant Pressure.

imparted in order to raise 1 lb. of air 1° is, as determined by experiment, .2375 H.U.; this specific heat at constant pressure we call c_p. Then for the process represented in Fig. 17 the total heat imparted is

$$Q=c_p(t_2-t_1).* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The amount of external work done is

$$U=144p(v_2-v_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and substituting from (17), we have

$$U=144\times .37(T_2-T_1)=53.3(T_2-T_1). \quad . \quad . \quad . \quad (20)$$

That is, for each degree that the temperature is raised, 53.3 or R ft.-lbs. of external work are done by the gas.

* This formula, as all those involving volumetric or thermic quantity which precede or follow, is for the unit weight of gas; for any weight G, (18) would take the form

$$Q=c_p(t_2-t_1)G.$$

The volume v of 1 lb. of gas under any conditions is called its specific volume.

Subtracting this external work from the total heat, we get the rate of heat-expenditure for internal work to be,

$$c_v = c_p - AR = .2375 - \frac{53.3}{778}$$

$$= .2375 - .0685 = .1690. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

(*c*) THE PRESSURE-VOLUME MEASURE OF WORK.—The method used in Eq. (19) for expressing work is based on the following considerations:

If we imagine the body of gas to be enclosed in a cylinder with an area of cross-section (or of piston) of 1 sq. ft., then every cubic foot of volume-change will cause the piston to move 1 ft.; and this movement will be against the pressure per square foot, $144p$ or P: evidently, then, $P \times (v_2 - v_1)$ meets literally the definition of work as force $\times$ distance. But it is also apparent that changing the size of the piston, which, for a given weight of gas, would vary the force upon the piston in inverse ratio to the distance moved by it, would not change the product Pv: so that, having used the simplest case to establish the idea, we see that the result would be the same for any dimensions, or for other methods of confining the gas even for a portion of free air, confined simply by the surrounding atmosphere. In this Pv-method of computing work, all measurements must be made in terms of the factors of the work-unit. Thus in the foot-pound system, pressures must be in lbs. and on the sq. ft., and volumes in cu. ft. Using kilogram-meters, pressure must be in kg. per sq. m., volume in cu. m. For lbs. per sq. ft. we shall regularly use the symbol P, putting Eq. (17) into the form, for work computation,

$$Pv = RT = 53.3T. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

EXAMPLE 1. Let 1 lb. of air at 100° F. and atmospheric pressure be heated to 200°; find values for total heat, external work, and internal work.

The total heat is .2375 × 100 = 23.75 B.T.U.

The initial volume is

$$v_1 = 12.39 \times \frac{560}{492} = 14.10 \text{ cu. ft.}$$

and the increase in volume is

$$v_1 \times \frac{T_2 - T_1}{T_1} = 14.10 \times \frac{100}{560} = 2.518 \text{ cu. ft.}$$

The pressure is $144 \times 14.7 = 2117$ lbs. per sq. ft.; then

$$U = P(v_2 - v_1) = 2117 \times 2.518 = 5331 \text{ F.P.}$$

and

$$AU = 5331 \div 778 = 6.85 \text{ H.U.}$$

Finally, $$I = Q - AU = 16.90 \text{ H.U.}$$

(*d*) THE SPECIFIC HEAT FOR INTERNAL WORK, c_v in (21), is also called the specific heat at constant volume; because in heating under this latter condition, all the heat imparted to the gas goes to doing internal work or raising the temperature. A most important fact about this internal work is that it depends only upon the initial and final temperatures, and not at all upon the character of the process through which the gas passes. Always, then, the change in internal energy, and the heat supplied to produce it, are

$$I = c_v(t_2 - t_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

(*e*) ISOTHERMAL EXPANSION.—Consider now what happens when a portion of gas expands isothermally in a cylinder, from v_1 to v_2,

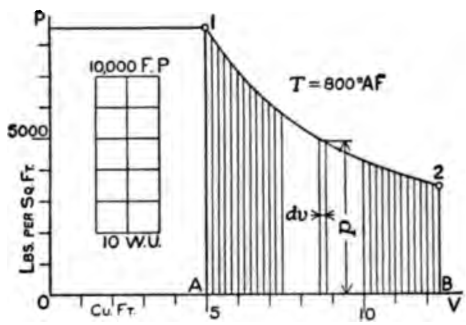

FIG. 18.—The Isothermal Curve.

at the temperature T. Since T_2 does not differ from T_1, the internal work is zero. The external work is represented by the area under the curve, between the limiting ordinates 1A and 2B

in Fig. 18. The element of work, for a change of volume dv, is evidently Pdv: and the total work is $\int Pdv$ between the limits v_2 and v_1. The equation of the curve being

$$Pv=P_1v_1,$$

we have

$$P=\frac{P_1v_1}{v},$$

whence

$$\int_{v_1}^{v_2} Pdv=P_1v_1\int_{v_1}^{v_2}\frac{dv}{v}=P_1v_1\log_e\frac{v_2}{v_1},$$

or

$$U=Pv\log_e r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where r is the ratio of expansion, always a number greater than unity (therefore the inverse ratio of compression), and equal to either $v_2\div v_1$ or $p_1\div p_2$. Then substituting from (22), we have

$$U=RT\log_e r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Now all the heat that need be supplied is that required for doing this external work; for

$$\begin{aligned} Q&=I+AU \\ &=0+ART\log_e r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26) \end{aligned}$$

In the reverse operation of compression, work is done upon the gas, and heat must be abstracted in order to keep the temperature from rising.

A table of natural or hyperbolic logarithms, for use in this calculation, will be found in Table I., col. 3. The relation to common logs is

$$\log_e r=2.3026\log r. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

EXAMPLE 2.—A cylinder 2 ft. in diameter and with the piston 1 ft. from the cylinder-head is filled with air at 80° F. and 3 atmos. pressure: what work will be done in expansion to three times the initial volume?

Here $P_1=3\times 144\times 14.7=6350$ lbs. per sq. ft.;

$v_1=3.1416$ cu. ft.;

$r=3$; $\log_e r=1.0986$.

Then $U = 6350 \times 3.142 \times 1.0986$
$= 21{,}916$ F.P.

And $21{,}916 \div 778 = 27.2$ H.U. must be supplied to do this work and keep the temperature constant.

(*f*) Graphical Representation of Work.—In Fig. 18 we have an example of the Pv method of representing work, applied to the case of variable pressure. For the actual curves given by heat-engines, and drawn by the indicator, mechanical integration, by means of the planimeter, is generally used. Having found the mean pressure acting through any change of volume, we use the general method

$$U = P_m(v_2 - v_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Or, we may work out a scale of relation of work to area, like that shown in Fig. 18, and multiply the area-measurements by this scale.

(*g*) A General Law of Expansion, which has some important applications, is expressed by the equation

$$pv^n = C, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where the index n takes particular values for different cases.

For this curve,

$$\begin{aligned} U &= \int_{v_1}^{v_2} P dv = P_1 v_1^n \int_{v_1}^{v_2} \frac{dv}{v^n} \\ &= \frac{1}{1-n} P_1 v_1^n [v_2^{1-n} - v_1^{1-n}] \\ &= \frac{1}{n-1} [P_1 v_1^n v_1^{1-n} - P_2 v_2^n v_2^{1-n}] \\ &= \frac{P_1 v_1 - P_2 v_2}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30) \end{aligned}$$

Putting this in terms of T, by (22), we have

$$\left. \begin{aligned} U &= \frac{R(T_1 - T_2)}{n-1}, \\ \text{or} \qquad AU &= \frac{AR(T_1 - T_2)}{n-1}. \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Now from (21), $AR=(c_p-c_v)$, whence,

$$AU=\frac{(c_p-c_v)(T_1-T_2)}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad (32)$$

As always, $I=c_v(T_2-T_1)$. Then

$$\begin{aligned} Q&=AU+I \\ &=\frac{(c_p-c_v)(T_1-T_2)}{n-1}-c_v(T_1-T_2) \\ &=\frac{(c_p-nc_v)(T_1-T_2)}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33) \end{aligned}$$

(*h*) ADIABATIC EXPANSION.—The most important particular case under the above law is that in which no heat is given to or taken from the gas as it expands or is compressed. Then $Q=0$, and from (33) we have

$$(c_p-nc_v)(T_1-T_2)=0.$$

Now T_1 and T_2 cannot be the same, therefore we must have

$$c_p-nc_v=0,$$

or

$$n=\frac{c_p}{c_v}=k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The equation of this "adiabatic" (no transfer) curve is then,

$$pv^k=C; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and the value of k, see (21),

$$k=\frac{.2375}{.1690}=1.406. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

(*i*) THE ADIABATIC CURVE is drawn in Fig. 19 for a considerable range of expansion and of compression, with the isothermal dotted in, through a common point A, for comparison. In expansion without heat-supply, the gas does work at the expense of its internal energy and loses temperature, and the curve drops below

the isothermal: conversely, in compression without heat-rejection, the external work done upon the gas adds itself to the internal energy, and the temperature rises. Comparing the two curves of expansion, we have that for a given volume, as OT, the point N on the adiabatic represents a smaller pressure in the gas than the point R on the isothermal; and that after expansion to a certain pressure, as OS, the gas under isothermal condition fills a larger volume than does that expanded adiabatically.

(*j*) Drawing the Exponential Curve.—The task of drawing an adiabatic or any other curve of the form $pv^n = C$, if it involved the calculation of a series of co-ordinates directly from the equation, would be very laborious. To facilitate this operation, a

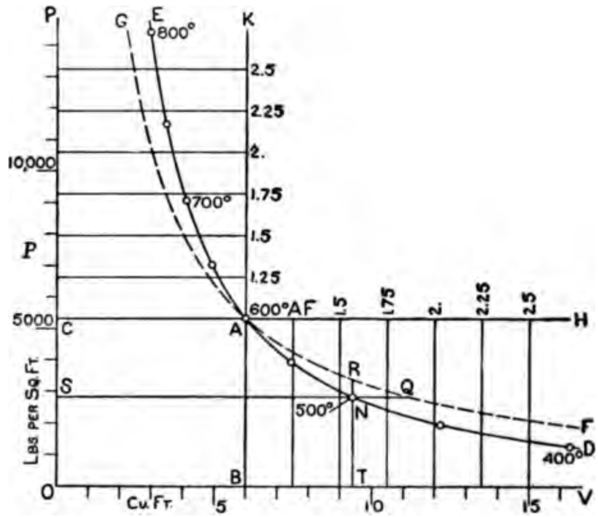

Fig. 19.—The Adiabatic Curve.

series of ratios is given in Table II., by means of which the ordinates can be found by simple multiplication. In expansion, we take v as the independent variable, locating a series of ordinates at intervals of one-fourth of the original volume v_1, or of CA in Fig. 19; then by multiplying the initial ordinate AB or p_1 by the factors in the table, we get the lengths of the successive ordinates. Thus in Fig. 19, as drawn, AB was 2.665″, representing 5330 lbs.

Now from (21) ... the base-line on

2.25 2.5

As always, ... .3198 .2757

... .852 .734 ins.

... dependent variable, and

... —To find the relation ... adiabatic expansion

... . (37)

... . (38)

... . (39)

... operation, the ... energy, the for- ... knowing the

... . (40)

After the meaning of the exponent k has been established as in (34), we use c and kc instead of c_v and c_p. Note the reversal of algebraic sign in the expression for I: the internal work is of course a negative quantity in this case.

Example 3.—A cylinder 2 ft. in diameter has the piston 1 ft. from the head, and is filled with air at 3 atmos. pressure and at 80° F.: if the air is expanded adiabatically to 3 times the initial volume, what will be the final pressure, the final temperature, and the external work done?

First, $p_1 = 44.1$ lbs. per sq. in.

$$p_1 = p_1 \left(\frac{v_1}{v_2}\right)^k$$

$$= 44.1 \times \left(\frac{1}{3}\right)^{1.406}$$

$$= 44.1 \times .2133 = 9.410$$

$$\log \tfrac{1}{3} = -1 \quad +.52288$$

$$1.406 \log \tfrac{1}{3} = -1.406 + .73517$$

$$= \bar{1}.32917$$

$$\log 44.1 = 1.64444$$

$$\log p_2 = 0.97361$$

as compared with 14.7 after isothermal expansion through the same ratio.

It is rather simpler, as a numerical operation, to put the expression for p_2 into the form

$$p_2 = p_1 \div r^k,$$

for we thereby get rid of the negative characteristic of $\log \left(\frac{v_1}{v_2}\right)$: then

$$\log 3 = 0.47712$$

$$1.406 \log 3 = 0.67083$$

$$\log 44.1 = 1.64444$$

$$-\,0.67083$$

$$\log p_2 = 0.97361$$

Next,

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$= T_1 \div r^{k-1}$$

$$= 540 \div 3^{.406}$$

$$= 540 \div 1.5621 = 345.7° \text{ AF.} = -114.3° \text{ F.}$$

$$\log 3 = 0.47712$$

$$.406 \log 3 = 0.19371$$

$$\log 540 = 2.73239$$

$$-\,0.19371$$

$$\log T_2 = 2.53868$$

To find the external work by (30), we have

$$U = 144\,\frac{44.1 \times 3.142 - 9.41 \times 9.425}{.406}$$

$$= 144\,\frac{138.55 - 88.69}{.406}$$

$$= \frac{144 \times 49.86}{.406} = 17{,}683 \text{ F.P.}$$

per sq. ft.; and the distances measured up from the successive ordinates were as follows:

$\frac{v}{v_1}=$	1.25	1.5	1.75	2.	2.2
$\frac{p}{p_1}=$	.7307	.5655	.4553	.3774	.31
$p=$	1.947	1.507	1.214	1.006	.8

For compression, p is taken as the independ values from column 10 of the table are used.

(*k*) Adiabatic Temperature-Range.—To fi between the limiting temperatures in an adi from p_1v_1 to p_2v_2, we proceed as follows:

$$p_1v_1^k=p_2v_2^k; \quad . \quad . \quad . \quad .$$

factor this into the form

$$p_1v_1\times v_1^{k-1}=p_2v_2\times v_2^{k-1},$$

then substitute from (22) and get

$$RT_1v_1^{k-1}=RT_2v_2^{k-1},$$

or

$$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{k-1}=\left(\frac{v_1}{v_2}\right)^{.406} \quad . \quad .$$

Again, from (37),

$$\frac{v_1}{v_2}=\left(\frac{p_2}{p_1}\right)^{\frac{1}{k}},$$

and in terms of pressures, (38) changes to

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}=\left(\frac{p_2}{p_1}\right.$$

(*l*) External Work.—Since, in t external work is equal to the change mula for the latter gives t limiting temperatu

original condition. This continuity of process, or closure of cycle, is essential if the working of the engine is to be continuous.

(*c*) In general, the major divisions of the cycle are, an expansion during which heat is received from some source at high temperature and external work is done; and a compression during which external work is done upon, or received by, the working-medium and heat is rejected to an outside body at low temperature. Each of these main divisions usually contains two distinct processes, and may contain more than two.

(*d*) Using the terminology of the ordinary cylinder and piston engine, the cycle consists of an out- or working-stroke and a return- or compression-stroke. Obviously, it is desirable that, for a given amount of heat supplied, the effective difference between the positive work of expansion and the negative work of compression shall be as large as possible; for the ratio of this effective work to the heat received measures the efficiency of the heat-engine. Since the pressure of a given portion of gas, confined in a certain space, is proportional to the absolute temperature, and since the same range of volumes is passed through in both strokes, the evident requirement for high efficiency is, that the expansion shall take place at the highest possible, and the compression at the lowest possible, temperature.

(*e*) If a maintained uniform high temperature of the source of heat is the upper limit, and if a similar constant temperature of the cold absorbing body or "source of cold" is the lower limit, then it appears that isothermal operations at these respective temperatures best meet the requirement just stated. But besides expansion at a constant high temperature and compression at a constant low temperature, there must be a drop from the high to the low temperature in one part of the cycle, and a return from the low to the high in another. For these operations, the adiabatic process, involving no transfer of heat to or from the medium, naturally suggests itself.

(*f*) This cycle—commonly called the Carnot cycle—is represented in Fig. 20. The engine uses a confined body of air, which is alternately heated and cooled in the cylinder. All the surfaces in contact with the air must be thermally neutral, that is, must

have no capacity for heat; and all except the cylinder-head must be perfectly non-conducting; the latter is a perfect conductor, but is provided with a cover of the neutral material. These conditions are imposed in order that adiabatic operations may be carried

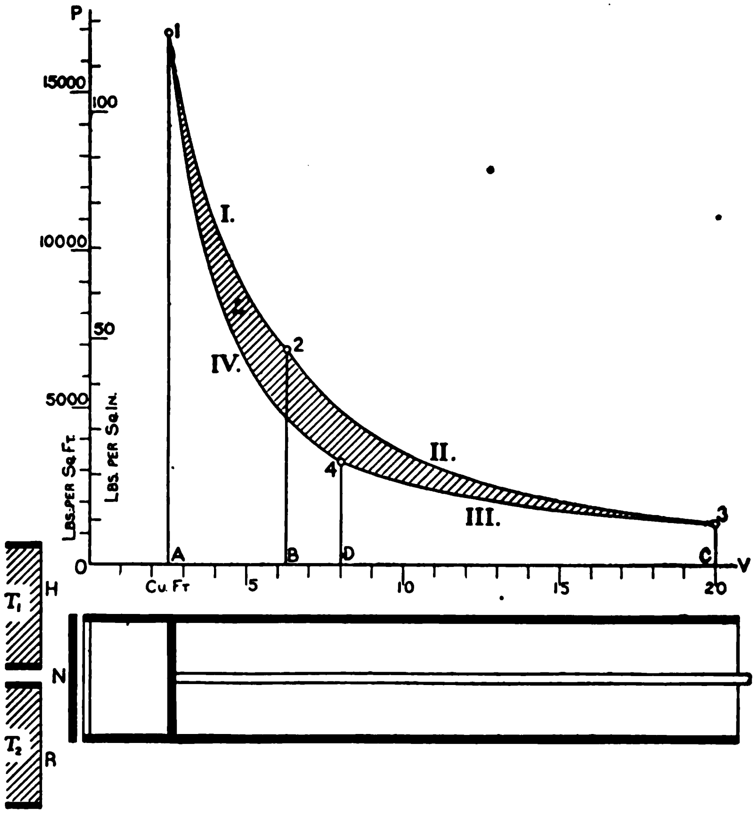

FIG. 20.—The Ideal Heat-engine.

out. There is also a source of heat H, or heat-reservoir at high temperature; and a heat-receiver R, at low temperature: either of these can be brought into contact with the cylinder-head, for the isothermal operations; and it is understood that the heat can pass instantaneously to and from the air when a way is opened. Mechanically, we assume that provision is made for exerting a

force along the piston-rod always equal and opposite to the pressure of the air upon the piston, and for regulating the movement of the piston.

(g) The description of the cycle shown in Fig. 20, with expressions for all the quantities involved, is as follows:

Start with a unit weight of air at p_1, v_1, and T_1.

Phase I. of cycle: Isothermal expansion, at T_1, from 1 to 2:

$$p_1v_1 = p_2v_2;$$

$$Q_I = AU_I = ART_1 \log_e \frac{v_2}{v_1}.$$

Phase II. Adiabatic expansion, along 23, from T_1 to T_2:

$$p_2v_2^k = p_3v_3^k; \quad \text{and from (38)}$$

$$\frac{v_3}{v_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}}; \qquad (41)$$

$$Q_{II} = 0;$$

$$U_{II} = \frac{R}{k-1}(T_1 - T_2) \quad \text{(see 31).}$$

Phase III. Isothermal compression, at T_2, from 3 to 4:

$$p_3v_3 = p_4v_4;$$

$$Q_{III} = AU_{III} = RT_2 \log_e \frac{v_3}{v_4}.$$

Phase IV. Adiabatic compression, along 41, from T_2 to T_1:

$$p_4v_4^k = p_1v_1^k;$$

$$\frac{v_4}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}}; \qquad (42)$$

$$Q_{IV} = 0;$$

$$U_{IV} = \frac{R}{k-1}(T_1 - T_2).$$

From (41) and (42),

$$\frac{v_3}{v_2} = \frac{v_4}{v_1},$$

whence

$$\frac{v_3}{v_4} = \frac{v_2}{v_1} = r. \qquad (43)$$

have ... by the same adiabatics be perfe... is pro... are equal, and balance tions ...

$$T_1 - T_2);$$

... received, or the efficiency, is

$$- T_2 \qquad (44)$$

... amount of work that can be

$$(45)$$

... HEAT.—Every heat-engine ... part of it into work. ... The thermodynamic ... its intensity or tem- ... heat from a higher ... is possible. The ... the most efficient ... the upper limit, and ... If heat is available ... at a lower tem- ... potentiality; ... temperature above ... heat ...

... stated ... reversible ... that its heat- ... its heat-rejection ... imaginable re- ... takes ... the case ... pressure by heat

received from a source at T: it would no more be possible to force the heat back into the source by the reverse operation of compression at constant pressure, than it would be to make water run up-hill. That the adiabatic process is reversible is self-evident.

(j) THE ARGUMENT FROM REVERSIBILITY, probably containing more of logical deduction than any other statement of what is very nearly a fundamental idea, is as follows:

Suppose that we reverse the working of the apparatus illustrated in Fig. 20, following out the cycle in the order 14321: now a certain amount of heat R_1 is drawn from the receiver during the isothermal expansion 43, the effective work of the cycle is added to it, and the amount of heat H_1 is rejected into the heat-source. To drive this reversed heat-engine, or heat-pump, we employ another heat-engine, connected to the same source and receiver, drawing from the former the heat H_2 and rejecting to the latter the heat R_2—and which is, if possible, to have a higher thermal efficiency. Assuming that there are no mechanical losses, the work done by No. 2 must be the same as that received by No. 1, or

$$H_2-R_2=H_1-R_1.$$

In No. 1, the quantities H_1 and R_1 bear the same relation to each other and to the work of the cycle, whether the engine is operated in direct or in reversed order; then the efficiency of No. 1, as an engine, is

$$E_1=\frac{H_1-R_1}{H_1}.$$

And the efficiency of No. 2 is, of course,

$$E_2=\frac{H_2-R_2}{H_2}.$$

Now if it were possible for No. 2 to be more efficient than No. 1, we should have

$$\frac{H_2-R_2}{H_2}>\frac{H_1-R_1}{H_1}.$$

Since the numerators are equal in these expressions, the inequality could be produced only by making H_1 greater than H_2; this is manifestly impossible, for it would involve putting more heat

into the source than was taken from it, or a gradual transfer of heat from a cold body to a hot body, by a self-contained process.

(*k*) THE HYDRAULIC ANALOGY.—There is a close analogy between the apparatus just described and an arrangement consisting of a water-motor set to driving a pump, where both work between the same levels. The best that any water-motor can do is to transform into useful work all the energy lost by the water in sinking from the upper level to the lower; reversed to a pump, this perfect motor would require for driving it only an amount of work just equal to that of lifting the water from the lower level to the higher. No more efficient motor can be imagined.

The hydraulic analogy also applies to the efficiency deduced in Eq. (44): the total potential energy of position of a portion of water is that due to its total height h_1 above sea-level; if it is discharged from the motor at a level h_2, the absolute efficiency of the operation is

$$E=\frac{h_1-h_2}{h_1}.$$

It will not do, however, to carry this analogy into the details of the respective hydraulic and thermodynamic processes.

EXAMPLE 1.—The particular numerical values used in laying out Fig. 20 are as follows:

As primary data, (using 1 lb. of air),

$$T_1=800°,\quad T_2=500°;\qquad v_1=2.5,\quad v_2=20 \text{ cu. ft.}$$

Then for Phase I., $\quad pv=.37T_1=296,$

and for Phase III., $\quad pv=.37T_2=185.$

Then $p_1=296\div2.5=118.4$ lbs. per sq. in.

and

$$p_2=185\div20=9.25 \text{ lbs. per sq. in.}$$

The adiabatic ratio is

$$r_a=\left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}}=1.6^{\frac{1}{.406}}=3.182 \qquad \log 1.6=0.20412 \qquad \frac{\log 1.6}{.406}=0.50276$$

Now

$$v_2=20\div3.182=6.284, \qquad p_3=296\div6.284=47.10,$$

and

$$v_4=2.5\times3.182=7.95, \qquad p_4=185\div7.95=23.27.$$

The two isothermal ratios are

$$\frac{v_2}{v_1}=\frac{6.284}{2.5}=2.514 \quad \text{and} \quad \frac{v_3}{v_4}=\frac{20}{7.95}=2.516.$$

Using $r=2.515$ we get

$$U_1=144\,p_1v_1\log_e r=144\times 295\times .9223$$
$$=34{,}179 \text{ F.P.} \quad \text{or} \quad 50.36 \text{ H.U.}$$

Since the efficiency is .375, the useful work is

$$39{,}179\times .375=14{,}692 \text{ F.P.} \quad \text{or} \quad 18.88 \text{ H.U.}$$

The piston-displacement in this case, from v_1 to v_3, is 17.5 cu. ft.; and the work done per cu. ft. of this displacement is only $14{,}692\div 17.5=840$ F.P. Now by the method of Eq. (28) we find that the mean effective pressure in this operation—the difference between the mean forward-pressure on the out-stroke and the mean back-pressure on the return-stroke—is 840 lbs. per sq. ft. or 5.83 lbs. per sq. in. A noticeable fact in regard to this cycle is the small effective mechanical intensity of the operation; this means that the engine must have a large bulk for a given output of work.

EXAMPLE 2.—A heat-engine working between the limits $T_1=800°$ AF. and $T_2=500$ receives 8750 H.U. per horse-power-hour: what is its absolute efficiency, and what its relative efficiency, compared with the ideal engine?

The heat-value of 1 H.P.H.—see § 5 (f)—is 2545 H.U.; then the absolute efficiency, or ratio of work done to heat received, is

$$E=\frac{2545}{8750}=.2909.$$

The ideal efficiency is $E_0=.375$; then the relative efficiency is

$$E\div E_0=.2909\div .375=.773.$$

into the source than was taken [illegible] heat from a cold body to a hot b[illegible]

§ THE HYDRAULIC ANALOGY [illegible]

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[illegible] is

[illegible]

[illegible] $= 3.18$.

[illegible]

and

[illegible] $= 2.5$

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[illegible]erties of Steam.

[illegible] the operation of [illegible] water, the first [illegible]oiling-point. This [illegible] pressure at which [illegible] ordinary case, the [illegible] the boiler. As the [illegible] heat there is no [illegible] steam-formation is [illegible] 14.7 lbs. per sq. in.: [illegible] complex rela-[illegible]

[illegible] heat of water [illegible] greater than [illegible] water through [illegible] the number [illegible] steam, the first [illegible] water from 32° [illegible] is called the [illegible] the actual [illegible] than 32°, the [illegible] water at 32°; [illegible] the initial tempera-[illegible]

(6)

[illegible] steam-[illegible] addi-[illegible]

heat will not raise the temperature, but will change the into steam. Of course, the pressure can be kept constant by giving room for the continual expansion from water into steam. In the steam-plant, the moving piston of the engine, continually receiving steam behind it, permits this necessary expansion. Since the temperature remains constant, evaporation at constant pressure is an isothermal operation.

(*d*) Heat of Vaporization and Total Heat.—The heat used in producing this change of state from liquid to vapor does chiefly disgregation work; and a very large amount of heat is absorbed during the operation. Since this heat of vaporization disappears as sensible heat, apparently becoming concealed in the substance, it is commonly called latent heat. The symbol for it is r.

Now the total heat of formation of 1 lb. of steam, above water at 32°, is

$$H=q+r. \qquad (47)$$

But if the feed-temperature is t_0, then the heat required is

$$Q=q-q_0+r. \qquad (48)$$

(*e*) Heat for External Work.—The heat of vaporization is really expended in two ways: the larger part, as just stated, does disgregation work, in overcoming internal molecular forces; a smaller part does external work, in overcoming the confining pressure through the increase of volume from water to steam. If w be the volume of one pound of water, and s the volume of the pound of steam—these are both "specific volumes"—then the increase of volume is

$$u=s-w. \qquad (49)$$

From this we get the external work to be

$$U=144\,pu=Pu. \qquad (50)$$

A fair average value for the weight of 1 cu. ft. of water at the temperature of steam-formation is 56 lbs.; this makes $w=.0175$ cu. ft.—a quantity so small relative to s that it can be neglected except in exact calculations.

(*f*) INNER LATENT HEAT.—Having calculated from APu the heat-value of the external work of formation, we get the inner latent heat l by subtraction from the total latent heat r, so that

$$l = r - APu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

(*g*) PARTIAL EVAPORATION.—The limits of the evaporation process under the conditions p and t are, at the beginning, 1 lb. of water, of the volume w, containing the heat q; at the end, 1 lb. of steam, of the volume s, containing the heat $q+r$. At any intermediate point in the operation, where the fraction x of the pound has changed to steam, while the remainder $(1-x)$ is yet water, the volume of the mixture is

$$v = w + xu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

or, very nearly, simply

$$v = xs. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

At the same time the heat-content is

$$Q = q + xr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

For the general case of an initial temperature t_0 and incomplete evaporation, to the degree x, the formula for the heat of steam-formation is

$$Q = q - q_0 + xr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Frequently, however, it is more convenient to make the computation by the method

$$Q = H - q_0 - (1-x)r, \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

subtracting from the total heat H, first the heat already in the water at the beginning of the process, second the heat not put into the unevaporated water at the end.

§ 10. Steam Constants and Relations.

(*a*) DETERMINATION OF THE STEAM-QUANTITIES.—The numerical values of the various steam-quantities have been found by a great number of careful experiments—notably those of Regnault —extending over a wide range of conditions. The laws connecting

them do not have the logical simplicity of those for perfect gases, but are almost entirely empirical: the results of the experiments were very carefully plotted, and equations found for the curves passing through these points. The relation between p and t and

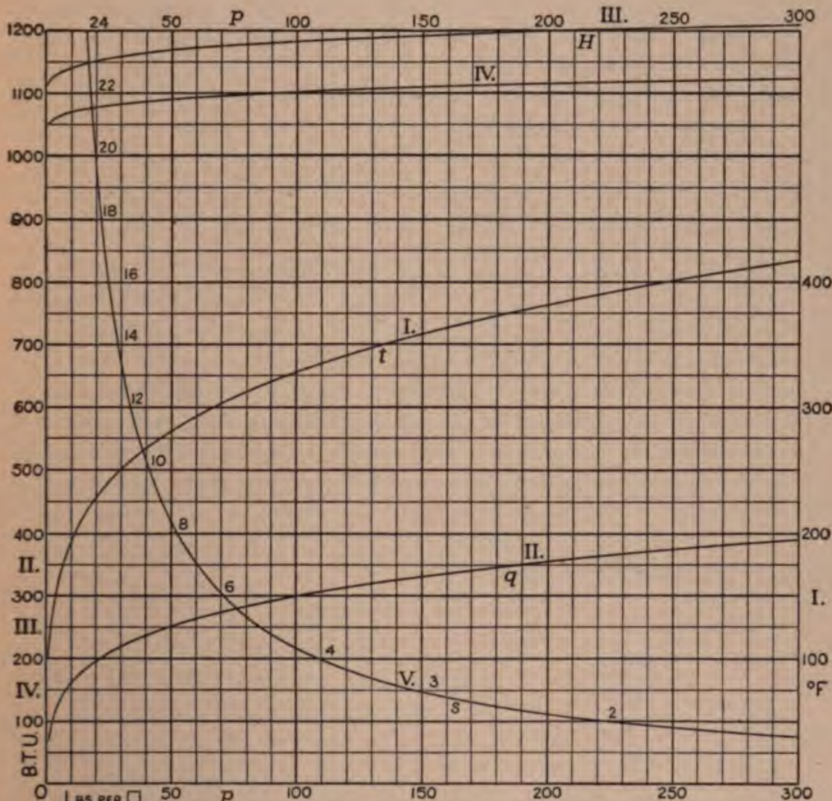


FIG. 21.—Steam-curves on p.

the values of q and H or r are fundamental determinations, which could be made only by experiment: the volume s can be calculated by a method whose deduction belongs to advanced Thermodynamics, besides being found by experiment: and APu is, of course, a derived quantity, as is l.

(b) STEAM AT ATMOSPHERIC PRESSURE.—The particular values for steam at 212° and 14.7 lbs. per sq. in. are

$$q = 180.9 \text{ H.U., which is 0.9 greater than } (t-32);$$
$$r = 965.7 \text{ H.U.};$$
$$H = 1146.6 \text{ H.U.}; \quad \text{and} \quad s = 26.56 \text{ cu. ft.}$$

These three heat-values should be memorized, because the heat-formulas which we use when steam tables are not available are so shaped as to give the departure from these numbers for a given departure of t from 212°.

(c) GRAPHICAL STEAM-TABLE.—The quantities enumerated in (a), taken from the Steam-table (Table IV.), are laid out on a pressure-base in Fig. 21. The curves are as follows:

Curve I. shows t, to the scale at the right.

Curve II. gives q and III. gives H, both to the scale at the left; then r is included between II. and III.

Curve IV. is got by measuring the external work APu downward from III.; so that the inner latent heat l is between II. and IV.

Curve V. gives the volume of 1 lb. of steam, to the scale marked along the curve.

This graphical steam-table is on too small a scale to be used for getting numerical values; it is intended to show how the several quantities vary, and especially to illustrate the relative magnitude of the different heat-quantities.

(d) RELATION BETWEEN PRESSURE AND TEMPERATURE.—It will be noted that t changes very rapidly with low values of p, but at a slow rate over the range of usual boiler-pressures. The equation of curve I., found by trial, is of complex form, and is of no practical use for ordinary determinations; so that reference to the Steam-table is the only practical way to get t from p or the reverse. It will be well to remember the following few values, so as to have some idea of the temperature corresponding to any pressure:

p, by gage =	50	100	150	200
t = ...	298	338	366	388

In all our work up to this point, we have taken p to be absolute, or the total pressure above zero. Actual pressure-gages

measure, however, not above zero, but above the atmosphere: and 14.7 must be added to their indications to get absolute pressure.

(*e*) VARIATION OF PRESSURE WITH TEMPERATURE.—In Fig. 22, curve I. shows the converse of Fig. 21 I., or how p varies on t:

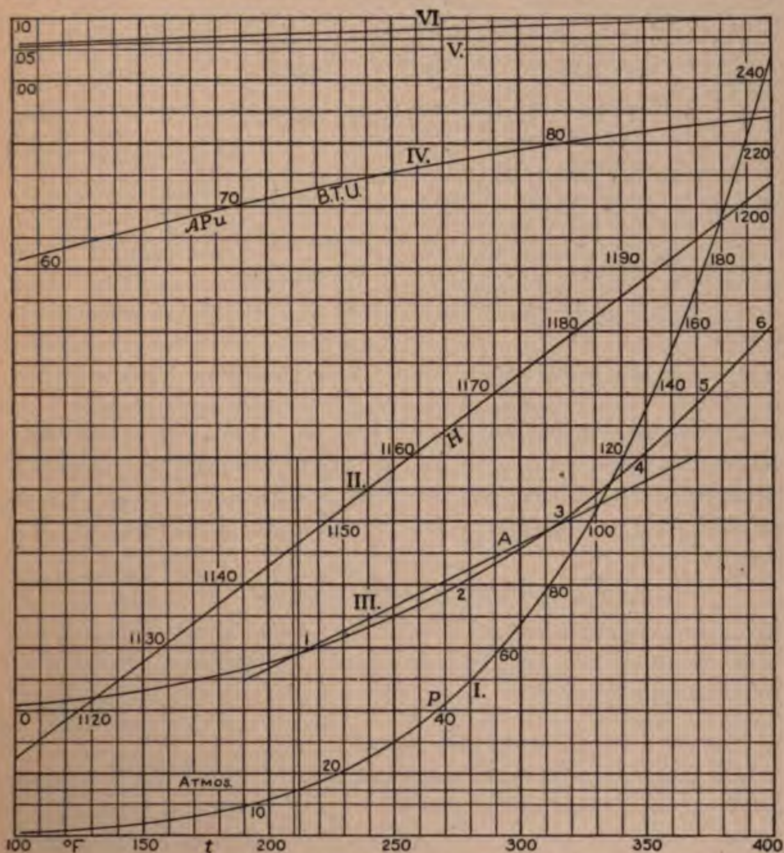


FIG. 22.—Steam-values on t.

note how rapidly the pressure rises with the temperature in the higher part of the scale. The line of atmospheric pressure is drawn in, and we see clearly how the left-hand part of the curve extends into the vacuum range: this illustrates the principle on which the working of the vacuum-condenser depends.

(*f*) FORMULA FOR TOTAL HEAT.—The simplest and most useful steam-heat formula is that for the total heat,

$$H = 1146.6 + .305(t - 212). \quad . \quad . \quad . \quad . \quad . \quad (57)$$

This is an exact formula, applying to all ranges of temperature: for low ranges it may be more convenient in the form

$$H = 1081.9 + .305(t - 32).. \quad . \quad . \quad . \quad . \quad . \quad (57')$$

In Fig. 22, the straight line II. shows how H varies: by the expedient of plotting only the upper end of H, we get a diagram that is to a large enough scale to be useful for finding numerical values, approximately, even on this reduced figure.

(*g*) HEAT OF THE LIQUID.—The exact formula for the water-heat is,

$$q = (t - 32) + .000{,}011(t - 32)^2 + .000{,}000{,}093(t - 32)^3. \quad (58)$$

In Fig. 22, curve III. is got by plotting, to the large scale indicated, the excess of q over $(t - 32)$: this curve for $[q - (t - 32)]$, when we know t, can be used for getting the exact value of q quite as effectively as the Steam-table. An approximate formula for q is,

$$q = 180.9 + 1.02(t - 212); \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

its effect is shown by the straight line A, Fig. 22 III., and it appears to be exact enough for all practical purposes over the range from 10 lbs. to 105 lbs. absolute, or from 190° to 330°; the error, within these limits, being less than .2 H.U.

(*h*) APPROXIMATE FORMULA FOR LATENT HEAT.—Now we have $r = H - q$; or, subtracting (59) from (57), the approximate formula

$$r = 965.7 - .715(t - 212); \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

which is good for the same range as (59). When we note that the total heat increases by only .305 H.U. per degree, while the heat of the liquid increases by a little more than 1 H.U., we see why the latent heat must decrease as the temperature of formation rises.

(*i*) THE RELATION BETWEEN THE PRESSURE AND THE VOLUME of one pound of steam, represented in curve V. of Fig. 21, is given very closely by the empirical equation,

$$ps^{1.065} = C. \qquad (61)$$

For p in lbs. per sq. in. and s in cu. ft., the value of C is 483.

(*j*) VALUE OF THE EXTERNAL WORK.—The curves at the top of Fig. 22 are all concerned with the external work of steam-formation, AU or APu. Curve IV. shows the value of AU in H.U. per lb. of steam, to the scale marked. Curves V. and VI. show two ratios, the first the ratio of AU to the total heat H, which begins at .055 and rises to .07; the second the ratio of AU to the latent heat r, ranging from .06 to .10. These are given in order to emphasize the smallness of the fraction of heat that goes to do external work during evaporation; and will be referred to when the performance of the engine is taken up.

EXAMPLE 1.—If steam is made at a gage-pressure of 85.8 lbs. per sq. in., from feed-water at 110° F., and contains 1.7 per cent. of moisture, how much heat is utilized from the fuel in the making of each pound?

The absolute pressure is 100.5 lbs., and the corresponding temperature 328.0°; at 110°, using Fig. 22 III.,

$$\begin{aligned}
q_0 &= 110 - 32 + 0.1 = 78.1; \text{ similarly} \\
q &= 328.0 - 32 + 3.3 = 299.3 \text{ (compare with Table)}; \\
H &= 1146.6 + .305 \times 116 = 1146.6 + 35.4 = 1182.0; \\
r &= 1182 - 299.3 = 882.7; \\
x &= .983; \qquad (1 - x) = .017.
\end{aligned}$$

Now

$$\begin{aligned}
Q &= q - q_0 + xr \qquad (55) \\
&= 299.3 - 78.1 + .983 \times 882.7 \\
&= 221.2 + 867.5 = 1088.7; \qquad \text{(A)}
\end{aligned}$$

or

$$\begin{aligned}
Q &= H - q_0 - (1 - x)r \qquad (56) \\
&= 1182.0 - 78.1 - .017 \times 882.7 \\
&= 1103.9 - 15.2 = 1088.7. \qquad \text{(B)}
\end{aligned}$$

For this last method we should need to get only H and q_0, and find r from (60):

$$\begin{aligned}
r &= 965.7 - .715 \times 116 \\
&= 965.7 - 82.9 = 882.8.
\end{aligned}$$

In (B), the first item, 1103.9, is the heat in 1 lb. of dry steam at p and from t_0; which is frequently desired for comparison with the final valu of Q.

EXAMPLE 2.—Steam is made at 150 lbs. abs., from feed-water a 210°, completely evaporated. How much heat is spent in raising th water, in inner work, and in outer work?

First, $q_0 = 210 - 32 + .9 = 178.9;$

$t = 358.2;$

$q = 358.2 - 32 + 4.4 = 330.6.$

Then $q - q_0 = 151.7.$

Second, $H = 1146.6 + .305 \times 146.2$

$= 1146.6 + 44.6 = 1191.2;$

$r = 1191.2 - 330.6 = 860.6.$

Now suppose that we are required to find the outer work withou using the table at all: from (61) we have

$150\ s^{1.065} = 483$	$\log 483 = 2.68395$
$s = 2.998$	$\log 150 = 2.17609$
(In table, 3.001)	$1.065)0.50786$
	$\log s = 0.47687$

Now $U = 144pu = 144 \times 150 \times (2.998 - .018)$

$= 21{,}600 \times 2.980 = 64{,}368$ F.P.,

or

$AU = 82.74$ H.U.

Then for the inner work we have,

$$l = r - APu$$
$$= 860.6 - 82.7 = 777.9 \text{ H.U.}$$

The discrepancy between the value of AU found here and that in th Table is due partly to the difference in s, partly to the fact that in th Table w is taken .016 all through.

In both these examples, the least possible use is made of th Steam-table, in order to give practice with the formulas. In ger eral, however, the Table is to be used wherever it can be. Fc most purposes, especially for working up the results of test: values carried to one decimal place are sufficiently accurate.

EXAMPLE 3.—A cylinder 2 ft. in diameter by 3 ft. long is filled wit steam at 87.5 lbs. abs.: what is the weight of this steam?

The volume of the cylinder is

$$3.1416 \times 3 = 9.425 \text{ cu. ft.}$$

The weight per cu. ft. of the steam, from the Table, is .2025 lbs. The total weight is

$$9.425 \times .2010 = 1.894 \text{ lbs.}$$

Or, the volume per pound is 4.975 cu. ft., and

$$9.425 \div 4.975 = 1.894 \text{ lbs.}$$

§ 11. The Different States of Steam.

(*a*) The steam or water-vapor which we have so far been considering is what is known as saturated steam. This may be defined as steam in the state in which it is formed from water, or steam as it exists in the presence of water. If we separate the steam from water, and then supply more heat to it, it will become superheated and will approach a perfect gas in behavior.

(*b*) Graphical Representation of Saturated and Superheated Conditions.—The distinction between these two states can best be illustrated by the method of Fig. 23. For a particular pressure p or OM, we lay off to scale the volume w=MW of one pound of water, and the volume s=MS of the same water when just all turned into steam. Doing this for a series of different pressures, we get two sets of points, through which the curves W_1W, S_1S can be drawn: and these curves are the boundaries of the saturated state of steam. The W-locus separates steam from water, the S-locus divides saturated steam from superheated. Any point in the intermediate space shows the relation between the pressure and volume of a unit weight of a mixture of steam and water in a certain proportion. The figure is drawn true to scale, except that the water-volume MW is laid out ten times its proper size, so that it may be visible.

(*c*) Curve of Constant Steam-weight.—The curve S_1S_2, or any similar curve V_1V_2, drawn for a particular constant steam-fraction x (in the figure for 0.5), is called a curve of constant steam-weight. Besides being a mere curve of relation, as laid out, it also represents what would take place if a body of steam and

In (B), the first item, 1103.9, is ... from t_0; which is frequently desired ... of Q.

EXAMPLE 2.—Steam is made at ... 210°, completely evaporated. ... water, in inner work, and in ...

First, q_0 ...

t ...

q ...

Then $q - q$ ...

Second, H ...

Now suppose that we ... using the table at all: from ...

150 ... 483

$s = 2.998$

(In table, 3.001)

Now ...

or ...

Then for the ...

The discrepancy ... Table is due partly ... Table ... is taken ...

In both the ... Steam-table ... eral ... most ... values carried ...

EXAMPLE 3.— ... steam at 87.5 ...

... pressure and temperature ... and the heat con... ... and no steam ... quality-fraction x

... curve.

... constant-weight ... as well as saturated. ... reason for this is best ... the superheated side. ... consider the peculiarities

... after the water has all ... heat and keep this ... and the steam expand ... definition of superheated ... pressure, has a higher ... than saturated steam. ... to S′, we might keep ... supply heat; then pressure ... a second definition is, ... volume, a higher tempera...

... STEAM.—Besides the two ... the saturation curve to any ... by isothermal expansion, ... pressure at which the satura... ... at H—provided that this ... pressure at the start. This ... equilateral hyperbola; but the ... disappears as the steam gets ... The complete isothermal ... constant pressure line WS for the ... curve SX very nearly ($pv = C$). ... point on the saturation curve ... lines.

... ONE TO THE OTHER.—Now if

we start at any superheated point H, and abstract heat at constant pressure or at constant volume, or in any other manner, the steam will behave like a gas till the curve SS is reached; and then there will be an abrupt change in properties. Thus, along HS

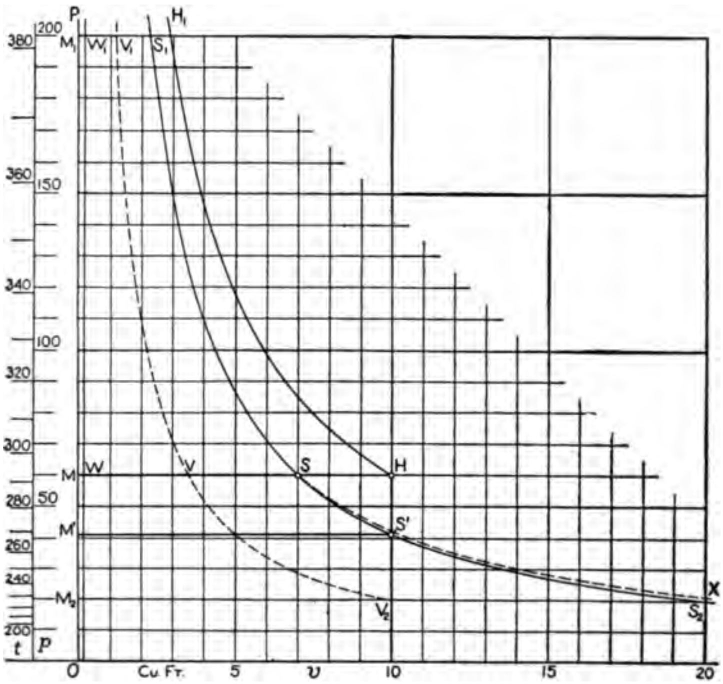


FIG. 23.—Different States of Steam.

temperature falls and volume shrinks, to S; then temperature remains constant, and volume is diminished by condensation. Along HS′, temperature falls and pressure drops, to S′; then pressure and temperature fall together in the different saturation-relation, as the steam condenses. From the superheated side, then, SS is logically the "saturation curve"; from the other side it may well be called the "dry-steam curve."

§ 12. Superheated Steam.

(*a*) A GENERAL FORMULA FOR SUPERHEATED STEAM, devised by Zeuner, and covering the space to the right of the saturation curve in Fig. 23, is

$$pv = .6496T - 22.58\sqrt[4]{p}, \quad . \quad . \quad . \quad . \quad . \quad (62)$$

where p is in lbs. per sq. in., v in cu. ft., and T in °AF. This serves for dry saturated steam as well as superheated: for if we substitute corresponding saturation values of p and T, the v we get is the specific steam volume s. Some examples will make clear the use of this formula.

(*b*) In Fig. 23, for the point S, p is 60 lbs., t is 292.5°, and s from the Table, 7.087 cu. ft. Now T is 752.5° and $\sqrt[4]{60} = 2.783$: then by (62),

$$\begin{aligned} 60\,v &= .6496 \times 752.5 - 22.58 \times 2.783 \\ &= 488.82 - 62.84 = 425.98; \\ v &= 7.085, \text{ a close agreement with } s. \end{aligned}$$

(*c*) For the point H, v is taken 10 cu. ft., and the corresponding temperature is to be found. We have

$$60 \times 10 = .6496T - 62.84,$$

$$T = \frac{662.84}{.6496} = 1020.4°,$$

or

$$t = 560.4°.$$

For a point on the isothermal HH_1, as that at 180 lbs. pressure, we have

$$\begin{aligned} 180\,v &= 662.84 - 22.58 \times 3.663 \\ &= 662.84 - 82.71 = 580.13; \\ v &= 3.223 \text{ cu. ft.} \end{aligned}$$

If the curve were a hyperbola, this value would be

$$v = \frac{60 \times 10}{180} = 3.333.$$

From the manner in which p is involved in (62), it appears that the isothermal for superheated steam will fall within the hyperbola in compression, without it in expansion.

(*d*) Expansion at Constant Pressure.—The most important practical case with superheated steam is expansion at constant pressure—for where superheated steam is used in an engine, it is made by passing steam from the boiler over heated surfaces on its way to the engine. In (62), call the saturation-temperature T_s, and get

$$ps = .6496T_s - 22.58\sqrt[4]{p},$$

then subtracting this from the general form

$$pv = .6496T - 22.58\sqrt[4]{p}$$

we get

$$p(v-s) = .6496(T-T_s) \text{ or } .6496(t-t_s). \quad . \quad . \quad . \quad (63)$$

So that in this action superheated steam behaves like a gas, the change of volume being proportional to the change of temperature, for any particular pressure. Note that, if we use the value of s from the Table in (63), any difference between this s and that found by (62) will enter as an error into the value of v; but this error is practically negligible.

(*e*) The Specific Heat of Superheated Steam in this expansion under constant pressure is 0.48. In Fig. 23, for instance, the pound of steam passes from 292.5° at S to 560.4° at H, or through 267.9°: the heat required is $267.9 \times .48 = 128.6$ H.U. To make 7.09 cu. ft. of saturated steam at 60 lbs., after the water had been heated up to 292.5°, took 908.2 H.U., or 128.1 H.U. per cu. ft.; to add 2.91 cu. ft. to this volume by superheating to 560.4° takes 128.6 H.U., or only 44.2 H.U. per cu. ft. The external work done per cu. ft. is the same in both parts of the operation. For mechanical purposes it appears, then, that a good deal might be gained by superheating the steam.

§ 13. The Adiabatic Behavior of St

(*a*) For steam, as for any other expansive s
law holds that in adiabatic expansion the ex
at the expense of the heat-energy stored in th
ture of steam and water, the physical chang
release of this heat depend upon the comp
If the fraction of the water is small, some ste
in order to yield up the necessary heat: but
ponderates, then the external work is relati
free by the drop in temperature of the wa
to do this work, and a part of the wate
rated.

... to the

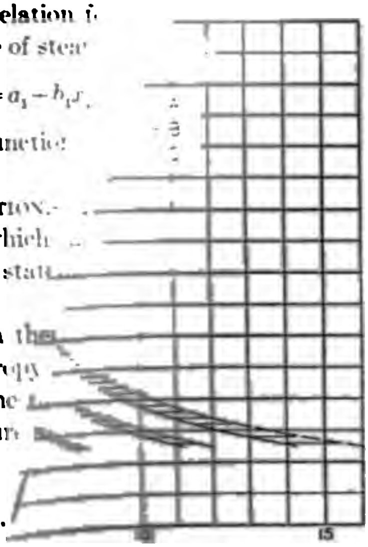
(*b*) The exact equation of relation f
(or compression) of any mixture of stea

$$a - bx = a_1 - b_1x_1 \quad . \quad . \quad (70)$$

where a and b are adiabatic functio
units of the Steam-table.

(*c*) THE ADIABATIC EQUATION.
form ... based on principles which
in Chapter VI. A condensed stat
follows:

When heat is imparted to a the
process a property called entropy
defined as the summation of the
heats to the absolute temperatur



N

... complete operation
... estimated above
... varies: and w

C. Adiabatic by Eq. (71).

... for steam initially dry
... point are:

N ...

... = 2.325 cu. ft.;

... = 2.325;

... = 1.0408.

... entropy of the liq

·l of steam when
· Table

·.367.

.71

·ls = 9.509 cu. ft.

·· in Fig. 24 are given in
..itatively, the action of con-
·iabatic expansion. The initial
.5. and .00: successive computed
·low:

\ ALUES OF x, FIG. 24.

II.	III.	IV.	V.
.75	.50	.25	.00
.7464	.5018	.2572	.0125
.7420	.5034	.2647	.0261
.7372	.5051	.2730	.0409
.7312	.5067	.2821	.0576
.7236	.5080	.2924	.0768
.7129	.5087	.3045	.1003
.7009	.5084	.3158	.1232

·! spaces in the figure, between each adiabatic and
:·ding curve of constant steam-weight, show clearly
.sation with steam preponderating, the evaporation
.vater-weight is greatest. It appears that for x a little
·han 0.5 the adiabatic would agree with the constant-
·urve—note the reversal of action shown by the last value
..nn III.

I·N COMPRESSION, these actions are all reversed in direc-
that is, there is evaporation when moderately wet steam is
·ressed, condensation when there is relatively only a little
··n. If the operation begins with dry steam, or if steam with
· a little moisture is dried by compression, then adiabatic
npression of this dry steam will superheat it.

An approximate formula, if we take $c=1$, is

$$N_1=\log_e \frac{T}{492}. \quad . \quad . \quad .$$

Eq. (68) was used in calculating the a-colum[illegible] table.

(e) THE RELATION BETWEEN PRESSURE AN[illegible] only indirectly by Eq. (64). Starting with volume, the latter depending upon x_1 accordi[illegible] find x for the new pressure, then work back [illegible] new volume. Eq. (64) is to be used in the for[illegible]

$$x=\frac{a_1+b_1x_1-a}{b}. \quad . \quad .$$

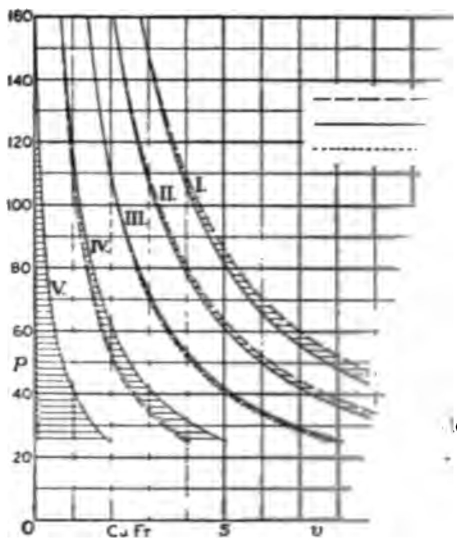


FIG. 24.—Adiabati[illegible]
A. Constant steam-weight. B. Exact [illegible]

[illegible] 3
[illegible] 4
5.137
7.403
11.305

[illegible] 70) and (52); [illegible] for the smaller [illegible] departs rapidly [illegible] wn in, from 80 lbs. [illegible] It appears that [illegible] end below $x=0.65$. [illegible] 0 lbs. pressure.

EXAMPLE 1.—Curve I. in Fig. 2[illegible] and saturated. The values at the st[illegible]

$p_1 = 160$ lbs. abs.;
$x_1 = 1.000$;
$a_1 = .5211$;

$x_1 = .75$
$v_1 = 2.123$
$n = 1.110$

$\log\left(\frac{p_1}{p}\right)\frac{1}{n} = 0.38376$
$\log v_1 = 0.32695$

$\log v = 0.71071$

Suppose that we wish to find the volume of the pound of steam when expanded adiabatically to $p=40$ lbs. We get from the Table

$$a=.3933, \qquad b=1.2743, \qquad s=10.367.$$

Then

$$x=\frac{.5211+1.0408-.3933}{1.2743}=\frac{1.1686}{1.2743}=.9171$$

and

$$v=.9171\times 10.349+.018=9.491+.018=9.509 \text{ cu. ft.}$$

(*f*) Adiabatic Curves.—The curves in Fig. 24 are given in order to show, graphically and quantitatively, the action of condensation or evaporation during adiabatic expansion. The initial values of x are 1.00, .75, .50, .25, and .00: successive computed values are given in the table below:

Table 13 A. Values of x, Fig. 24.

p	I.	II.	III.	IV.	V.
160	1.00	.75	.50	.25	.00
140	.9910	.7464	.5018	.2572	.0125
120	.9808	.7420	.5034	.2647	.0261
100	.9693	.7372	.5051	.2730	.0409
80	.9558	.7312	.5067	.2821	.0576
60	.9392	.7236	.5090	.2924	.0768
40	.9170	.7129	.5087	.3045	.1003
25	.8936	.7009	.5084	.3158	.1232

The shaded spaces in the figure, between each adiabatic and the corresponding curve of constant steam-weight, show clearly the condensation with steam preponderating, the evaporation when the water-weight is greatest. It appears that for x a little greater than 0.5 the adiabatic would agree with the constant-weight curve—note the reversal of action shown by the last value in column III.

(*g*) In Compression, these actions are all reversed in direction: that is, there is evaporation when moderately wet steam is compressed, condensation when there is relatively only a little steam. If the operation begins with dry steam, or if steam with only a little moisture is dried by compression, then adiabatic compression of this dry steam will superheat it.

(*h*) AN APPROXIMATE, EMPIRICAL FORMULA—
for the adiabatic curve of saturated steam is got
equation $pv^n=C$, using a special index which
initial steam-weight according to the rule

$$n=1.035+0.1x. \quad . \quad . \quad .$$

In order to show how nearly correct this fo
worked out by it are given in the table below, in
those found by the method of Example 1: these
ditions of curves I. and II. in Fig. 24.

TABLE 13 B. VALUES OF v, FIG. 24

p	I. $x=1.00$.		II.
	A.	B.	A.
160	2.825	2.825	2.123
140	3.173	3.177	2.395
120	3.630	3.640	2.751
100	4.256	4.275	3.242
80	5.173	5.203	3.962
60	6.657	6.704	5.133
40	9.508	9.583	7.399
25	14.402	14.499	11.301

Here the A-columns contain results got by
while those in the B-columns are from (71).
values of x used in Fig. 24, the curve of Eq. (71
from the true adiabatic: these curves are drawn
down to 25 lbs., in the dotted line marked C
the effective range of Eq. (71) does not extend

EXAMPLE 2.—Find v by (71) for curve II. at 60

$$v=v_1\left(\frac{p_1}{p}\right)^{\frac{1}{n}} \qquad \begin{aligned} p_1&=160 \\ p&=60 \\ s_1&=2.825 \end{aligned}$$

$$\log p_1=2.20412 \qquad \log\log\frac{p_1}{p}=9.62938$$

$$\log p\ =1.77815 \qquad \log n=0.04532$$

$$\log\frac{p_1}{p}=0.42597 \qquad \log\log\left(\frac{p_1}{p}\right)\frac{1}{n}=9.58406$$

$$v=5.137$$

(*i*) USE OF TABLE II.—In comparison with the exact method, with a and b worked out and tabulated, this method is at a decided disadvantage as to ease of computation. The results are, however, quite accurate enough for graphical work, and the values entered in Table II. can be used to good effect when an adiabatic curve is to be drawn on a steam diagram—following the method explained in § 7 (*j*), under Fig. 19.

(*j*) THE ADIABATIC EQUATION FOR SUPERHEATED STEAM is similar to that for air, but k has the value 1.333 instead of 1.406, so that

$$pv^{1.333}=C. \qquad (72)$$

(*k*) EXTERNAL WORK.—As stated in (*a*), the external work in adiabatic expansion is done at the expense of the internal heat-energy of the steam. This internal energy does not include the heat used in the external work of evaporation: so that whereas the total heat above 32°, for any condition x, is $Q=q+xr$, the internal energy is

$$I=q+xl. \qquad (73)$$

Now for any two points on an adiabatic curve, we have

$$I_1=q_1+x_1l_1, \qquad I_2=q_2+x_2l_2;$$

then the heat lost, or external work done, is

$$AU=q_1+x_1l_1-q_2-x_2l_2. \qquad (74)$$

§ 14. The Carnot Cycle with Steam.

(*a*) Assuming the same conditions of working as those of the ideal air-engine in § 8, and using a cycle with the same set of operations, we have for the ideal steam-engine the action illustrated in Fig. 25. Starting at 1, there is in the cylinder 1 lb. of water at the temperature t_1 and pressure p_1: and isothermal expansion, at constant pressure, to complete evaporation at 2, constitutes the first phase. Then follows adiabatic expansion to the lower limit of pressure and temperature at 3, completing the out-stroke. On the return-stroke, we have first isothermal compression, with

rapid rejection of heat, then adiabatic compression, along 41, to the original state.

(*b*) UNIVERSALITY OF THE CARNOT CYCLE.—It is a general principle of thermodynamics that, since no peculiar properties of the working-medium are involved in the expression for the efficiency of an engine with the Carnot cycle, therefore this efficiency

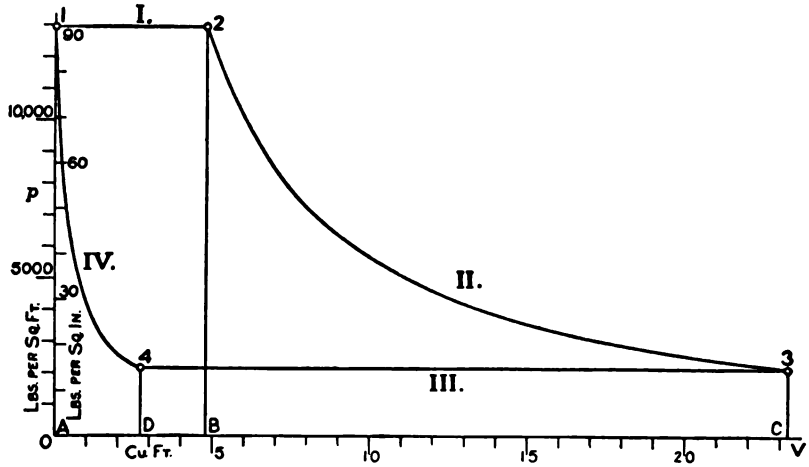


FIG. 25.—The Ideal Steam-cycle.

is independent of the medium used. Then with steam, as with air, the limit of efficiency is

$$E=\frac{T_1-T_2}{T_1} \text{ or } \frac{t_1-t_2}{t_1+460}. \quad . \quad . \quad . \quad . \quad . \quad (75)$$

And since the heat received during Phase I. is r_1, the useful work of the cycle, per pound of steam and expressed in heat-units, is

$$AU=Er_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

It would be very difficult, if not impossible, to develop an algebraic proof for the ideal efficiency from the form of the steam-cycle, because the adiabatic relations are of such complex form. But the general principle that the effective output is not dependent upon the medium can, perhaps, be made to appear more con-

vincing by the working out of a particular case, with numerical values.

(c) Numerical Determinations for a Particular Case.—The primary conditions in Fig. 25 are:

$$p_1=p_2=90 \text{ lbs. per sq. in.};$$
$$p_3=p_4=15 \text{ `` `` `` ``}.$$

We will use the subscripts 1, 2, 3, 4, on the p's, v's, and x's, to correspond with the points 1, 2, 3, 4. But on all the heat symbols, sub 1 will refer to the upper isothermal and sub 2 to the lower. The values taken from the table are:

$t_1=320.0°$	$T_1=780.0°$	$t_2=213.0°$	$T_2=673.0°$
$q_1=291.2$	$r_1=888.4$	$q_2=181.9$	$r_2=965.0$
$l_1=807.9$		$l_2=892.7$	
$a=.46535$	$b_1=1.1389$	$a_2=.31479$	$b_2=1.4339$
$s_1=4.845$	$u_1=4.827$	$s_2=26.07$	$u_2=26.053$

Get the volumes first:

$$x_1=0; \qquad x_2=1.00;$$
$$v_1=w_1=.018; \qquad v_2=s_1=4.845;$$
$$x_3=\frac{a_1+b_1-a_2}{b_2}=\frac{1.2895}{1.4339}=.8993;$$
$$v_3=x_3s_2=.8993\times 26.07=23.445,$$

or, more exactly,

$$v_3=x_3u_2+w_2=.8993\times 26.053+.017$$
$$=23.430+.017=23.447;$$

whence it appears that this refinement is entirely superfluous here, where x is nearly 1.

Again,

$$x_4=\frac{a_1-a_2}{b_2}=\frac{.15056}{1.4339}=.10500;$$
$$v_4=x_4u_2+w_2=2.736+.017=2.753,$$

or

$$v_4=x_4s_2=2.737.$$

Here, of course, the exact method is necessary.

(*d*) HEAT QUANTITIES.—Next, determine the heat-quantities at the critical points:

At 1, the internal heat and the total heat in the water are the same, so that

$$Q_1 = I_1 = q_1 = 291.2 \text{ H.U.}$$

At 2, the total heat is

$$Q_2 = q_1 + r_1 = 1179.6,$$

while the internal heat is

$$I_2 = q_1 + l_1 = 1099.1.$$

At 3,

$$Q_3 = q_2 + x_3 r_2 = 181.9 + 867.8 = 1049.7;$$
$$I_3 = q_2 + x_3 l_2 = 181.9 + 802.8 = 984.7.$$

At 4,

$$Q_4 = q_2 + x_4 r_2 = 181.9 + 101.1 = 283.0;$$
$$I_4 = q_2 + x_4 l_2 = 181.9 + 93.7 = 275.6.$$

The heat received in Phase I. is

$$Q_2 - Q_1 = r_1 = 888.4 \text{ H.U.}$$

The heat rejected in Phase III. is

$$Q_3 - Q_4 = 1049.7 - 283.0 = 766.7 \text{ H.U.}$$

Now

$$\frac{888.4}{766.7} = 1.1585$$

and

$$\frac{T_1}{T_2} = \frac{780}{673} = 1.1585.$$

Whence we see that the two heat-quantities bear to each other the proper relation. Further, the heat transformed into useful work is

$$AU = 888.4 - 766.7 = 121.7 \text{ H.U.}$$

(*e*) EFFECTIVE WORK OF CYCLE.—We will now work out from the figure a value for the effective work, to see how closely it will agree with the 121.7 H.U. just found.

In Phase I., the positive external work, reduced to heat-units, is

$$\frac{144p_1(v_2-v_1)}{778}=\frac{144\times 90\times 4.827}{778}=80.41.$$

In Phase III., the negative work is

$$\frac{144p_3(v_3-v_4)}{778}=\frac{144\times 15\times 20.692}{778}=57.45.$$

In Phase II., the heat transformed into external work is

$$I_2-I_3=1099.1-984.7=114.4.$$

And in Phase IV., the external work transformed into heat is

$$I_1-I_4=291.2-275.6=15.6.$$

Combining these four quantities, we get:

I.	+ 80.4	III.	−57.5
II.	+114.4	IV.	−15.6
	+194.8		−72.1
	− 72.1		
	+121.7		

Note that the relation between the two adiabatic works in this case is entirely different from that which holds with a perfect gas; and further that with a steam-isothermal, the heat imparted does *not* all go to do external work.

(*f*) Comparison of Steam with Air as a Medium.—A very important feature of the operation shown in Fig. 25—as compared with that in Fig. 20—which constitutes the tremendous practical advantage of the steam-engine over the hot-air engine as a machine, is the much greater mechanical intensity of the operation. In the case just analyzed, the piston-displacement per pound of steam is 23.25 cu. ft., and the effective work is 121.7 H.U. or 94,683 F.P.: this gives 5.234 H.U. or 4072 F.P. per cu. ft., against only 840 F.P. for Fig. 20, as found in Example 2, § 8. Of course, the steam-engine has generally a comparatively small range of temperature: the ideal efficiency is only .137 for the case discussed above.

(*d*) HEAT QUANTITIES.—Next, determ[illegible]
at the critical points:

At 1, the internal heat and the total [illegible] same, so that

$$Q_1 = I_1 = q_1 = 291.2 \text{ H.U.}$$

At 2, the total heat is

$$Q_2 = q_1 + r_1 = 1179.6,$$

while the internal heat is

$$I_2 = q_1 + l_1 = 1099.1.$$

At 3,

$$Q_3 = q_2 + x_3 r_2 = 181.9[illegible]$$
$$I_3 = q_2 + x_3 l_2 = 181.[illegible]$$

At 4,

$$Q_4 = q_2 + x_4 r_2 = 18[illegible]$$
$$I_4 = q_2 + x_4 l_2 = 18[illegible]$$

The heat received in Pha[illegible]

$$Q_2 - Q_1 = r_1 [illegible]$$

The heat rejected in Ph[illegible]

$$Q_3 - Q_4 = 1[illegible]$$

Now

and

Whence we see th[illegible] the proper relation[illegible] work is [illegible]

(*e*) EFFE[illegible] the figure [illegible] agree with th[illegible]

[illegible]-engine.

[illegible]eam-engine differs [illegible] have been discussing [illegible] functions of heat [illegible]ation are separated [illegible] to the boiler, engine, [illegible]nsider, then, the work [illegible] engine proper is only

[illegible]—For the actual plant, [illegible]sion is with the feed- [illegible] IV. of the ideal cycle is [illegible] That is, the operation [illegible] been condensed, the [illegible] by condensation is com- [illegible] pressure and tempera- [illegible]part in the steam-plant. [illegible] in a condenser or by [illegible] engine can do is to secure [illegible] temperature of the exhaust [illegible] the feed-pump—takes the [illegible] boiler: this closes the cycle, [illegible] concerned is an open one. The [illegible] engine, whereby a portion of [illegible] fills the clearance-space, has [illegible] to the mechanical action of [illegible] only a secondary effect in a

[illegible] SEPARATION begins therefore [illegible] both heating and evapora- [illegible] to t_1, then the operation [illegible] from this condition are [illegible] by imperfect regulation [illegible] to the minor fluctuations on [illegible] of steam into the cylinder.

As pointed out in § 9 (*c*), the steam, rising from the surface of the water in the boiler, has to keep pushing some other steam ahead of it till it gets to the cylinder, where the external work of evaporation is really done upon the piston, in the form of pressure acting upon a moving part of the confining surface.

(*d*) ADIABATIC EXPANSION IMPOSSIBLE.—But when the steam gets to the engine, it encounters, not cylinder-walls that are non-conducting and non-absorbent, or that are at its own temperature, but metal walls that have been considerably cooled by contact with the exhaust steam. During the admission period, then, a part of the steam is condensed, its heat going into the surfaces of the cylinder and piston: it therefore loses volume, one pound filling only the space AB, instead of A1, in Fig. 26.

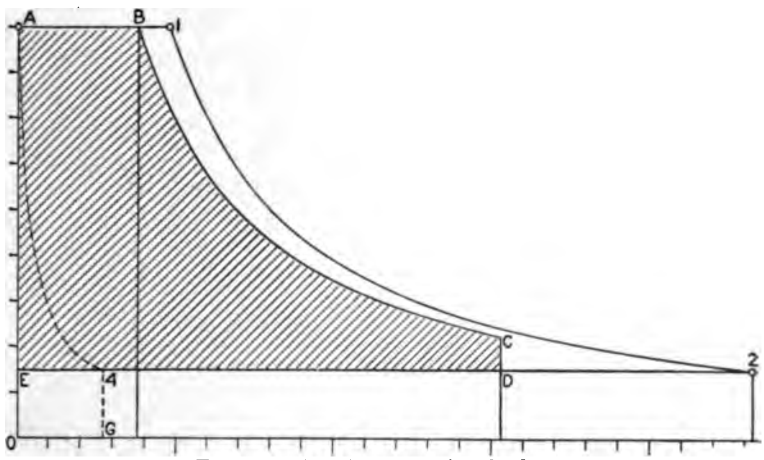


FIG. 26.—The Steam-engine Cycle.

(*e*) THERMAL ACTION OF THE CYLINDER-WALLS.—After the steam gets into the cylinder and communication with the steam-pipe is cut off, an expansion takes place which is far from being adiabatic. As the wet steam drops in pressure, its temperature falls too; the flow of heat from steam to metal, which has been rapidly diminishing during the latter part of the admission period, presently reverses; and during the rest of the expansion and all of the exhaust period, heat is passing from the metal to the steam.

Upon the expansion this has the effect, that instead of some steam being condensed to furnish heat for external work, as under adiabatic conditions, there is so much heat supplied by the cylinder-walls and by the hot water present in the cylinder, that usually there is some evaporation. The expansion line BC rises above the curve of constant steam-weight, approximating the equilateral hyperbola: and we have the very important and useful conclusion, from experience, that for many purposes—especially for the preliminary laying out of steam diagrams—the working fluid in the steam-engine may be taken to follow the law $pv=C$ in its expansion and compression. Of course, this simple working is the result of very complex actions and reactions; which can be determined, more or less completely, only by actual tests of the engine.

(*f*) INCOMPLETE EXPANSION.—Chiefly for mechanical reasons, the expansion is not usually complete: that is, it is stopped at some point C, before the exhaust pressure is reached; and, by the opening of the valve, the pressure is allowed to drop from C to D before the piston begins its return stroke. But this steam released from C to D immediately falls to the exhaust pressure, so that very nearly the whole operation of heat-abstraction is isothermal at the lower limit. We have then that the actual cycle agrees closely with the ideal in two of its phases, but departs widely from it in the other two.

(*g*) Before taking up the discussion of the actual working of the steam in the engine, we will develop, and reduce to numerical values, the expression for its limit of effective performance, as set forth in Fig. 26.

§ 16. The Limit of Performance of the Engine with Saturated Steam.

(*a*) With the Carnot cycle, the heat received per pound of steam is

$$Q=r_1, \qquad (77)$$

and the effective work done, represented by the area A124A in Fig. 26, is

$$AU=\frac{T_1-T_2}{T_1}r_1. \qquad (78)$$

But the cycle which really forms the limit of the actual steam-engine is that whose outline is A12EA in Fig. 26. Here the heat imparted includes the raising of the water from t_2 to t_1, so that its amount is

$$Q' = q_1 - q_2 + r_1 \quad \text{or} \quad H_1 - q_2. \quad . \quad . \quad . \quad . \quad (79)$$

The corresponding work-output can be found by calculating the work-equivalent of the area A4EA, and adding it to U from (78). The method of doing this can best be shown by an example.

(*b*) Calculation for Table 16 A. Let $p_1 = 240$ lbs., $p_2 = 15$ lbs.; then from the Table we get

$t_1 = 397.3°$	$T_1 = 857.3°$	$t_2 = 213°$	$T_2 = 673°$
$q_1 = 371.2$	$r_1 = 831.9$	$q_2 = 181.9$	$l_2 = 892.7$
$a_1 = .5633$		$a_2 = .3148$	$b_2 = 1.4339$
		$s_2 = 26.07$	

For convenience, we shall refer to the Carnot cycle as Cycle A, and to the cycle without adiabatic compression as Cycle B.

Now for Cycle A,

$$E = \frac{T_1 - T_2}{T_1} = \frac{184.3}{857.3} = .2150;$$

$$AU = Er_1 = .2150 \times 831.9 = 178.84.$$

For Cycle B, we first find

$$x_4 = \frac{a_1 - a_2}{b_2} = \frac{.2485}{1.4339} = .1733.$$

Now the internal energy at 4 is

$$I_4 = q_2 + x_4 l_2 = 181.9 + .1733 \times 892.7$$
$$= 181.9 + 154.7 = 336.6 \text{ H.U.}$$

At A the internal energy is q_1, so that the external work of Phase IV., the area A4GOA, is

$$q_1 - q_2 - x_4 l_2 = 371.2 - 336.6 = 34.6 \text{ H.U.}$$

Of this, however, only the part A4EA is to be added to AU above. To get E4GO, we have

$$v_4 = x_4 s_2 = .1733 \times 26.07 = 4.519 \text{ cu. ft.};$$

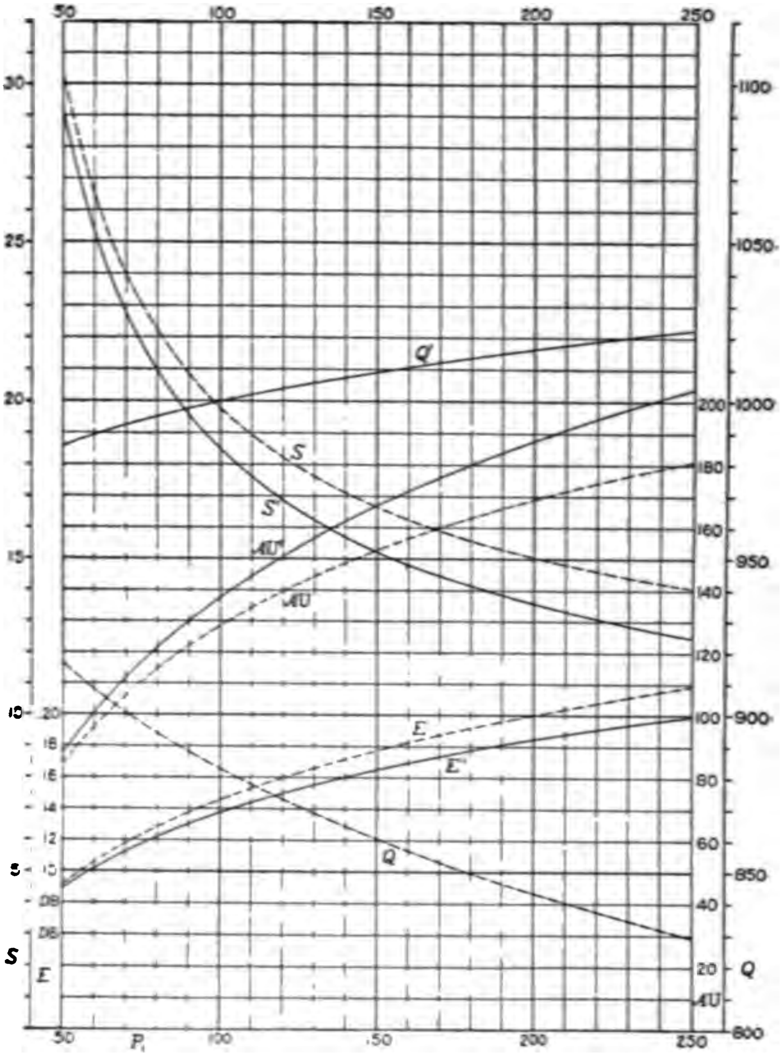


FIG. 27.—Limit Curves for Non-condensing Engines.
See page 84.

TABLE 16 A. LIMIT VALUES FOR NON-CONDENSING ENGINES.

Boiler-pressure, Lbs. Abs.	Heat Imparted, B.T.U. per Lb.		Heat Converted, B.T.U. per Lb.		Absolute Efficiency.		Pounds of Steam per H.P. Hour.	
p_1	Q	Q'	AU	AU'	E	E'	S	S'
50	916.6	985.7	84.0	87.5	.092	.089	30.30	29.10
60	908.2	989.3	96.0	100.5	.106	.102	26.53	25.31
70	900.9	992.4	106.0	111.7	.118	.113	24.02	22.78
80	894.3	995.1	114.5	121.4	.128	.122	22.23	20.97
90	888.4	997.7	121.9	129.9	.137	.130	20.90	19.60
100	882.9	999.9	128.5	137.5	.145	.137	19.81	18.51
110	877.9	1002.1	134.3	144.4	.153	.144	18.95	17.63
120	873.2	1004.0	139.5	150.8	.160	.150	18.24	16.88
130	868.7	1005.8	144.3	156.6	.166	.156	17.64	16.26
140	864.6	1007.6	148.7	162.0	.172	.161	17.12	15.71
150	860.6	1009.3	152.7	167.0	.177	.166	16.68	15.25
160	856.9	1010.8	156.4	171.6	.182	.170	16.28	14.84
170	853.3	1012.3	159.9	176.0	.187	.174	15.92	14.46
180	849.9	1013.8	163.1	180.2	.192	.178	15.60	14.12
190	846.6	1015.1	166.1	184.1	.196	.181	15.32	13.82
200	843.4	1016.4	168.9	187.8	.200	.185	15.07	13.56
210	840.4	1017.7	171.6	191.3	.204	.188	14.84	13.30
220	837.4	1018.9	174.1	194.7	.208	.191	14.63	13.07
230	834.6	1020.1	176.5	197.9	.212	.194	14.43	12.86
240	831.9	1021.2	178.8	201.0	.215	.197	14.24	12.66
250	829.1	1022.3	181.0	204.0	.218	.200	14.06	12.48

then

$$\frac{144p_4v_4}{778}=\frac{144\times15\times4.519}{778}=12.54 \text{ H.U.}$$

Subtracting this 12.54 from the 34.60 just found, we have 22.06 H.U. to be added to AU; then for this cycle,

$$AU'=178.84+22.06=200.9 \text{ H.U.*}$$

Now the heat imparted in this second case is,

$$Q'=r_1+q_1-q_2=1203.1-181.9=1021.2 \text{ H.U.}$$

* A simpler way of getting AU' is given in § 24 (*b*).

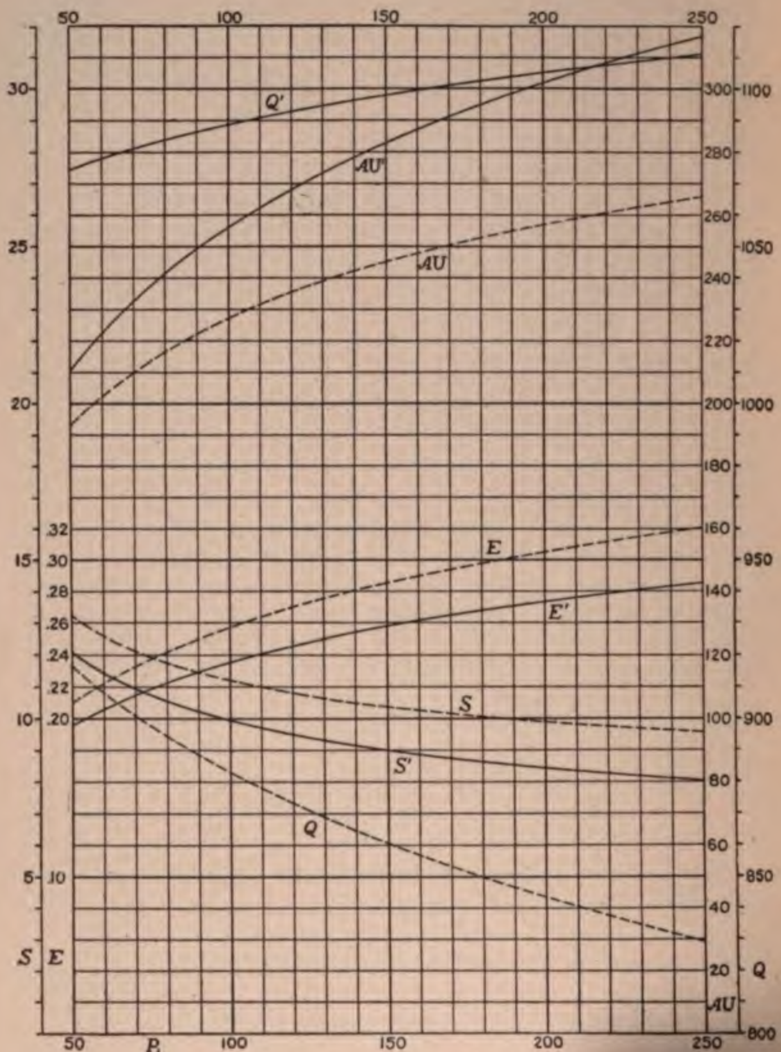


FIG. 28.—Limit Curves for Condensing Engines.
See page 84.

TABLE 16 B. LIMIT VALUES FOR CONDENSING ENGINES.

Boiler-pressure, Lbs. Abs.	Heat Imparted, B.T.U. per Lb.		Heat Converted, B.T.U. per Lb.		Absolute Efficiency.		Pounds of Steam per H.P. Hour.	
p_1	Q	Q'	AU	AU'	E	E'	S	S'
50	916.6	1074.4	192.6	210.5	.210	.196	13.21	12.09
60	903.2	1078.0	202.1	222.6	.223	.206	12.59	11.43
70	900.9	1031.1	209.9	232.7	.233	.215	12.13	10.93
80	894.3	1033.8	216.5	241.5	.242	.223	11.76	10.54
90	888.4	1036.4	222.1	249.3	.250	.229	11.46	10.21
100	882.9	1038.6	227.1	256.3	.257	.235	11.21	9.94
110	877.9	1090.8	231.5	262.6	.263	.240	11.00	9.69
120	873.2	1092.7	235.5	268.2	.269	.245	10.81	9.49
130	868.7	1094.6	239.1	273.5	.275	.250	10.65	9.31
140	864.6	1096.3	242.3	278.4	.280	.254	10.50	9.14
150	860.6	1098.0	245.2	283.0	.285	.258	10.38	8.99
160	856.9	1099.5	248.0	287.2	.289	.261	10.26	8.86
170	853.3	1101.0	250.6	291.2	.294	.265	10.16	8.74
180	849.9	1102.5	253.0	295.0	.298	.268	10.06	8.63
190	846.6	1103.8	255.2	298.6	.301	.271	9.97	8.52
200	843.4	1105.1	257.2	302.0	.305	.273	9.90	8.43
210	840.4	1106.4	259.1	305.3	.308	.276	9.82	8.34
220	837.4	1107.6	260.9	308.4	.312	.278	9.75	8.25
230	834.6	1108.8	262.6	311.5	.315	.281	9.69	8.17
240	831.9	1109.9	264.2	314.1	.318	.283	9.63	8.10
250	829.1	1111.0	265.7	316.8	.321	.285	9.58	8.03

The efficiency is therefore

$$E'=\frac{200.9}{1021.2}=.1966.$$

Last of all, the number of pounds of steam required per H.P. hour is, for the two cases—one H.P.H. = 2545 H.U.—

$$S=\frac{2545}{178.8}=14.23, \quad S'=\frac{2545}{200.9}=12.66.$$

But while the less efficient operation uses fewer pounds of steam per H.P.H., it of course requires more heat, the respective amounts being

$$SQ\ =11{,}834 \text{ H.U. for Cycle A,}$$

$$S'Q'=12{,}925 \text{ H.U. for Cycle B.}$$

(*c*) IN FIGS. 27 AND 28 AND IN THE ACCOMPANYING TABLES, 16 A and 16 B, results similar to those just worked out are given for the whole range of practice and for the two classes into which engines are divided as to their lower limit of working—that is, for non-condensing and for condensing engines. The respective lower limits are:

For non-condensing engines,

$$p_0 = 15 \text{ lbs. abs.,} \qquad t_0 = 213^\circ \text{ F.}$$

For condensing engines,

$$p_0 = 1.94 \text{ lbs. abs.,} \qquad t_0 = 125^\circ \text{ F.}$$

While the condenser can be, and usually is, at a lower temperature than 125°, it is not possible to realize the full vacuum due to the steam temperature, because a small amount of air is always present. The assumed figures represent about the best attainable conditions in each case.

As to the change in subscript, we shall hereafter use sub 2 for the conditions at the end of expansion, at C in Fig. 26; then sub 0 will denote the exhaust conditions.

(*d*) IN THE CURVES AND TABLES, the several quantities represented or given are:

Q = the heat imparted.
AU = the heat converted into effective work.
E = the absolute efficiency, or the ratio of AU to Q.
S = the number of pounds of steam required per H.P. hour.

The first two are, of course, per pound of steam.

The plain symbols and the dotted curves refer to the Carnot cycle, Cycle A.

The primed symbols and the full-line curves refer to the cycle without adiabatic compression, Cycle B.

The method of computing all these quantities is set forth in the example worked out in (*b*).

These tables form a standard of comparison, by which the performance of engines may be judged, and will frequently be referred to.

(*e*) Work-value of Adiabatic Compression.—A noticeable fact here shown is the considerable effective work-value of the adiabatic compression—which in the last case in Table 16 B rises to 51.1 H.U. Compare with this the work of forcing the feed-water into the boiler, found as follows:

At 125° F., 1 cu. ft. of water weighs 61.7 lbs., and the volume of 1 lb. is .0162 cu. ft. Including the drawing of the water from the condenser, and neglecting the small positive pressure in the latter, we assume that this volume must be displaced against the full pressure of 250 lbs. per sq. in. The work required is

$$U = 144 \times 250 \times .0162 = 584 \text{ F.P.},$$

equivalent to .750 H.U.

It appears that this latter quantity is relatively insignificant.

§ 17. The Ideal Steam Diagram.

(*a*) The Modifications which the ideal Carnot cycle undergoes when subjected to the conditions of actual working in the steam-engine, as brought out in connection with Fig. 26, are three:

1. The expansion is changed in character by the thermal action of the cylinder-walls.

2. The expansion is stopped before the lower pressure-limit is reached, or is incomplete.

3. The adiabatic compression of the whole "charge" of working substance is entirely omitted.

As the result of these changes, the ideal steam diagram, with which the actual diagram made by the indicator is to be compared, takes the form ABCDE, drawn first in Fig. 26 and reproduced in Fig. 29: in which the expansion curve BC is of the form $[pv=C]$. This diagram is, of course, based on the entire absence of clearance-space in the cylinder, and on ideally perfect action of the valve.

(*b*) The Determining Dimensions of this diagram are:

1. The initial pressure and volume, p_1 and v_1, at the point B, where expansion begins.

2. The final volume v_2, or [illegible]io of expansion, $r = v_2 \div v_1$.

3. The back-pressure during the return-stroke, p_0.

The effective work represented by the diagram is found as follows:

The work of the full-pressure expansion, from A to B, or the work during admission—the area ABGO—is

$$U_1 = 144 p_1 v_1.$$

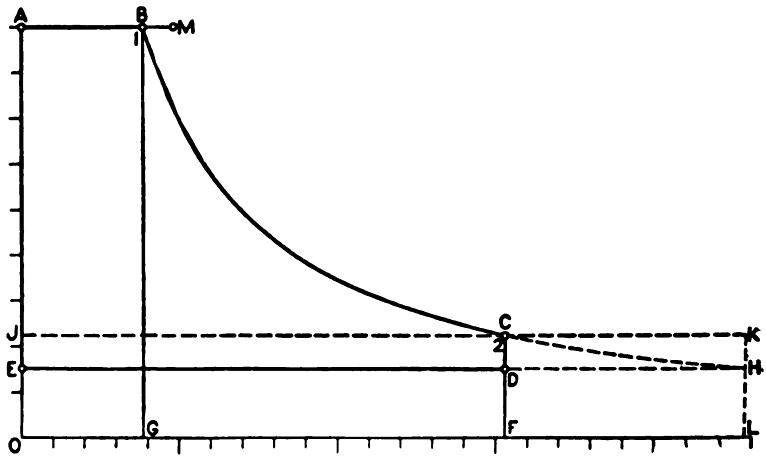


FIG. 29.—The Ideal Steam Diagram.

The work of the expansion in the cylinder, from B to C, area BCFG, is

$$U_2 = 144 p_1 v_1 \log_e r.$$

Then the total or gross work, area ABCFO, is

$$U_T = 144 p_1 v_1 (1 + \log_e r). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

The negative work in the return-stroke, area DEOF, is

$$U_B = 144 p_0 v_2 = 144 p_0 r v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Now the net or effective work, area ABCDE,

$$U = 144 v_1 [p_1 (1 + \log_e r) - p_0 r].$$

(c) Specific Volume at Cut-off.—Continuing to use the system of measuring the work-performance of one unit's weight of the medium, we have that the initial volume v_1 becomes determinate when we know the amount of the cylinder-condensation at cut-off. In Fig. 29, AM is the volume of 1 lb. of dry saturated steam; then

$$m_1 = \mathrm{BM} \div \mathrm{AM} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

is the fraction of condensation; and its complement

$$x_1 = \mathrm{AB} \div \mathrm{AM} = 1 - m_1$$

determines v_1 through the relation

$$v_1 = x_1 s_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

The value of m can be determined by an engine test in which the weight of steam used is found by actual measurement, and compared with the steam shown by the diagram: and may be pretty closely estimated from the results of tests previously made under similar conditions and recorded.

(d) The Limit of Expansion.—The proper terminal pressure p_2—from which, in connection with the known or assumed character of the expansion curve, the ratio of expansion is derived—is determined by considerations which are also illustrated in Fig. 29:

Suppose all the frictional resistances in the engine to be reduced to an equivalent pressure on the piston, measured in the same terms as the steam-pressure p, and represented by the extra height EJ added to the back-pressure in Fig. 29—where JE is made just equal to CD. Now if the piston travelled beyond CF, to KL, the work done upon it by the steam during this out-stroke would be given by the area CHLF; and during the return-stroke, from L to F, the engine would have to do, in overcoming back-pressure plus friction, the work KCFL. The result would be a net loss of work represented by the area KCH.

The conclusion is obvious.

(e) Mean Pressures of the Ideal Diagram.—Since the work done in the cylinder is usually measured in terms of the whole

piston-displacement v_2, rather than the cut-off volume v_1, Eq. (82) is frequently more convenient if changed into the form

$$U=144v_2\left[p_1\frac{1+\log_e r}{r}-p_0\right]. \quad . \quad . \quad . \quad . \quad . \quad (85)$$

The expression $p_1\frac{1+\log_e r}{r}$ gives the mean total pressure during the forward-stroke, or the mean height of the figure ABCFO; calling this p_{m0} and letting p_m represent the mean effective pressure, we have

$$p_m=p_{m0}-p_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

Then (85) becomes

$$U=144p_m v_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

Values of the ratio

$$\frac{p_{m0}}{p_1}=\frac{1+\log_e r}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

are given in Table I.

(*f*) We have now established the fundamental principles on which all discussion of the thermodynamic performance of the steam-plant is based: and are ready to take up the consideration of the actual behavior of the steam in the cylinder, which forms the subject of the next chapter.

EXAMPLE.—In Fig. 29, with $p_1=90$ lbs. abs., we take m to be .20 and get

$$v_1=x_1s_1=.80\times 4.845=3.876 \text{ cu. ft.},$$

also

$$U_1=144\times 90\times 3.876=50,233 \text{ F.P.}$$

Further, r is taken to be 4, then

$$U_2=50,233\times 1.3863=69,637 \text{ F.P.}$$

and

$$U_T=50,233+69,637=119,870 \text{ F.P.}$$

Now $v_2=3.876\times 4=15.504$, and $p_0=15$; then

$$U_B=144\times 15\times 15.504=33,488 \text{ F.P.}$$

Finally,

$$U = 119{,}870 - 33{,}488 = 86{,}382 \text{ F.P.}$$

The mean total pressure is

$$\begin{aligned} p_{mo} &= U_T \div 144 v_2 \\ &= 119{,}870 \div 2233 = 53.69 \text{ lbs. per sq. in.} \end{aligned}$$

And the mean effective pressure is

$$p_m = 53.69 - 15 = 38.69 \text{ lbs. per sq. in.}$$

Working out the ratio of p_{mo} to p_1 directly from $\log_e r$, we have

$$(1 + 1.3863) \div 4 = .5966;$$

and multiplying by p_1, we get

$$p_{mo} = 90 \times .5966 = 53.69 \text{ lbs. per sq. in.}$$

From this we can compute the effective work through (86) and (87), the whole operation being much shorter and easier than that used above.

CHAPTER IV.

THE ACTION OF THE STEAM IN THE ENGINE.

§ 18. The Indicator Diagram.

(*a*) Comparison of Actual with Ideal Diagram.—The particulars in which the actual steam diagram differs from the ideal form given in Fig. 29 are shown in Fig. 30, where the indicator diagram ABCDEF represents the behavior of the steam in the cylinder, and is to be compared with the circumscribed ideal diagram GHLQT, or more properly JHLQP. This indicator diagram is characteristic of the type of engine described in § 2; that is, of the high-speed, non-condensing engine with large clearance and with the steam-distribution controlled by a single slide-valve, so that high compression is unavoidable with early cut-off—a class which includes also the ordinary simple locomotive. This type is here selected for illustrative purposes because the modifications in the form of the diagram, caused by mechanical actions, are especially marked. Our present point of view is the acceptance of the diagram as an accomplished fact, considering in general terms the conditions which produce certain effects, but postponing the intimate analysis of these conditions until the correlative subjects, such as the thermal action of the cylinder-walls and the working of the valve-gear, are taken up in detail.

The diagram in Fig. 30, while modelled on that given by the indicator, is not a true indicator diagram; the latter has certain peculiarities, due to the instrument itself, which are better left out for present purposes. Further, the curves of expansion and compression are here exact hyperbolas.

(*b*) Reference Lines.—In Fig. 30, the following reference or auxiliary lines are drawn:

The vacuum-line ON is parallel to the atmosphere-line PQ, at the distance 14.7 lbs. per sq. in., to scale, below it.

The end-lines GM and KN touch the diagram, marking off the length MN which represents the stroke of the piston, or, to a suitable scale, the volume displaced by the piston in one stroke, also called the nominal cylinder-volume.

The distance MO is measured off to represent the clearance-volume to this same scale, and the clearance-line or pressure-axis OJ is drawn.

The boiler-pressure line JK is at the proper distance above PQ.

Finally, the expansion-curve is produced upward to meet the line of boiler-pressure at H, and downward to the end-line at L.

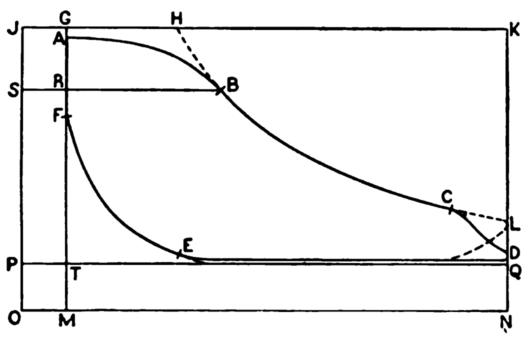


FIG. 30.—The Indicator Diagram.

(c) ADMISSION.—Starting at A, we note first that the highest pressure in the cylinder is below the boiler-pressure. Most of this loss is due to the resistances which the steam-pipe and the valves in it offer to the current of steam; though a part of the drop may be caused by the engine-valve and the ports.

As the piston advances and the valve begins to close, the pressure falls off. Complete closure, or mechanical cut-off, is marked by the point B where the convex admission-curve merges into the concave expansion-curve—or where the curvature reverses in direction. But expressed in terms of the amount of steam admitted, rather than by the position of the piston when the edge of the valve meets the edge of the port, the effective cut-off

is at H: the volume JH measures, at boiler-pressure, the total amount of steam present in the cylinder.

On account of all the resistances which the steam has to overcome in getting to the engine, there is then a loss of possible effective work represented by the area GHBA.

(*d*) CUT-OFF.—There are several ways of stating when the cut-off takes place:

From the purely mechanical point of view, we have that the valve closes when the piston is at the distance RB along the stroke: that is, the ratio of apparent cut-off by the valve is

$$C_{VA}=\frac{RB}{MN}. \qquad (89)$$

But in order to have a ratio of volumes, we must include the clearance; the volume back of the piston at cut-off is SB, while that at full stroke is ON. Then the real ratio of cut-off by the valve is

$$C_{VR}=\frac{SB}{ON}. \qquad (90)$$

All distances and volumes are expressed as fractions of the stroke or of the nominal cylinder-volume, represented by MN, which is taken as unity. If we let i stand for the clearance-fraction, or the ratio of OM to MN, the relation results,

$$C_{VR}=\frac{C_{VA}+i}{1+i}. \qquad (91)$$

In the figure, $C_{VA}=.35$ and $i=.10$; then $C_{VR}=.45\div1.10=.409$.

Finally, as the ratio of the initial volume of all the steam in the cylinder, when at boiler-pressure, to its final volume when the stroke is completed, we have the effective real cut-off

$$C_{ER}=\frac{JH}{ON}. \qquad (92)$$

(*e*) EXPANSION.—Of the expansion-curve BC little need be said here: the conditions affecting its shape have been set forth in general terms in § 15 (*e*); in detail, they form a subject of very

considerable magnitude, the treatment of which will be found in Part II. It may be remarked that the curve is seldom a true hyperbola, either falling below or rising above that curve, according to the conditions of working in the particular case.

(*f*) RELEASE.—The port is opened for exhaust before the stroke is completed, and the pressure falls along the release-line CD as the steam is discharged. In this case, about half the steam escapes from C to D, the rest from D to E. On account of this early release there is a loss of work represented by CLD. This loss is practically unavoidable, for if the valve were kept closed till the end of the stroke it could not be opened rapidly enough to avoid the dotted release-curve.

The same causes that produce a loss of pressure during admission are responsible for the excess back-pressure, which lifts the exhaust-line DE above PQ, and deducts another small portion from the area of the ideal diagram.

(*g*) COMPRESSION.—All the departures from the ideal diagram which we have so far considered are due simply to the difference between actual and ideal methods of handling the steam, or of getting it to and from the cylinder: that is, the steam must always lose some pressure in getting through the pipes and passages, and the valve cannot instantaneously open and close fully. But in the compression curve we encounter a change which is more fundamental. The area JGFEP, taken from the gross area JHLQP as the result of clearance and compression, does not represent loss of mechanical energy in the same way as do AGHB, CLD, and DEQ.

§ 19. Clearance and Compression.

(*a*) WORKING STEAM AND CLEARANCE STEAM.—A clear understanding of the effects of clearance and compression can be gained by thinking of the steam in the cylinder as made up of two parts—the live or working steam which enters the cylinder to do work and escapes after the work is done, and the dead or clearance steam which, as a certain quantity, remains in the cylinder indefinitely, alternately expanding and contracting as the pressure

changes. We can even imagine the latter body of steam to be separated from the working steam by a sort of light diaphragm, which will move back and forth in the cylinder like a loose auxiliary piston. When compression begins, at E in Fig. 31, this diaphragm is against the piston, and so remains up to F; then the piston stops, and the compression of the dead steam is continued along the curve FU by the fresh entering steam.

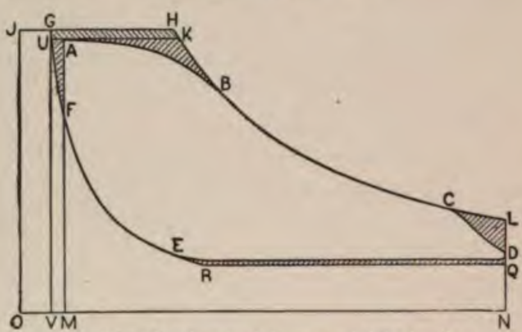


FIG. 31.—Mechanical Losses of Work-effect.

(b) FREE EXPANSION.—In all our discussions of expansive action, up to the end of the last chapter, we have assumed that the condition stated in the definition of external work in § 7 (a)—namely, that the external pressure shall be always equal to the internal stress—has been perfectly fulfilled: in other words, that a condition of static equilibrium has existed throughout the whole body of working substance. But when, as during the supplementary compression from F to U, steam flows through a contracted passage from a high-pressure chamber (the steam-chest) into one where the pressure is lower (the cylinder), there is a loss of effective work. The unbalanced part of the higher pressure accelerates the flow of the steam, acting against the inertia of the latter; so that ordinary pressure-work is transformed into kinetic energy of the swiftly moving current of steam. Then as the steam comes to rest in the cylinder, its velocity being dissipated in eddies, this mechanical, kinetic energy is transformed back into heat, from which form only a fraction can possibly be reconverted through a subsequent thermodynamic operation.

(c) KINETIC LOSSES.—In this particular case, the part VM of the clearance-space is filled with new steam, while the dead steam fills the part OV. In filling MV, the whole external pressure-work AMVU is expended; but only the part MFUV, actually stored up by the compression of the clearance-steam, is ready to be given back in the form of pressure-work upon the piston during the expansion. Then the area AFU represents the loss of available energy caused by incomplete compression.

The other losses shown by shaded areas in Fig. 31 are of this same character: the steam-pipe loss, GHKU, represents pressure that was used up in producing wasted velocities, either in the form of eddy-currents caused by the drag of the pipe-walls upon the steam-current, or of similar effects resulting from the passage of the steam around bends or through contractions. Likewise, AKB, CLD, and DEQ represent work which might have been utilized against the main resistance of the engine, but which actually was expended in moving the steam.

The whole subject of the kinetics of steam is fully discussed in the next chapter.

(d) EXPANSION OF THE CLEARANCE-STEAM.—Returning now to the main question of the effect of clearance and compression upon work-performance, we see that as soon as the pressure begins to fall, toward or after cut-off, the clearance-steam will expand with the working steam, following the curve UFE down to the lowest pressure: so that the effective volume of the active steam is to be measured, at any pressure, from this curve—or from the position of the imaginary diaphragm separating the two bodies of steam. Since this curve extends far to the right after it gets below the level of the release-point C, the result of this action is to diminish very considerably the amount of working expansion realized in the cylinder.

(e) EXPANSION OF THE WORKING STEAM.—This effect can best be shown by the method of Fig. 32, which is derived from Fig. 31, the reference-curve GFE being rectified and the volume-abscissas shifted horizontally to correspond. The performance of the clearance-steam is now shown by [illegible] JP, and can be left out of co[illegible]tion: and the e[illegible]ormance of the

working steam, freed from complications, is given by the distorted diagram ABCDEF. It appears that the true ratio of expansion is most fairly given by comparing the maximum volume RT with the initial volume GH.

In Fig. 30, the apparent effective cut-off ratio, GH to MN, is 0.25; the ratio of volumes, including clearance, JH to ON, is 0.318: but the true effective ratio, GH to RT in Fig. 32, is 0.354, or the

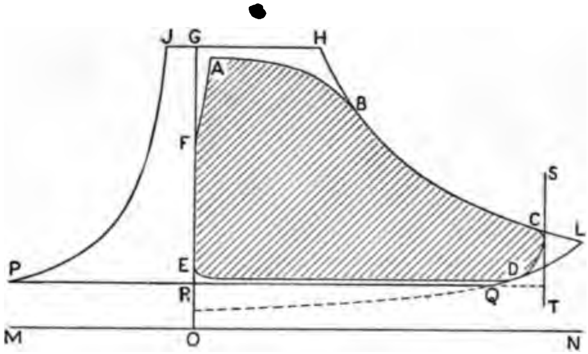


FIG. 32.—The Diagram of True Expansion.

ratio of expansion is 2.82. It is merely a coincidence that this happens to agree with the apparent cut-off by the valve, RB to MN in Fig. 30.

This method of changing the shape of the diagram so as entirely to eliminate the clearance-steam is especially useful when the several indicator diagrams from a compound engine are to be combined; because the weights of dead steam in the successive cylinders are likely to be quite different. With the simple engine, however, after we have used the illustration in Fig. 32 to gain a clear idea of the steam-action, we can go back to the actual diagram and get from it all the information given by the derived diagram.

That the curve HBL in Fig. 32 is a hyperbola referred to the axes OG and ON should be evident at sight; but a rigorous proof of this fact is as follows:

In Fig. 31, let the equation of the curve HBL be $pv=C$, and

that of GFR be $pv'=C'$: then for the difference of volumes we have

$$p(v-v')=C-C', \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

and this difference is the abscissa in Fig. 32.

(*f*) THE EFFECTIVE RATIO OF EXPANSION.—Returning to the primary steam diagram, we have illustrated first in Fig. 33 a method of applying the criterion just developed for determining the true ratio of expansion. The compression-hyperbola EF is transferred bodily over to the position ST, where it is tangent to the release-curve CD—this transfer being most easily made by means of a template of the curve. Then this curve is identical with the straight end-line ST of Fig. 32, all horizontal distances between it and the original curve EF being the same. The ratio of this distance ET to GH, which measures the volume of the working steam at boiler-pressure, is the degree of expansion effectively realized.

It might be considered better to locate the end-line ST in Fig. 32, or the curve ST in Fig. 33, in such a position that the area enclosed by this line, the expansion-curve produced, and the back-pressure line—a shape like the squared-off end of the ideal steam diagram of Fig. 29—would be equal to the area of the rounded nose of the actual diagram. This would give a more exact measure of the effective expansion; and in most cases the curve could be located and sketched in by eye with accuracy sufficient for practical purposes.

With the diagram fastened down on the drawing-board and the scale kept horizontal by the T-square, the maximum distance between the compression-curve FE and the release-curve CD can be measured by trial, without going to the trouble of drawing the curve ST.

(*g*) THE MOST ADVANTAGEOUS COMPRESSION.—Fig. 33 illustrates also a method of finding the amount of compression that will give the highest ratio of work done to steam used for a particular set of conditions. Having the clearance fixed, and assuming that the line ABCD is not changed by varying the compression, we draw a number of different compression-hyperbolas, covering the range from zero up to full compression—the latter term meaning, com-

pression up to the admission-pressure. Then for each case the work-area ABCDEFA is measured with a planimeter and the ratio found which it bears to the steam-volume at boiler-pressure, GH. From the figure, as drawn, the following values were found:

Case No.......	1	2	3	4	5
Work area, square inches...	15.05	16.08	16.96	17.60	17.85
Steam vol., GH, inches.....	2.62	2.77	2.92	3.07	3.21
Ratio....................	5.75	5.80	5.81	5.73	5.56

Laying off these ratio-values to scale from JP as a base-line, we get the curve IK; and it appears that the maximum of efficiency will be secured when the compression is about midway between curves 2 and 3 in the figure, rising to the height R.

This graphical method is decidedly preferable to an attempt to get a general mathematical expression for the ratio of work done to steam used: such an expression would be very complicated, and could be made only approximate at best.

It must be borne in mind that certain minor influences are here neglected. With any actual valve-gear, changing the time when compression begins will also modify, more or less, the shape of the release-curve. Further, it is likely that the amount of cylinder-condensation will vary slightly with the compression, so that the steam used will not always be measured to quite the same scale by the intercept GH. But these influences will have only a secondary effect upon the shape of the curve IK.

(*h*) GENERAL CONSIDERATIONS AS TO CLEARANCE.—The two evils due to extreme conditions, between which a middle course must be chosen in order to get the best result, are as follows:

Low compression leads to too great a loss due to filling the clearance-space, represented by the area between AF and GF in Fig. 33.

Too high compression, by reducing the effective ratio of expansion, cuts away work-area at the other end of the diagram. It goes without saying that the compression should not be carried above the admission-pressure—which sometimes happens when the valve is not properly proportioned.

A little study of Fig. 33 will show that as the cut-off becomes earlier, and the release-pressure at C lower, the compression should be greater; but as C rises with late cut-off, the amount of compression should diminish. In the latter case not only does the greater height of C increase the loss of area due to a shortening of the effective length of the diagram by high compression; but, on the other hand, an absolute increase in the waste-area AFG may not signify a relative increase, when we consider the larger quantity of steam going through the cylinder per stroke at late cut-off.

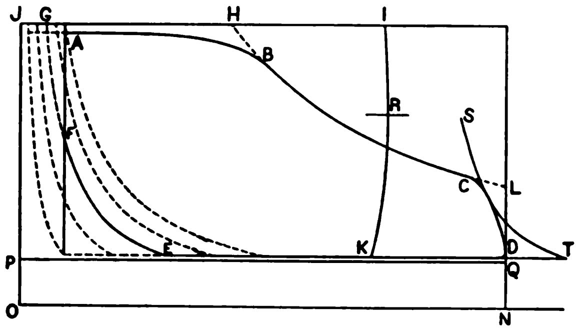


FIG. 33.—The Effect of Compression.

With the general statement that the single slide-valve gives just this desired compensating effect, increasing the compression as the cut-off is made earlier, we will drop this matter until the subject of valve-action is taken up.

(*i*) MEASURING THE CLEARANCE.—The clearance-volume in any engine can either be calculated from measured dimensions or found by an experimental method which consists in filling the space with water and measuring the amount required. It can also be estimated from the diagram, on the assumption that the compression-curve is an equilateral hyperbola. One way is to reverse the construction given in Fig. 14, choosing two points A and B on the curve as in Fig. 34 I., drawing the rectangle ACBD, and finding where the diagonal CD cuts the vacuum-line. A property of the hyperbola which gives a rather more convenient method is, that if a secant line EBAF be drawn across the curve and its axes, the intercepts BE and AF will be equal.

The application of the first construction to an actual indicator diagram—taken from Fig. 37—is illustrated in Fig. 34 II.: it appears that the determination is not very definite, so that it must be supplemented by a knowledge of probable proportions and by the use of good judgment. Referring to Fig. 37, we see

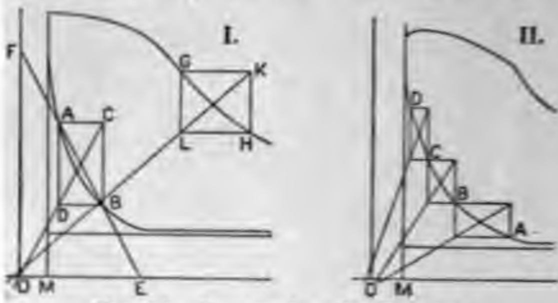


FIG. 34.—Constructions for Clearance.

that the true hyperbola drawn from the origin here found agrees quite well with the compression-curve.

The construction for clearance might be applied to the expansion-curve, as shown at I.; but it is evident that, the farther the rectangle GH is from the origin, the greater will be the effect of a given deflection of the radial KL; so that, aside from any question as to the character of the curves, it is better to use that which is nearer the axes.

As to the value of the clearance-fraction, the statement that for engines of the Corliss type it ranges from .02 to .06, while in high-speed, slide-valve engines the range is from .06 to .12, is well justified by practice: in some extreme cases the clearance is as much as 20 per cent. of the nominal cylinder-volume.

§ 20. Work Done and Steam Used.

(a) MEAN EFFECTIVE PRESSURE.—In determining by means of the indicator diagram the work done or the power developed by the steam in the engine, we do not use the diagram as a direct measure of work, through a work-scale like that illustrated in Fig. 18; but follow rather the simpler process suggested by Eq. (86).

Considering the diagram in Fig. 35, we have that during the forward stroke, from M to N, the steam does upon the piston an amount of work represented by the area ABCDNM; reducing this figure to an equivalent rectangle, we get the mean total pressure p_{mf} or GM. Similarly, during the return stroke, the work done upon the steam by the piston is DEFMN, and the corresponding mean back-pressure is p_{mb} or MK. Then the mean

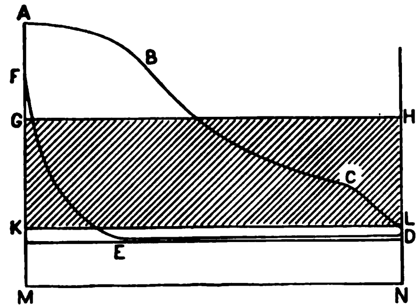


FIG. 35.—Gross and Effective Work.

effective pressure, or the mean intercepted ordinate of the figure ABCDEF, is GK, or

$$p_m = p_{mf} - p_{mb}. \qquad (94)$$

In practice, we do not derive the M.E.P. from p_{mf} and p_{mb}, but find it directly by measuring the area of the enclosed diagram, dividing by the length so as to get the mean height, then multiplying this mean height by the pressure-scale.

(b) WORK PER REVOLUTION.—Although p_m actually represents the difference between the two quantities of work done upon the piston in the two strokes which make up one revolution (one work is positive, the other negative), we treat it as if it were simply an unbalanced pressure acting upon the piston through the forward stroke. Then if A is the area of the piston in square inches and S the length of the stroke in inches, we have for the work done in one end of the cylinder per revolution

$$U = p_m A \frac{S}{12} \text{ F.P.} \qquad (95)$$

In the other end of the cylinder there is done a similar amount of effective work, but generally not quite the same: how nearly alike the two M.E.P.'s will be depends upon valve-action, and they may be very different if the valve-gear is in bad adjustment: further, the area of the piston is reduced on one side by the cross-section of the piston-rod, so that generally the two A's are not the same. In any case, the sum of the two separate U's gives the total work per revolution.

Note that the use of the ordinary M.E.P. does not give the work per stroke: to get this, we should have to subtract from the forward-pressure work on one face of the piston the simultaneous back-pressure work on the other face. But while the separate works per stroke would not, in general, be the same as the works in the two cylinder-ends, the sum of either two would give the same total work.

(c) HORSE-POWER.—Letting N be the number of revolutions or of double-strokes per minute, we have for the indicated horse-power developed in one end of the cylinder

$$H=\frac{p_m ASN}{12\times 33000}. \quad \quad (96)$$

Having found the several partial I.H.P.'s, as H_1, H_2, etc., we get the total power by taking their sum.

The constant part of this formula,

$$\text{E.C.}=\frac{AS}{12\times 33000}, \quad \quad (97)$$

which we call the engine constant, can be worked out once for all in any particular case and kept on record. It can most conveniently be found with the help of Table VI.

We distinguish two measures of the power of an engine: first, the indicated power, or the rate of work-performance by the steam in the cylinder, as shown by the indicator; second, the effective power delivered by the machine, after its own internal frictional resistances have been overcome. The ratio of effective to indicated power is the mechanical efficiency of the engine.

(*d*) Measurement of Steam Used.—To pass from work per revolution or I.H.P., which is purely a measure of mechanical performance, to the thermodynamic question as to how much work is done by one pound of steam or, conversely, how many pounds of steam are required per horse-power-hour, we must have a measure of the amount of steam used by the engine per stroke or per hour. This can be determined actually and exactly only by a test in which the steam is weighed (or measured) as water, either as feed-water before it goes into the boiler, or after it has been brought back to water by means of a surface-condenser. The former method is available when the output of one or more boilers can be wholly devoted to the engine tested; and is liable to error caused by insufficiently accurate determination of the amount of water in the boiler at the beginning and at the end of the test. The second method gives the surest results, but calls for a supply and arrangement of apparatus which is not usually available except in a special laboratory or testing plant. In any case, a test of this sort involves a great deal of work, and some easier way of at least estimating pretty closely the steam-consumption of the engine is very desirable. A method of doing this, foreshadowed in Fig. 29 and § 17 (*c*), will now be fully developed.

(*e*) Indicated Steam-consumption.—From the indicator diagram we can get a close measurement of the "steam" that is

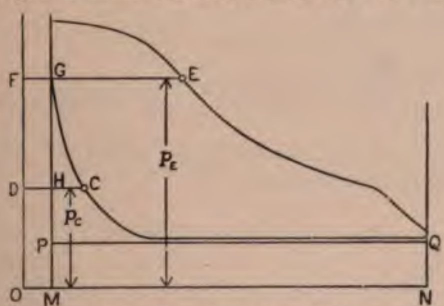


Fig. 36.—Data for I.S.C.

present in the cylinder as steam—in distinction from that part of the working medium which goes through the engine as water, on account of cylinder-condensation or of initial moisture in the steam.

Corresponding to any point on the expansion- or compression-curve, there is in the cylinder a definite volume of steam whose pressure is known, so that its weight per cu. ft. is determinable from the Steam-table. Suppose that we wish to find the indicated or apparent steam-consumption shown by the diagram in Fig. 36. For a point E on the expansion-curve, we have that the space filled with steam of the pressure p_E and specific weight d_E is made up of two parts—the volume back of the piston from its initial position, or GE, and the clearance-volume FG: expressing this whole volume as a fraction of the piston-displacement in one stroke, represented by MN, we use

$$\frac{\mathrm{FE}}{\mathrm{MN}}=e.$$

Then the volume out to E is

$$v_E=eV,$$

and the weight of saturated steam in the cylinder is

$$w_E=Ved_E.$$

This measures the total steam, both working-steam and clearance-steam: for the latter alone, taking a point C on the compression-curve, we have

$$\frac{\mathrm{DC}}{\mathrm{MN}}=c, \quad v_C=cV, \quad w_C=Vcd_C.$$

Then for the working-steam, or the weight of steam apparently used per revolution, we get

$$w=V(ed_E-cd_C). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

Letting K stand for the expression in parenthesis, this formula becomes

$$w=KV; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

and we see that K may be defined as the indicated steam-consumption per cubic foot of piston-displacement.

The simplest case of this determination is encountered when the points E and C lie on the same horizontal line, so that $d_E = d_C$; for then

$$K = (e-c)d_E \quad \text{and} \quad (e-c) = \frac{\mathrm{EC}}{\mathrm{MN}}. \quad . \quad . \quad . \quad (100)$$

(*f*) I. S. C. Per Hour and per Horse-power-hour.—Now the piston makes 60 N out-strokes in one hour, N being the num ber of revolutions per minute, or the R.P.M.; then the I.S.C. in pounds per hour is

$$W = 60KNV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

This is for one end of the cylinder, or for one diagram: a similar value would be found for the other end, and the sum of the two would give the total steam shown by the indicator.

If we let V_P stand for the total piston-displacement in cubic feet per hour in both directions, so that

$$V_P = 60N(V_1 + V_2), \quad . \quad . \quad . \quad . \quad . \quad (102)$$

and find a mean value of K from the two diagrams, then the simplest way to get the I.S.C. per hour, total, is to substitute in

$$W = K_m V_P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

In terms of stroke and piston-area,

$$V = \frac{AS}{1728} \quad \text{and} \quad W = K\frac{ASN}{28.8}$$

from (101); dividing this by the formula for I.H.P. in (96), we get

$$W_H = \frac{W}{H} = \frac{13750}{p_m}K, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where W_H stands for I.S.C. per H.P.H.

This method is generally used where isolated diagrams are worked up, while that of (101) and (103) is better for long tests with uniform conditions, especially where the actual steam-consumption is also being measured.

Note that when two values of W_H have been found from a pair of diagrams, their *mean*, not their sum, is to be taken to get a

result for the whole engine; because each value is the ratio of the steam passing through one end to the work done by that steam.

The point E may be located anywhere along the expansion-curve, but is usually taken either just after cut-off or just before release. On account of re-evaporation during expansion, the latter position will usually give a larger I.S.C. than the former.

(*g*) WORKING UP INDICATOR DIAGRAMS.—The application of the methods developed in this and the two preceding sections will now be illustrated by the complete work-up of a pair of diagrams, shown in Fig. 37.

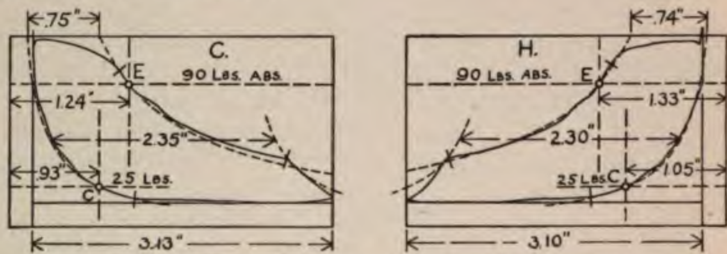


FIG. 37.—Diagrams from a High-speed Steam-engine.

The fundamental data from the engine are:

Diameter of cylinder.	14″
Stroke of piston.	14″
Diameter of piston-rod.	2.25″

These are more concisely stated by giving the size of the engine as $14\times14-2.25$, diameter of piston being always given first.

The derived constants are:

Piston-area, $\begin{cases}\text{head end } 153.9 \text{ sq. ins.}\\ \text{crank end } 150.0 \text{ " "}\end{cases}$

Engine constant, $\begin{cases}\text{head end } .005446\\ \text{crank end } .005303\end{cases}$ see (97).

From the diagrams, by the method of Fig. 34, the clearance was found to be about 8 per cent.; and this exact value is used, no direct determination having been made.

The three important valve-action "events," cut-off, release, and exhaust-closure, are marked by short cross-lines, as in Fig. 10.

Then the position of the piston for each of these events, expressed by a stroke-fraction estimated from the zero end (as distinguished from the full-stroke end) of each diagram, is given by the following table:

TABLE 20 A. LOCATION OF VALVE EVENTS.

Event.	Cut-off.	Release.	Compression.
Head end	.31	.86	.37
Crank end	.29	.85	.36

The diagrams were taken with a spring of 60-scale in the indicator, so that the scale of the ordinate is 60 lbs. per sq. in. to the inch. In precise work the indicator-spring must be calibrated by comparison with some standard of pressure, and the exact scale thus found is then used in the calculation of results; here, however, the nominal scale will be used.

The general observations particular to this set of diagrams were:

Steam-pressure .... 105 lbs. by gage.
Speed of engine .... 225 R.P.M.

All the reference lines given in Fig. 30 are drawn on both diagrams; further, hyperbolas are passed through selected points on both the expansion- and the compression-curves, for comparison with these curves. Then the method of Fig. 33 is used to get the realized effective ratio of expansion, which comes out:

$$\text{Head, } \frac{2.30}{.74}=3.08; \quad \text{Crank, } \frac{2.35}{.75}=3.13.$$

The manner of calculating the horse-power is outlined in the following table:

TABLE 20 B. CALCULATION OF I.H.P.

Cylinder End.	Area Diagram.	Length Diagram.	Mean Height.	M.E.P.	I.H.P.	
					Ends.	Total.
H	2.50	3.10	.806	48.35	59.25	116.25
C	2.40	3.13	.796	47.70	57.00	

Using the E.C., the I.H.P. for one end is got by multiplying this constant by the R.P.M. and by the M.E.P., as indicated by (96); thus for head end,

$$H = .005446 \times 225 \times 48.35 = 59.25.$$

To get the I.S.C. we use the points through which the hyperbolas are drawn; from the dimensions marked on the figures the following table of intermediate and final results is worked out.

TABLE 20 C. CALCULATION OF I.S.C.

Cylinder End.	e.	c.	d_E. d_C.	ed_E.	cd_C.
H.............	.429	.339	.2064	.08853	.02101
C.............	.396	.297	.0621	.08170	.01845

Cylinder End.	K.	$\frac{13750}{\text{M.E.P.}}$	I.S.C. Ends.	I.S.C. Mean.
H.............	.06742	284.5	19.16	18.68
C.............	.06325	288.0	18.20	

Here formula (102) is used after K has been found, and the result is in pounds per H.P.H. To get the total steam per hour we can combine the final results, so that

$$W = 116.25 \times 18.68 = 2171.$$

Or, after finding the K's, we compute:

		Head.	Crank.	
Cylinder-volume,	V =	1.246	1.215	cu. ft.
Piston-displacement,	V_P =	16820	16400	per hour.

$$\text{Then } W = \left\{ \begin{array}{l} \text{Head } 16820 \times .06742 = 1134 \\ \text{Crank } 16400 \times .06325 = 1037 \end{array} \right\} = 2171 \text{ lbs.}$$

Note: The slide-rule is especially adapted to computations based upon data of the degree of accuracy here attainable.

§ 21. Estimating Cylinder-condensation.

(*a*) FORMULA FOR FRACTION OF STEAM CONDENSED.—From a study of the published records of a large number of engine tests, the writer has devised the formula given below, by means of which the fraction of the total steam that is not shown by the indicator can be pretty closely estimated: this fraction corresponds to m_1 in Eq. (83). The formula is rational as to the elements involved, but empirical as to the amount of influence which each exerts: as here set forth it is limited to cylinders which are not steam-jacketed; and it is justified only to the degree in which results found by it agree with the data upon which it is founded.

The formula is

$$m = \frac{.27}{\sqrt[3]{N}}\sqrt{\frac{sT}{pe}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (105)$$

where

N = R.P.M. of engine;

s = a constant obtained by dividing the surface of the nominal cylinder in square feet by its volume in cubic feet;

T = a special function of the pressure, analogous to temperature, shown in Fig. 38 and given in Table 21 A;

p is the absolute pressure at the point E, Fig. 36, for which the I.S.C. is found, and

e has the same meaning as in § 20 (*e*), being the ratio of the volume at E, Fig. 36, to the nominal cylinder volume.

(*b*) INFLUENCES AFFECTING CYLINDER-CONDENSATION.—There are three major influences to be considered; these are:

1. The amount of metal surface exposed per unit weight of steam present, proportional to s/pe.

2. The total range of temperature in the cylinder, from admission to exhaust, represented by T.

3. The time of the whole operation, proportional to $1/N$.

(*c*) THE SURFACE EFFECT.—Of the total interior surface of the cylinder, that part which may be called the clearance-surface—

including the cylinder-head and the piston-face, with the steam-passages—is what is chiefly active in causing condensation. If we may assume that the cylinder-bore is of secondary importance, then the effective surface, to which is exposed the steam admitted in any one stroke, is practically constant. As a convenient and fairly approximate measure of this surface, we may take it to be proportional to the inside surface of the nominal cylinder, or to S' from

$$S' = 2\frac{\pi}{4}\frac{D^2}{144} + \pi\frac{DS}{144}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

where S and D are the stroke and diameter in inches, and S' is in square feet.

Of course, variation in the ratio of stroke to diameter, in the type of valve-arrangement, and in the shape and size of the steam-passages will affect the validity of this measure. But for the general run of slide-valve engines the variation is not great; and in the case of engines of the Corliss type, the smallness of the port-surface is neutralized by the large amount of surface around the exhaust-valve, which has a relatively high condensing effect. The range of data available is less complete than might be desired; but so far as can be seen, this rough assumption as to the size-effect seems to be well justified.

(*d*) THE RELATIVE SURFACE EXPOSED.—The weight of steam apparently present in the cylinder at cut-off is that required to fill the fraction e of the nominal volume V at the pressure p: it is got by dividing the actual volume eV by the specific steam-volume v. Since p and v (using v instead of s in Eq. (61)) are related by the equation $pv^{1.065} = C$, not greatly different from $pv = C$, we may take v to be approximately proportional to $1 \div p$, and get

$$\frac{eV}{v} \propto peV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

Now for a quantity proportional to the surface exposed per pound of steam we have

$$S' \div peV = \frac{S'}{V}\frac{1}{pe} = \frac{s}{pe}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

The constant $s=S'\div V$ may be called the specific surface of the cylinder, or its surface per cubic foot of volume. Dividing (106) by V, we get

$$s=\frac{\pi}{144}D\left(\frac{D}{2}+S\right)\div\frac{\pi}{4}\frac{D^2S}{1728}$$

$$=\left(\frac{D}{2}+S\right)\frac{4\times 12}{DS}=\frac{12}{D}\left(2\frac{D}{S}+4\right). \quad . \quad . \quad . \quad (109)$$

In (108) we find the ratio of the cylinder-surface to the indicated steam: it would be more logical to use the total steam; but this could be done only by greatly complicating the formula, or by using the trial method, with successive substitutions. This formula works quite well if it is not carried to the extreme of very early cut-off, where the condensation becomes relatively so large that a close prediction of its amount can hardly be expected.

(*d*) Temperature and Time Effects.—It is apparent that the amount of heat-interchange between steam and cylinder-walls will depend upon the range of temperature in the cylinder. But it was found that the very large increase in range caused by dropping to condenser temperatures exerted too great an influence in the formula; while in the first cylinder of a multiple-expansion engine the range was too small to account for the condensation. To use the range of pressure would give an error in just the opposite direction, making the moisture figure out too small with condensing engines, too large with compounds.

To get around this difficulty, at the same time avoiding any complex mathematical expression in terms of temperature, the artificial function T was laid out by trial; this is shown in Fig. 38, plotted on p as a base, with the temperature-curve (the same as Curve I. in Fig. 21) dotted in for comparison. To get T for the formula, look up T_1 and T_2, corresponding to the highest and lowest pressures in the engine, then

$$T=T_1-T_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

Do not confuse this T with the absolute temperature.

Again, the time permitted for the whole operation of the cycle will have an influence upon the condensation, diminishing both

the impartation of heat to the metal during admission, and the cooling of the cylinder by the exhaust steam. The time-effect was found to be well represented by using the cube-root of N.

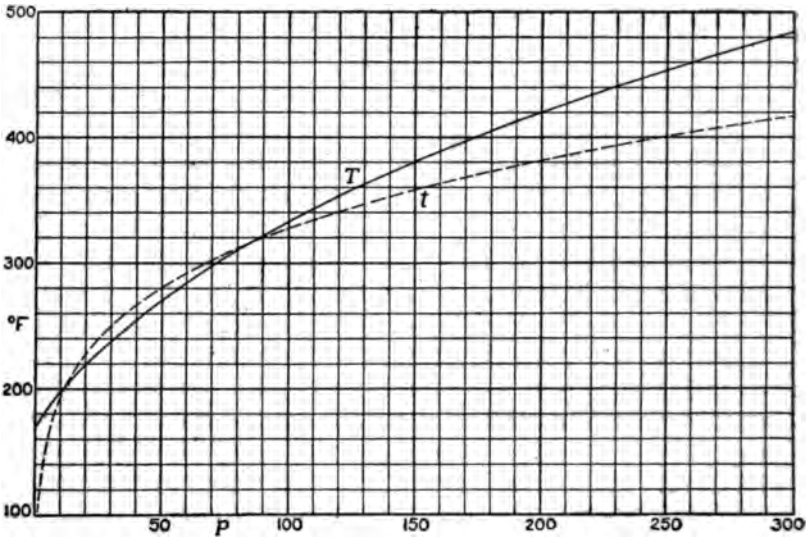


FIG. 38.—The Temperature Function T.

TABLE 21 A. VALUES OF T FOR EQUATION (105).

p	T	p	T	p	T	p	T
0	170	45	262	115	348	185	409
1	175	50	269.5	120	353	190	413
2	179	55	277	125	358	195	416.5
3	183	60	284	130	362.5	200	420
4	186	65	291	135	367	210	427
6	191	70	297.5	140	371.5	220	434
8	196	75	304	145	376	230	441
10	200	80	310	150	380.5	240	447.5
15	210	85	316	155	385	250	454
20	220	90	321.5	160	389	260	460.5
25	229	95	327	165	393	270	467
30	238	100	332.5	170	397	280	473
35	246	105	338	175	401	290	479
40	254	110	343	180	405	300	485

p is absolute pressure in pounds per square inch.
T is the temperative function in degrees F.

EXAMPLE.—Applying this formula to the diagrams worked up in the last section, we get

$$s=\frac{12}{14}(2+4)=5.14;$$

$$\begin{array}{lll} N=225; & \sqrt[3]{N}=6.08; & \\ p_1=102; & T_1=334.7; & \\ p_2=15; & T_2=210.0; & T=124.7; \\ p=90; & \text{mean value of } e=.412. & \end{array}$$

Then

$$m=\frac{.27}{6.08}\sqrt{\frac{5.14\times 124.7}{90\times .412}}=.185.$$

Now, of the total steam, the I.S.C. represents the fraction $(1-m)$: then for this case we have $18.68\div .815=22.9$ lbs. per H.P.H. as the probable actual steam-consumption.

(*e*) RESULTS FROM ENGINE TESTS.—In Table 21 B are given results from a number of different tests, selected partly with a view of giving some idea of the performance of engines, chiefly to exhibit the application of the cylinder-condensation formula. The symbols at the heads of the columns have the following meanings:

2 Des. = designation of the particular test in the original published table of results; most of the examples here given are taken from series of tests of the particular engine.

3, 4, diameter and stroke of the engine-piston.

5 N = R.P.M. of engine.

6 P = pressure of steam before it gets to engine, taken either at the boiler or, where so determined, from the steam-pipe.

7 p_1 = highest pressure in cylinder during admission.

8 p_2 = lowest back-pressure during exhaust.

(All these are absolute pressures.)

9 e_A = apparent cut-off, corresponding to C_{VA} in § 18 (*d*).

10 p_m = M.E.P., average for the two ends.

11 H = I.H.P. of engine.

12 S = measured steam-consumption, in pounds per H.P.H. The letter F means that the steam was measured as feed-water, C that it was weighed after condensation.

TABLE 21 B. RESULTS FROM ENGINE TESTS AND

1	2	3	4	5	6	7	8	9	10	11
No.	Des.	Diam.	Str.	N	P	p_1	p_2	e_A	p_m	H
1	D	58	105	6.6	——	49.0	2.0	.38	34.4	317
2	B	58	105	6.6	——	40.6	1.3	.32	25.0	229
3	5	36	96	13.9	——	34.3	2.7	.25	17.4	118
4	E	40	72	39.6	64.9	——	5.7	.29	25.9	406
5	J	34	60	59.9	97.0	——	1.1	.17	37.2	608
6	C	32	60	58.6	81.0	——	2.9	.32	40.0	567
7	F	"	"	59.1	84.8	——	3.1	.27	38.3	548
8	I	20	48	61.7	99.4	——	2.6	.18	41.8	195
9	L	20	48	60.3	81.9	——	2.8	.14	22.8	104
10	D	28	60	64.8	115.7	——	18.9	.32	41.2	493
11	A	23	60	74.7	87.0	——	17.5	.37	33.1	309
12	H	28	48	53.2	84.3	——	20.6	.18	17.7	139
13	B	12	36	72.7	105.1	——	17.0	.34	42.2	73.5
14	1	17	24	45.6	144	119	15.4	.19	29.0	128
15	7	"	"	71.4	143	133	16.2	.19	31.0	212
16	13	"	"	43.8	145	141	16.2	.27	52.6	223
17	20	Duplex		47.6	139	129	19.5	.47	65.4	453
18	9	17	30	61.9	104.9	98.9	16	.226	44.6	91.6
19	11	"	"	8.6	107.0	107.0	16	.271	58.0	16.6
20	15	"	"	86.0	106.6	94.3	16	.107	26.3	75.1
21	28	"	"	8.6	108.4	108.4	16	.153	42.2	12.1
22	7	16	18	40.3	——	86.5	2.5	.114	35.5	51.0
23	[illegible]	Duplex		40.3	——	91.6	16.7	.114	25.1	36.0
24	[illegible]	9	36	85.4	112	108	6.0	.195	54.7	52.4
25	[illegible]	"	"	85.6	120	116	16.8	.141	40.4	57.7
26	[illegible]	"	"	84.7	129	125	29.8	.268	47.9	88.6
27		11½	20	126	135	124	17.5	.177	38.5	80.3
28	[illegible]	8	24	61.1	85.7	79.0	16	.053	10.6	3.9
29	[illegible]	"	"	59.4	86.7	84.1	16	.200	28.2	10.0
30	[illegible]	11	6	408	65.5	50.7	14.5	.437	22.6	19.8
31	[illegible]	"	"	406	136.7	106.3	15.0	.216	38.4	33.6
32	[illegible]	"	"	138	120.1	108.7	15.5	.216	44.3	13.1
33	[illegible]	10	6	405	119.3	97.8	48.6	.470	33.0	28.7
34	[illegible]	"	"	402	148.4	130.0	51.0	.362	38.6	33.3
35	[illegible]	"	"	212	119.6	109.1	50.2	.470	40.2	18.3
36	[illegible]	7	6	400	166.4	149.7	103.4	.647	39.5	34.6

APPLICATION OF CONDENSATION FORMULA. TABLE 21 B.

12 *S*	13 *S'*	14 *s*	15 *t*	16 *T*	17 *p*	18 *e*	19 *m*	20 *m'*	
30.3 F	20.9	1.06	154	89.0	45.0	.46 C	.312 R	.306	1
30.6 "	19.3	1.06	158	78.8	36.1	.40 "	.374 "	.344	2
34.5 "	20.8	1.60	121	63.1	33.3	.33 "	.396 "	.372	3
24.3 F	19.2	1.50	130	99.5	65	.31 R	.21 C	.215	4
18.5 "	12.4	1.81	220	147	97	.19 "	.33 "	.263	5
19.9 "	16.1	1.90	173	125	81	.34 "	.19 "	.205	6
19.5 "	15.2	"	174	130	85	.29 "	.22 "	.219	7
19.2 "	14.2	2.90	192	147	99	.20 "	.26 "	.316	8
21.2 "	13.8	2.90	176	127	82	.16 "	.35 "	.366	9
25.8 F	21.2	2.11	113	131	116	.33 R	.18 C	.181	10
27.8 "	23.4	2.49	97	103	87	.38 "	.16 "	.179	11
34.0 "	23.8	2.21	86	94	84	.20 "	.30 "	.253	12
29.3 "	20.8	4.67	112	127	105	.36 "	.29 "	.253	13
30.2 F	21.4	3.83	124	133	100.4	.285 C	.301 C	.292	14
29.0 "	21.2	"	127	145	98.8	.285 "	.271 "	.259	15
26.8 "	20.3	"	132	152	115.2	.368 "	.242 "	.259	16
28.3 "	24.0	"	118	140	102.4	.566 "	.151 "	.204	17
27.5 C	20.2	3.62	111	120	92.8	.297 C	.267 C	.272	18
36.1 "	19.8	"	116	128	104.5	.342 "	.450 "	.474	19
28.0 "	19.0	"	107	114	89.0	.178 "	.320 "	.313	20
38.7 "	22.0	"	117	129	106.9	.224 "	.568 "	.583	21
34.7 F	19.4	4.33	183	137	86.5	.184 C	.441 R	.480	22
42.6 "	29.7	"	107	110	91.6	.184 "	.304 "	.418	23
23.6 C	15.0	6.00	163	150	103.1	.271 C	.365 C	.347	24
16.4 "	10.1	"	124	135	112.0	.218 "	.385 "	.354	25
15.3 "	11.2	"	94	120	119.5	.345 "	.268 "	.257	26
25.6 F	19.4	4.51	123	142	115.0	.294 C	.243—	.235	27
57.9 C	18.9	7.0	95	97	70.4	.093 "	.674 C	.692	28
37.5 "	19.1	"	99	103	70.4	.240 "	.491 "	.452	29
36.0 F	29.0	7.5	70	63.7	42.8	.497 C	.193 C	.173	30
26.0 "	18.3	"	119	129	87.1	.276 "	.296 "	.232	31
31.2 "	17.3	"	119	130	93.1	.276 "	.445 "	.322	32
22.8 F	20.5	9.0	46	65.4	88.3	.570 C	.102 C	.125	33
20.3 "	17.4	"	65	92.2	112.2	.462 "	.143 "	.147	34
23.1 "	18.4	"	53	72.3	92.7	.570 "	.203 "	.159	35
19.4 F	18.3	11.0	38	44.3	146.1	.772 C	.055 C	.076	36

13 S' = I.S.C. in same terms as S, determined here from S through the moisture-fraction m in Col. 19.

14 s = surface-coefficient of engine, by (109).

15 t = range of temperature in cylinder, given for comparison with T.

16 T = range of the artificial temperature of Fig. 38 and Table 21 A, according to Eq. (110).

17 p = absolute pressure at cut-off.

18 e = ratio of volume at cut-off to nominal cylinder-volume, as in (105) and in § 20 (e). Where marked C, e has this meaning, being equal to e_A in Col. 9 + fraction of clearance: but where marked R, e is the reciprocal of what is given as the ratio of expansion, the data being in that form.

19 m = fraction of cylinder-condensation, or of steam not shown by the indicator, from actual measurement. This result is marked C where the I.S.C. was taken at cut-off, R where it was taken at release; the former giving a result properly comparable with m'.

20 m' = fraction of cylinder-condensation, computed from Eq. (105).

The clearances of these engines are as follows:

Nos 1, 2, 3	.08	No. 27	.117
4–13	not given	28–29	.04
14–17	.099	30–32	.06
18–21	.071	33–35	.10
22–23	.07	36	.125
24–26	.076		

(f) REFERENCES.—The sources of the data set forth in Table 21 B will now be given.

Nos. 1–3. These are selected from tests made on three U. S. gunboats, in the early 1860's, by boards of U. S. Naval Engineers, and published by Isherwood in his "Experimental Researches in Steam Engineering." No. 1 is one of seven tests made on the *Mackinaw*, No. 2 is selected from the fifteen tests on the *Eutaw*, and No. 3 is one of seven on the *Michigan*. These were all paddle-

wheel steamers, and the tests were made with the vessels fast to the dock and simply pushing water backward. These examples represent the early type of marine practice, with large and slow-moving simple engines, using low-pressure steam.

Nos. 4–13 are a group of isolated tests of factory engines of the Corliss type, made by Mr. Geo. H. Barrus, reported to the American Society of Mechanical Engineers in 1889, and printed in Vol. XI. of the Transactions, page 170: only two, Nos. 6 and 7, are from the same engine. The data are not so full for these tests as for most of those available, but they give valuable information. In subdivision, the first group is made up of condensing engines, the second of non-condensing.

Nos. 14–17 are from the first locomotive installed for experimental purposes at Purdue University, and were reported to the A.S.M.E. by Prof. W. F. M. Goss in 1893, Trans., Vol. XIV., 826. This was, of course, a duplex engine. The tests given are those in which there was the least drop of pressure from the boiler to the cylinder, but it will be noted that even in these the throttling effect is considerable.

Nos. 18–21 represent a long series of tests made by Profs. J. E. Denton and D. S. Jacobus on the steam end of an air-compressor, in Nov., 1888, with especial view to the determination of cylinder-condensation. The engine had a double-slide valve-gear; and this, with the easily variable and controlled load, facilitated the covering of a wide range of conditions. See Trans. A.S.M.E., Vol. X., 722.

Nos. 22 and 23 are from a set of sixteen tests made by Major T. English on a two-cylinder simple engine driving pumps. The full report, with a complete thermal analysis, is published in the Proceedings of the British Institution of Mechanical Engineers for 1887, at page 478.

Nos. 24–26 are sample tests from the triple-expansion Corliss engine in the laboratory of Sibley College, Cornell University, selected from an extensive report of results made by Prof. R. C. Carpenter in 1895, and published in Trans. A.S.M.E., Vol. XVI., 913. The tests here given are all from the high-pressure cylinder, and represent three conditions of running: No. 24 is for the high-

pressure cylinder as a simple engine; for 25, the high and intermediate were run as a two-stage compound; and for No. 26 the whole engine was tested. The M.E.P. in Col. 10 is for the H.P. cylinder in all three cases.

No. 27 is an isolated test of a Hoadly portable engine, made in 1876, and described in Peabody's Thermodynamics of the Steam Engine, page 266.

Nos. 28 and 29 are from a small Corliss engine at the Massachusetts Institute of Technology, also taken from Peabody's Thermodynamics, page 386.

Nos. 30–36 are selected tests of the Willans central-valve engine, taken from the exhaustive report made by Mr. P. W. Willans, published in the Proceedings of the British Institution of Civil Engineers, 1888, Vol. XCIII., 128. This is a triple-expansion engine of peculiar design; and the examples given represent three different conditions of running. Nos. 30 to 32 are from the low-pressure cylinder run as a simple engine; for 33 to 35 the intermediate and low constituted a plain compound; while the last test is from the triple engine. Here the M.E.P. given is for the whole engine, "reduced" to the low-pressure piston, by a method which will be developed in § 23.

(*g*) DISCUSSION OF TABLE 21 B.—In this Table, the important controlling conditions are given in Cols. 5, 6 or 7, 8 and 9, and the most important results of the tests in Cols. 11 and 12; the question of prime interest here being, How many pounds of steam were used per horse-power-hour? This information will enable the reader to form some quantitative idea of the economic performance of engines of the classes here represented. It will be noted that the lowest value of S given is about 15, while the best that has been attained in an engine using saturated steam is about 11. But this triple-expansion engine (No. 26) is of small size, and was not run under its conditions of highest efficiency, chiefly in that the steam-jackets were out of use. The object of this Table being especially to illustrate the application of Formula (105), the choice of data was somewhat limited.

A large amount of information along these lines, including the full tables of results from which a few examples are here abstracted,

will be collated and discussed in Part II., where the matter of the actual performance of engines will be very fully gone into.

The range of conditions covered in any particular group of tests given in this Table is too small for a critical examination of Eq. (105). But so far as the conditions which vary with the engine are concerned, there seems to be a quite fair agreement between m and m'.

It must be understood this formula cannot be expected to give precise results. The elements involved are so numerous, and there are so many secondary, unaccountable influences, that the actual performance of engines is likely to show rather freakish small variations from any uniform law: and besides this, there is room for a good many small errors in engine tests, which in some cases are likely to accumulate in one direction. Further, individual peculiarities of the engine may have quite an influence.

In the group of Corliss-engine tests, Nos. 4 to 13, the largest variation is in No. 5, where m' is .067 less than m, which difference is equivalent to 20 per cent. of the latter. If we had found S' from the indicator cards, getting 12.4 as here, and then divided by $(1-m')$, the estimated value of S would have been $12.4 \div .737 = 16.8$, as against 18.5 actual: and the fraction of error would be, $(18.5-16.8) \div 18.5 = .092$.

Groups 18–21, 24–26, 28–29, made up of tests carried out under laboratory conditions, show very close agreement. The engine in Nos. 30–36 is of such a peculiar form that it would be expected to give rather special results.

The fraction of moisture in the steam coming to the engine was determined in only a part of these tests; where given, it ranged from 1 to 2 per cent., showing what is sometimes called "commercially" dry steam.

(*h*) Effective Value of the Condensation Formula.—The range of data here set forth is too narrow to serve as a basis of judgment on this point: but, as the result of extensive applications of the formula, the writer is of the opinion that, if the engine is working properly and the test is accurate, the estimated and the measured value of the steam-consumption ought not to differ by more than 10 per cent. of the latter. Some very extensive sets of

experiments keep well within this limit, notably those made under the best conditions as to method and skill; others show considerably wider departures. But, in general, the approach to or the overstepping of this limit is to be considered good cause for suspicion of the accuracy of the engine-test, or good reason to look for extra losses, as by leakage, in the engine. This degree of closeness seems to apply over a range of cut-off from .16 to .65 of the stroke. An approximation of this quality is accurate enough to be of great practical utility.

The test-groups where, in a series, only one variable changes are comparatively few. Formula (105) was got, not so much by trying the effect of single variations, as by substituting, in a roughly formed equation, the whole set of values from variant tests. Consequently, it cannot be assumed that the manner in which any variable is involved in the formula is an exact measure of its influence in the engine, as there are chances for rather complex combinations of effect.

§ 22. Engine Economy.

(*a*) CONDITIONS OF EFFICIENCY.—Efficient thermodynamic performance in an engine requires a wide range of temperature and pressure, with which goes a large ratio of expansion. But these conditions are all favorable to a high rate of cylinder-condensation. Take for instance an engine with cylinder 20″ by 48″, at 80 R.P.M., with an admission-pressure of 120 lbs. abs., exhaust-pressure of 2 lbs. abs., and an expansion ratio of 10: then $s=2.9$, $\sqrt[3]{N}=4.31$, $e=0.1$ (approximately), p we take the same as the admission-pressure, $T=353-179=174$; and from (105),

$$m'=\frac{.27}{4.31}\sqrt{\frac{2.9\times174}{120\times0.1}}=.406.$$

With such an amount of cylinder-condensation, plus mechanical losses, the disposable work of Cycle B, Fig. 26, would be greatly reduced. So then, with a wide range of working and high expansion there must be used devices for diminishing the thermal action of the cylinder-walls.

ylinders.
ices, for supplying

to the engine.
out the whole expan-
d into two or more steps
the smaller, high-pressure
and then goes into the larger.
is exhausted. This method
perature of the steam which
ticular portion of metal surface
ine. But in practice the name
d to engines with only two stages
ee stages is called a triple-expansion
stages a quadruple-expansion. The
, and four-step compound also recom-

he three cylinders are, high-pressure, inter-
in a quadruple we have the first inter-
second intermediate. In very large engines,
times divided between two cylinders of equal
e use of an excessively large single cylinder, or
rtain desired distribution of work.
practice, the initial pressures corresponding to
rees of compounding are about as follows, the line of
ogress being from the lower to the higher value in

mple engines.	30 to 70 lbs. by gage.
Compound.	80 to 120 " " "
Triple.	140 to 180 " " "
Quadruple.	200 to 225 " " "

These are, of course, always condensing engines.
In stationary practice, there is a tendency toward higher
ssures for the same classes of engines, condensing compou s
ing run with boiler-gage pressures of 120 to 150 lbs.; while

the compound locomotive, necessarily non-condensing, uses steam at from 200 to 225 lbs.

(c) WORKING OF THE COMPOUND ENGINE.—In deciding upon the cylinder-proportions of multiple-expansion engines, several objects are to be kept in mind: it is desirable that nearly the same amount of work be done in each cylinder, that the range of temperature be pretty equally divided, and that mechanical losses in the handling of the steam be reduced to a minimum. The whole subject is very complex, and nothing but the simplest general principles will be given at this point.

As to the handling of the steam, we distinguish two typical cases:

First, if the two pistons begin and end their strokes at the same time, then the steam from the high-pressure can pass directly into the low-pressure cylinder, and we have what may well be called a direct-expansion compound.

Second, when the two strokes are not timed together, as when there are two cranks at right-angles to each other, then direct exhaust is not feasible, and an intermediate chamber, called the receiver, becomes necessary. This type we will call the receiver-compound engine.

Engines of the first class frequently have a receiver, but it is not essential, as it is in the second class.

The volume of the receiver enters, with those of the cylinders, into the determination of the action of the steam in the engine. A discussion of this action will be found in Vol. II.

Mechanical losses in getting the steam through the engine, from cylinder to cylinder, partly counterbalance the gain from reduced cylinder-condensation.

In the matter of the mechanical action of the engine, the compound has the great advantage over the simple engine with the same total ratio of expansion, that the variation of driving-force throughout the stroke is very much smaller; so that the mechanism receives a much less severe and more evenly distributed force-action.

It may be remarked that the cylinder of a simple engine, using steam under the same general conditions as the compound, would be of the same size as the low-pressure cylinder of the latter.

In the next section are given the methods of working up indicator diagrams from compound engines, so as to get from them results of the same kind as those worked out under Fig. 37.

(*d*) THE STEAM-JACKET.—Another way of reducing hurtful condensation is to surround the cylinder with steam of the full boiler-pressure, so as to maintain a higher temperature in the metal. Usually there is simply an annular space around the cylinder-bore, either formed in the casting, or made by inserting the cylinder-wall proper as a liner within the main casting. Frequently, however, the cylinder-heads are also steam-jacketed.

The underlying idea of this device is, that since the condensation of the entering steam is caused by its meeting cooler metal walls, this source of loss will be partly removed by making the walls warmer. Its effective value depends upon the question, answerable only by actual trial, whether the gain in work done by the steam in the cylinder is greater than the loss of steam by condensation in the jacket: under some conditions the gain in economy is considerable, in other cases the two effects just about balance. Of course, the water forming in the jacket is to be continually drained off and returned to the boiler with the least possible loss of heat.

Against the steam-jacket it may be urged that during about half the time of a revolution it is wasting heat upon the exhaust-steam, and that even if it does impart some heat during the expansion, so as to raise the curve, only a relatively small fraction of this heat can be utilized thermodynamically.

The principal argument in favor of the jacket is based on the fact that wet steam is a far better conductor of heat than is dry steam. Obviously, the heat-interchange within the cylinder depends not only upon the difference in temperature between steam and metal, but also, and in very great degree, upon the freedom with which the steam can yield and receive heat. A dry gas is a very poor conductor, and can be heated rapidly only when there is free convection or circulation, so that a fresh portion is continually coming into contact with the heating surface. But a vapor, which can condense on a cooling surface, will yield up heat freely, a film of liquid forming on the confining wall, and

readily absorbing and transmitting heat, so as to increase rapidly in amount; this surface-effect being of far greater magnitude than any action resulting from diffused condensation.

If now the jacket, by drying the cylinder-walls and by diminishing the wetness caused by the first condensation of the entering steam, can hinder the transfer of heat, it may effect a saving in working-steam very considerably overbalancing what it uses itself.

(*e*) THERMAL ACTION OF THE CYLINDER-WALLS.—This whole subject of thermal interchange is far too complex for the quantitative use of the simple laws of heat. To calculate on the basis of so many pounds of metal, of known specific heat, being heated or cooled so many degrees in each cycle is impracticable, because there is no possible way of finding out either the weight of metal involved in the action or the average temperature-range. Only by test under working conditions can results be got, only by reasoning closely based upon tests can results be predicted. But in regard to this subject, the following conclusions seem to be justified by general considerations:

Of the thick body of metal forming the cylinder, only the inner portion suffers much change in temperature during the steam-cycle: the surface is heated and cooled quite a good deal, perhaps approximating to the temperature of the entering and of the exhaust steam; but the range of change diminishes rapidly from the surface inward, and at no very great depth the metal will have a nearly constant temperature. In a cylinder not protected from the outside air, or with free radiation, this constant mean temperature will drop toward the outer surface of the metal: if the cylinder is well covered, the drop will be much less rapid, but there will still be an outward flow of heat. With a steam-jacket, the temperature will rise going outward, and the heat-flow will be inward.

The jacket cannot change the character of the temperature variation on the inner surface of the cylinder; the best that it can do is to diminish the amount of this variation and to raise the level of the mean about which the actual temperature oscillates.

A jacket on the barrel of the cylinder can have comparatively little effect, for it does not reach the surface with which the steam

first comes into contact, and only after the piston has travelled some distance is any considerable area of jacket-heated surface exposed. Back of the clearance-surface, as on the cylinder-head, the jacket will be more active, both in imparting heat usefully and in wasting heat on the exhaust-steam.

Size of cylinder and time of cycle both exert a great influence. It has already been pointed out that cylinder-condensation is very much a matter of surface action, water being formed on the surfaces of the body of steam, but not throughout its bulk—as distinguished from condensation due to work done, which would be evenly distributed. Then the larger the body of steam, the less will be the effect of the metal walls.

In large engines at high speed, the jacket becomes of little effect, and can very well be omitted in favor of other and more active ways of getting heat into the steam.

(*f*) THE RE-HEATING RECEIVER.—This device is now generally used in large compound engines, sometimes with jackets, frequently alone. The receiver between the cylinders is enlarged to surround a body of coils or tubes filled with boiler-steam. The working-steam, being broken up into small currents and brought into intimate contact with this heating-surface, receives heat much more effectively than through the rather clumsy device of the jacket.

The high-pressure cylinder gets nearly dry steam from the steam-pipe: the steam going from the high to the low cylinder is sure to contain a fraction of moisture due to the work done in the high cylinder, together with loss by radiation; and the re-heater is intended to evaporate this moisture, so that the low-pressure cylinder will also receive practically dry steam. The saving in condensation in the low cylinder more than compensates for the fact that this heat enters the cycle below the upper limit of temperature.

(*g*) SUPERHEATING the steam as it leaves the boiler is a very effective way of increasing the thermodynamic efficiency of the engine. The superheating may vary in amount from a few degrees' rise at the boiler, enough to insure dry steam at the engine or to decrease slightly the freedom of heat-interchange in the cylinder,

to a rise sufficient to modify quite materially the expansion-process, perhaps wholly eliminating cylinder-condensation.

A discussion of the use of superheated steam must follow two lines — the first a consideration of thermodynamic possibilities, the second a description of the practical difficulties met with in the working of the engine, on account of the much higher temperature of the steam.

As to the magnitude of the thermal action involved in superheating, it is small in comparison with the effect of a large amount of cylinder-condensation. The range of boiler-pressures in actual use is from 100 to 225 lbs. abs., with a corresponding saturation-temperature of from 330° to 390° F.; while the highest point to which the steam may be raised, with due regard to the safety of both engine and boiler, is about 600° F. We see then that the heat which can be added is from 100 to 130 B.T.U. per lb. of steam: and with a total latent heat in the neighborhood of 860 B.T.U., an equivalent amount of heat would be given to the cylinder-walls during admission with an initial condensation of from 11 to 15 per cent.—which are very moderate values.

If now superheating is to be thought of as merely a device for increasing the heat-content of the pound of steam, so that it can yield up heat without the formation of moisture, it would appear that the remedy is not sufficient. But when we consider the fact that the facility of heat-interchange may, incidentally, be greatly reduced, with a corresponding diminution in its amount, then we appreciate better the possibilities of the case. The effect of superheating is, in this respect, closely similar to that of the steam-jacket, as described in (*a*): and with a reasonable use of compounding, so as to decrease both the range of temperature and the ratio of expansion in the first cylinder, together with the re-heating receiver, a work-performance per pound of steam approximating what is given as the limit for saturated steam in Tables 16 A and 16 B may reasonably be expected. Of course, this does not imply a proportional gain in thermal efficiency, for the total heat of formation increases also, though at a less rapid rate.

(*h*) THE DIFFICULTIES IN OPERATION are encountered, in the engine, chiefly in connection with the matter of internal lubrication.

In the early attempts to utilize the advantages of superheating, in the 1860's, when steam-pressures were low, say from 20 to 50 lbs. by gage, and tallow was the common cylinder-lubricant, even a moderate superheat caused trouble by drying up and decomposing this animal oil. Compounding then began to come into general use, with an accompanying steady rise in boiler-pressure: and as these changes both decreased the desirability and increased the practical difficulty of using superheat, the latter line of progress was largely abandoned.

Later, in the 1890's, under the pressure of the high cost of fuel on the Continent of Europe, this matter was again taken up, and the use of high superheat worked out to a successful application, especially in Germany; the solution of the problem requiring both a high-grade mineral oil and certain changes in construction—such as the substitution of lift-valves for slide-valves, and some simplification of the form of the cylinder, so as to get rid of the unequal expansion that a wide range of temperature would cause in a complicated casting.

As to the apparatus added to the boiler, this "superheater" cannot be placed beyond the boiler, because the desired steam-temperature is above that at which the products of combustion should pass to the chimney. It is therefore placed within the hot-gas circulation of the boiler, the products of combustion being partially cooled by water-heating surface before they strike the steam-heating surface; but a second portion of water-heating surface is disposed beyond the superheater. The limit of allowable temperature is imposed by the need of a fair margin against burning out the tubes, and also by the fact that the metal is weakened by prolonged exposure to high temperature.

§ 23. Diagrams from Compound Engines.

(*a*) Description of the Engine.—A set of actual diagrams from a compound engine will now be completely worked up, the particular methods necessary being developed and applied as we proceed.

The dimensions and arrangement of the cylinders of the engine from which were taken the diagrams given in Fig. 40 are shown

by an outline sketch in Fig. 39. This engine is of the tandem-compound, direct-expansion type, the sequence of the cylinder-ends being indicated by arrows: actually, of course, the steam passes from the exhaust-port of the HP cylinder, through a pipe, to the steam-chest of the LP cylinder; this whole intermediate space constituting a receiver of moderate capacity. In the notation used to designate the diagrams, the first letter shows the cylin-

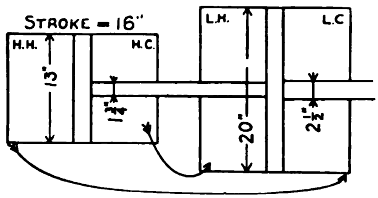


FIG. 39.—Cylinder-arrangement of Engine.

der, high or low, the second letter the end, head or crank; so that H.H. means "high head," and so on.

A complete description of this engine, besides more or less detail of construction, would set forth the manner of working of the valve-gear and of the governor: but for present purposes, where we are concerned merely with the working up of a set of diagrams, the data on Fig. 39 are sufficient.

(*b*) THE INDICATOR DIAGRAMS, as taken, but with lines of boiler-pressure, of condenser-pressure, and of perfect vacuum added, and with the valve-action events marked, are reproduced in Fig. 40. The general data are:

Steam-pressure, taken from steam-pipe near engine, =96 lbs. by gage or 111 lbs. absolute.

Vacuum in condenser = 22.2 ins. of mercury, below the pressure of the atmosphere, equivalent to about 3.9 lbs. absolute.

Speed of engine = 250 R.P.M.

Scale of springs, indicated on Fig. 40, 60 lbs. per inch on the HP diagrams and 20 on the LP.

Clearances, estimated from various diagrams, as follows, for the several cylinder-ends:

H.H.	.16	L.H.	.08
H.C.	.12	L.C.	.06

On these data the following remarks seem called for:

When the reading of a pressure-gage that has not been closely calibrated is to be changed to absolute pressure, it is quite close enough to use the round 15 lbs. for one atmosphere, instead of the more exact 14.7 lbs.

The vacuum-gage reads from the atmospheric pressure downward: the standard atmosphere is 29.9 ins. of mercury or 14.7 lbs. per sq. in.: so that 1 in. of mercury equals very nearly 0.5 lb. To get the absolute pressure in the condenser, subtract the gage-reading from 30 and divide by 2, getting here $7.8 \div 2 = 3.9$ lbs. This is a poor vacuum, as a good condenser should give from 26 to 27 ins.

The characteristic of diagrams taken at high speed—a wave-line effect due to the inertia of the moving-parts of the indicator—is quite marked on these LP diagrams, showing up even more clearly in Figs. 41 and 42, where hyperbolas are drawn on the curves: the true line of pressure-variation would trace a mean course through these waves.

It will be noted that there is quite an inequality of cut-off in the two ends of the LP cylinder.

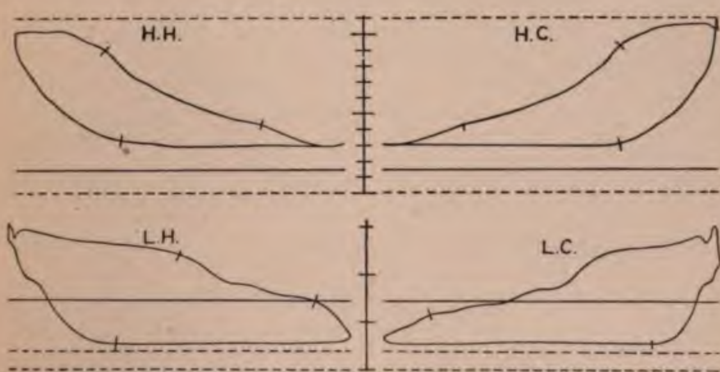


Fig. 40.—Indicator Diagrams from a Compound Engine.

(c) Calculation of I.H.P. and I.S.C.—The first operation is similar to that for a simple engine, the data and results being set forth in the following table, which needs no explanation:

TABLE 23 A. INDICATED HORSE-POWER.

Cylinder End.	Piston Area.	$\frac{AS}{396,000}$.	M.E.P.	I.H.P. Ends.	I.H.P. Cylinders.	I.H.P. Total.
H H	132.7	.005360	26.7	35.8	75.5	157.0
H C	130.3	.005266	30.15	39.7		
L H	311.8	.01260	14.22	44.8	81.5	
L C	309.3	.01249	11.75	36.7		

In calculating I.S.C. by the method of Eq. (99) or (101), the procedure is the same as with a simple engine. A single diagram is used, HP or LP as desired, in getting K, which is then the I.S.C. per cubic foot displaced by the piston in the cylinder-end from which the diagram was taken: and the total I.S.C. per revolution, per minute, or per hour is found as in (101). To find I.S.C. per H.P.H., we would take the sum of two HP determinations or of two LP determinations (from the two ends in either case), and divide by the total calculated I.H.P. The first part of the operation is given in the next table, the necessary dimensions to the E-points and C-points being marked on the diagrams in Figs. 41 and 42.

TABLE 23 B. CALCULATION OF K.

Cylinder End.	e	d_E	c	d_C	$ed_E - cd_C = K$
H H	.438	.2064	.316	.1189	.09050 − .03753 = .05297
H C	.433	"	.258	"	.09430 − .03062 = .06368
L H	.977	.0377	.244	.0262	.03685 − .00638 = .03047
L C	.714	"	.160	"	.02693 − .00419 = .02274

And applying Formula (101), we get,

TABLE 23 C. I.S.C. PER HOUR AND PER H.P.H.

Cylinder End.	Cylinder Volume.	$60NV$	K	W Ends.	W Totals.	$\frac{W}{H}$
H H	1.229	18,435	.0530	977	2131	13.57
H C	1.207	18,105	.0637	1154		
L H	2.887	43,305	.03047	1320	2296	14.61
L C	2.860	42,900	.02274	976		

But suppose that we wish to get W_H directly from the diagram by the method of Eq. (104): then we must take into account the fact that the steam which is metered in one cylinder does work in two (or more) cylinders: in other words, to get easily the ratio of indicated steam to horse-power, we must express both in terms of the same cylinder-dimensions. Let K be found, for instance, from an HP diagram: in this HP cylinder-end, the M.E.P. p_1 acts upon the piston-area A_1; and then the same steam does work in an LP cylinder-end represented by M.E.P. p_2 on area A_2. Now this latter amount of work would be done upon the HP piston by a "reduced" M.E.P. of the value

$$p_R = p_2\frac{A_2}{A_1} = Rp_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

acting on the area A_1. In this formula, R stands, in general, for the ratio of the area upon which the pressure is actual to that to which it is to be reduced. Finally, a total M.E.P. $p_m = p_1 + p_R$, introduced into Eq. (96), will represent the total I.H.P. developed by the steam in terms of the dimensions of the HP cylinder; which dimensions will then cancel out in the division leading to Eq. (104). The application of this second method is shown by the following table:

TABLE 23 D. I.S.C. PER H.P.H. DIRECTLY CALCULATED.

Cylinder End.		Volume Ratio.	Total M.E.P. $p + p_R = p_m$	$\frac{13750}{p_m}$	K	I.S.C. Ends.	I.S.C. Means.
H	H	$\frac{\text{H.H.}}{\text{L.C.}} = .429$	26.70+(27.36)=54.06	254.1	.0530	13.48	13.56
	C	$\frac{\text{H.C.}}{\text{L.H.}} = .418$	30.15+(34.00)=64.15	214.1	.0637	13.64	
L	H	$\frac{\text{L.H.}}{\text{H.C.}} = 2.39$	14.22+(12.60)=26.82	512.4	.03047	15.61	14.53
	C	$\frac{\text{L.C.}}{\text{H.H.}} = 2.33$	11.75+(11.47)=23.22	591.8	.02274	13.45	

Here the actual M.E.P.'s are first entered under p; then each is multiplied by the volume-ratio * beside it, and the result entered

* Volume-ratio, usually identical with the area-ratio R in (111), is strictly of more general application, for compound engines have been built in which the stroke was different in the two cylinders.

under p_R in the line belonging to the cylinder-end in which is carried out the other part of the cycle. Thus for H.H., $26.70 \times .429 = 11.47$, and this is entered opposite L.C.; and so on. The rest of the calculation is obvious. The difference between the two mean I.S.C.'s for the LP cylinder, one found here, the other in Table 23 C, is due to the fact that the mean of the ratios of partial quantities is not the same as the ratio of the sums: the method of Table 23 C is inherently the more correct.

With a three- or four-stage compound, there would be two or three reduced M.E.P.'s to be added to the actual M.E.P. in the cylinder-end for which the I.S.C. was being found.

(*d*) CHARACTER OF THE EXPANSION.—The most interesting information given by the result of this calculation—aside from its being a basis for an estimate of actual steam-consumption, according to the method of Eq. (105)—is in regard to the question as to whether the I.S.C. is greater at the beginning or at the end of the whole operation of expansion; the points E_1 and E_2 being located with this question in view. To know, first, what fraction of the entering steam will be condensed by the HP cylinder, second, whether and by how much the fraction of moisture is likely to increase or diminish as the expansion progresses, and how the various economy-expedients such as the steam-jacket and the reheating receiver will affect these actions, is to have a sound basis for proportioning the cylinders and predicting the performance of a compound engine. In Part II. the methods employed in getting the formula for cylinder-condensation will be further applied along the lines just indicated. At present, however, we are far more concerned with ways of getting results than with the results themselves.

For the investigation of the character of the total expansion, a most effective implement is the combined diagram, in which the separate indicator cards are brought to the same scales and to common reference-lines, so as to form the parts of a complete diagram of steam-performance. Two methods of combination will now be illustrated, the first fully defined by the statement in the last sentence, the second embodying the principle used in deriving Fig. 32.

(e) Diagrams Brought to a Common Clearance-line.—In Fig. 41 I., the indicator diagrams are prepared by dividing the length of each into ten equal parts and erecting ordinates at the division-points. Then in II. the volumes, first of the clearances, next of the cylinders, are laid out from OP to a convenient scale, and similar vertical division-lines drawn. In this case, the pressure-scale is the same as that of the LP diagram, so that ordinates from the latter can be transferred directly; while those from the HP diagram must be multiplied by three—or measured off three times.

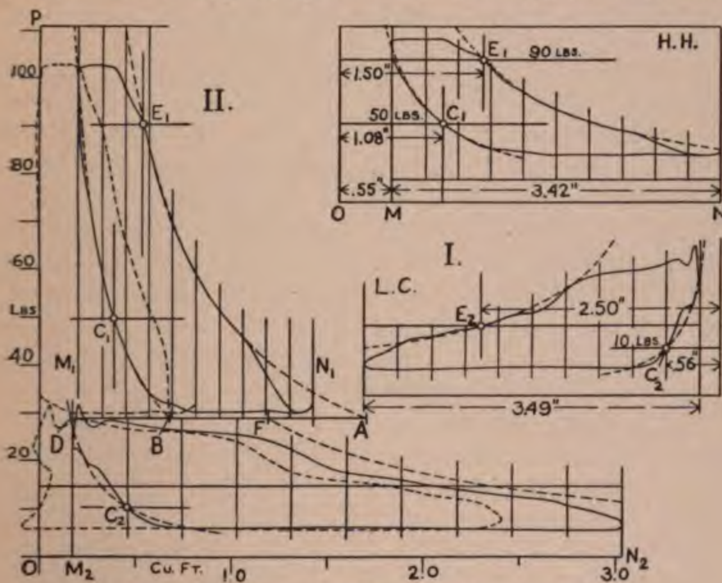


Fig. 41.—Diagrams Combined on Clearance-lines.

Using also intermediate ordinates where the changes in pressure are rapid, as at the ends of the diagrams, we get a series of points through which the new curves can be traced, giving the result shown by the full-line figures.

On the original diagrams, hyperbolas are drawn through both the E-points and the C-points: on the combined diagram, three of these are reproduced, all except that through E_2: instead of the latter we continue the curve through E_1, not by simply producing

this hyperbola, but so as to get a correct measure of the continuity of the expansion. A horizontal line AD is drawn between the diagrams, the two HP hyperbolas cutting it at A and B, while that through C_2 meets it at D: then the length DF is made equal to BA, and the expansion-hyperbola continued from F. Obviously, this is nearly as effective as the method of Fig. 32; for while it does not get rid of the clearance-steam, it does eliminate the difference between the two quantities of this steam in the successive cylinders.

Diagrams on the rectified compression-hyperbolas, like Fig. 32, are dotted in, without any of the construction used in getting them being shown: this would consist simply of a lot of horizontal lines, along which the volumes, measured from the reference-curves C_1B, DC_2, would be laid off from OP. The effect of indicator-inertia upon the LP cards is shown by the peculiar heel on the derived diagram; which is due to the fact that the indicator-piston lagged behind the steam-pressure at first, then swung ahead of it, and kept oscillating about the true pressure until its energy was absorbed in friction-work. The hyperbola drawn through C_2 should, as here, follow the mean of these waves, so that the areas between the two curves, on opposite sides of the hyperbola as a reference-line, shall be just about equal.

(*f*) DIRECT COMBINATION ON THE COMPRESSION HYPERBOLAS.—This is illustrated in Fig. 42, where a graphical construction is used for transforming the volumes to their new scales. The preliminary preparation consists in drawing a lot of similarly-spaced horizontal lines on the indicator diagrams and on the plane of the combined figure, in II. In order to transform HP volumes, we construct III. by laying off the actual length of the HP diagram at AB, making BC equal the volume of the HP cylinder, to the scale of the combined diagram (corresponding with M_1N_1 in Fig. 41 II.), and drawing the diagonal AC. Then if we take any abscissa as AD in I. and lay it off from A in III., the intercept DE will give the proper volume to be laid off as DE in II. Similarly, FA is the length of the LP diagram, AG is the corresponding volume to the scale of II., and F is the origin: FH is measured from F and HK is ready to be used in II. It is more convenient to use these pro-

portion-diagrams in III. than to work through the first method of combination as in Fig. 41.

In Fig. 42 II., a hyperbola is drawn through E_1, and cuts under the LP expansion-curve: but the corresponding hyperbola in Fig. 41 II., from F, is well above its LP curve. It appears, there-

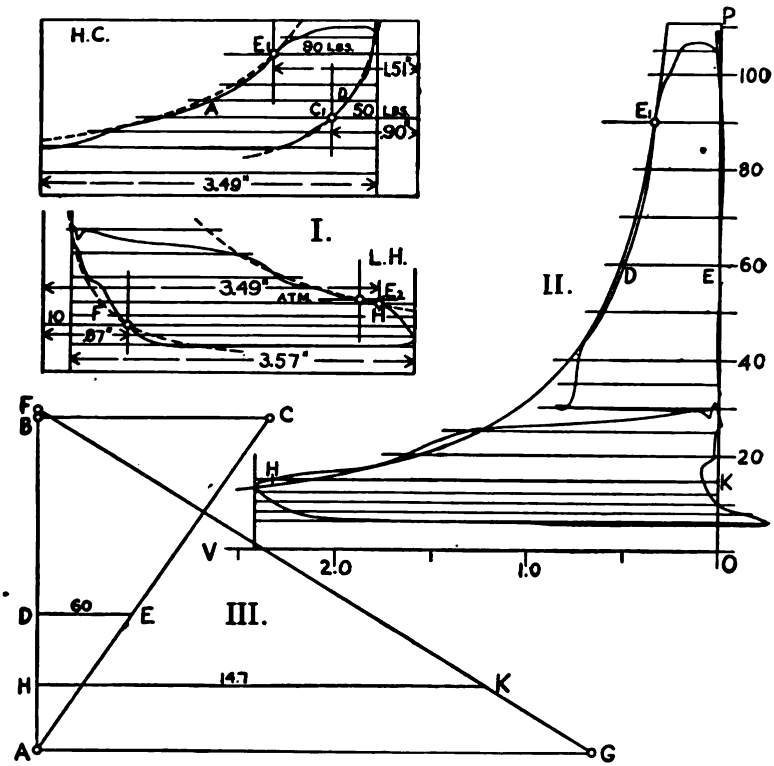


FIG. 42.—Diagrams Combined on Compression Curves.

fore, that the steam is not divided between the two ends of the LP cylinder in the same ratio as between the two ends of the HP cylinder. The only way, then, to get a fair criterion of the total expansion is to draw a mean of the two combined diagrams, as is done in Fig. 43.

(*g*) IN FIG. 43, the separate diagrams, in dotted line, are traced from Figs. 41 and 42, with the minor irregularities on the com-

pression side left out; and the average diagrams are partly drawn, in full line. Through the E-point are passed two curves, No. 1 the curve of constant steam-weight, $pv^{1.065}=C$, No. 2 the usual equilateral hyperbola, $pv=C$. The increasing value of the steam-fraction

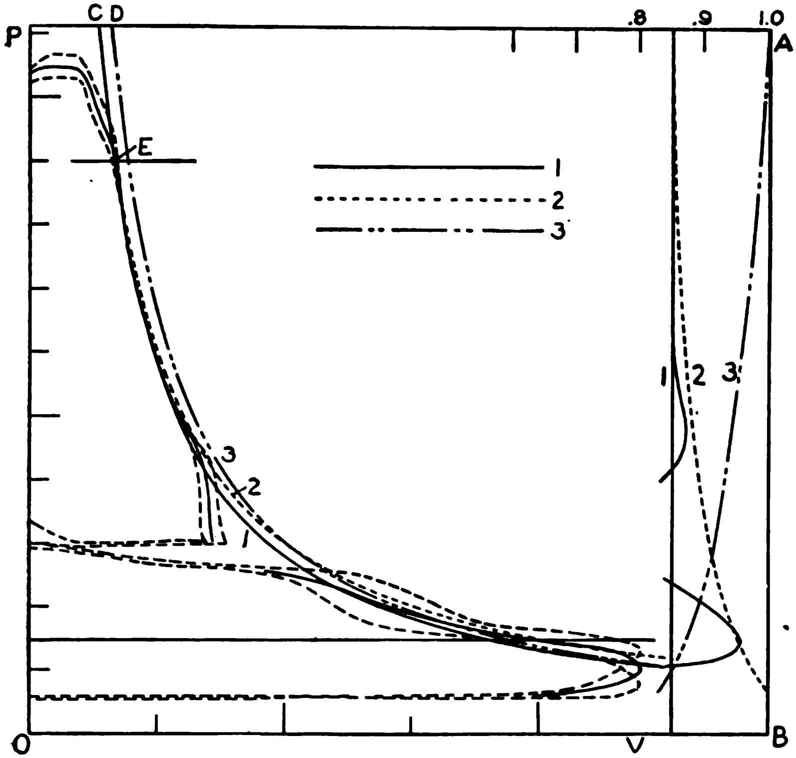


FIG. 43.—The Average Combined Diagram.

tion x is shown by the rise of the LP expansion-curve above the curve of constant weight.

According to Formula (105), the fraction of cylinder-condensation should be about 0.15: if this is correct, the actual steam-consumption is about $13.6 \div .85 = 16$ lbs. per H.P.H.; which is rather too good to be true, with the poor vacuum here evident. Using this result, however, for illustrative purposes, we carry curve 1 up to the steam-pressure line at C, then make DP : CP=

1.00 : .85, and from D draw an adiabatic for steam initially dry at D—using Table II. for this curve, as also for curve 1. This gives a comparison of the actual diagram with the form of Cycle B, Fig. 26.

The result of all this drawing is a rather confusing mess of lines, especially along the LP. expansion-curve; this crowding of the lines being an inherent defect, and a decided disadvantage, of the *pv* diagram, when a wide range of expansion is to be represented. A scheme for showing more clearly the character of the expansion, especially as to the important question of re-evaporation or increasing condensation, is set forth at the right side of Fig. 43. Here the abscissa is x, the steam-fraction, to the scale marked at the top. The line of unity, AB, is the standard of comparison, while line 1 is the curve of constant steam-weight for $x=.85$. The increasing condensation in adiabatic expansion is shown by the slant of line 3 to the left; and the re-evaporation necessary to an expansion along the hyperbola is similarly shown by the departure of curve 2 from line 1. These curves are not here drawn with any great precision. Finally, those parts of the actual expansion-curves that lie to the left of curve 1 are sketched in on this system, making very evident the proportion of re-evaporation.

The entropy-temperature diagram, described and explained in Chapter VI., gives the same information as these derived curves, and in a much more direct and convenient shape.

CHAPTER V.

THE DYNAMICS OF STEAM.

§ 24. The Steam-jet.

(*a*) Conditions of Flow.—With a current of steam or gas, as with any other moving body, velocity can be produced only by the action of force and the expenditure of energy: the force accelerates the mass, the energy is stored in the current in kinetic form. The conditions of perfect flow, under which all the work done in or upon the steam on account of the drop in pressure is changed into kinetic energy of the forward-moving jet, are illustrated in Fig. 44. The apparatus consists of the vessel A, the nozzle B, and the tube D; B has the throat or orifice C, then expands or flares to D. In A the pressure is p_1, in D it is p_2; and in entering and passing through the nozzle, the steam loses pressure and gains velocity. It is assumed that no work is lost in overcoming friction, or through any secondary action in the jet.

Fig. 44.—Conditions of Jet-formation.

(*b*) Work Expended in Acceleration.—Imagine a section-plane across the vessel at EF: steam passing this plane may be thought of as continually pushed forward by the steam behind it; and the work which it receives—and in steady flow would transmit simply and directly to the steam ahead of it, without change—is, per pound,

$$U_1 = 144p_1v_1 = 144p_1(x_1u_1 + w_1). \quad . \quad . \quad . \quad . \quad (112)$$

After the steam attains its full velocity in the tube D, and in passing any cross-plane, as GH, it must continually push ahead of it steam subjected to the lower pressure p_2, doing the work

$$U_2 = 144 p_2 v_2 = 144 p_2 (x_2 u_2 + w_2). \quad . \quad . \quad . \quad . \quad (113)$$

As the pressure drops along the nozzle, the steam expands; and if we imagine two cross-planes close together and enclosing a small body of steam between them, then the static pressure in the substance will be less at the right plane than at the left one. This small difference of pressure produces a corresponding acceleration, and the pressure-work of the expansion is thus gradually changed into kinetic energy of the jet.

The expansion, unless very special conditions are imposed, is essentially adiabatic: for after any particular action has become established, and each portion of the inner surface of the nozzle has assumed the temperature of the part of the jet which touches it, then only as heat is conducted along the nozzle or radiated from it will there be any chance for a transfer of heat between jet and guiding-surface.

The adiabatic expansion-work is—see Eq. (74), § 13 (k)—

$$U_E = 778(q_1 + x_1 l_1 - q_2 - x_2 l_2). \quad . \quad . \quad . \quad . \quad (114)$$

And the net work expended in accelerating the current, per pound of steam, is

$$U = U_1 + U_E - U_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

This is the same as the effective work of Cycle B, Fig. 26 and § 16 (b), corresponding to U' in Tables 16 A and 16 B; and if we reduce it to heat-units, using the symbol E for the energy AU given to the jet, we have

$$\begin{aligned} E &= AP_1(x_1 u_1 + w_1) - AP_2(x_2 u_2 + w_2) + q_1 + x_1 l_1 - q_2 - x_2 l_2 \\ &= q_1 + x_1 r_1 - q_2 - x_2 r_2 + A(P_1 w_1 - P_2 w_2). \quad . \quad . \quad . \quad . \quad . \quad (116) \end{aligned}$$

This equation embodies the simpler method of calculating the net work of Cycle B, referred to under § 16 (b); except that in that case the last term, $A(P_1 w_1 - P_2 w_2)$, which represents the water-volume effect, was omitted.

46. THE ADIABATIC TABLE.—To facilitate the application of Eq. (116) and the determination of the energy imparted to the jet under any conditions within the range of practice, the numerical quantities given in Table V. have been worked out. Each column is for the initial pressure at the top; and each group of values is for an expansion from this p_1 to the p_2 marked at the side. That is, the lower line of the cycle, D'C' in Fig. 45, is taken successively at one pressure after another, the top line remaining at AB. This table can best be explained by showing how one set of values was computed:

Take the case of an initial pressure of 150 lbs. per sq. in. absolute and a terminal pressure of 15 lbs. abs.: the diagram for the operation is given in Fig. 45; and the steam is supposed to be dry and saturated at B, so that $x_1 = 1$.

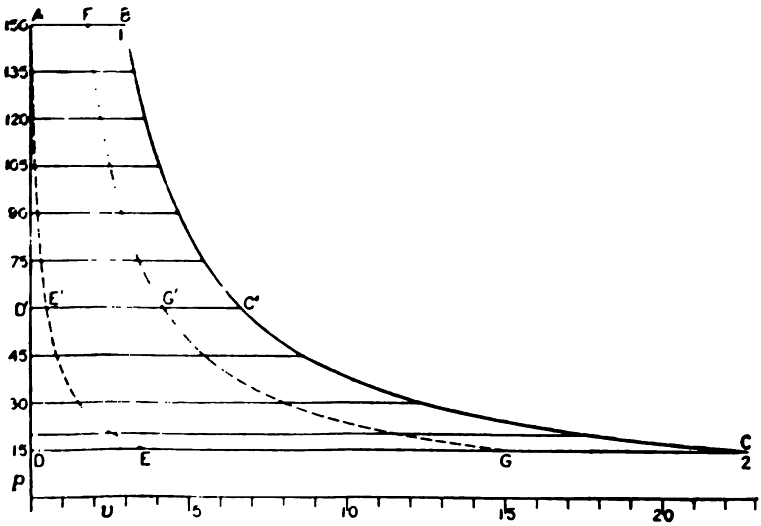


FIG. 45.—Adiabatic Diagram.

From the Steam-table,

$p_1 = 150$	$q_1 = 330.56$	$p_2 = 15$	$q_2 = 181.94$
$a_1 = .51466$	$r_1 = 860.62$	$a_2 = .31479$	$r_2 = 964.97$
$b_1 = 1.56656$		$b_2 = 1.4339$	

Then $$x_2 = \frac{a_1 + b_1 - a_2}{b_2} = \frac{1.2518}{1.4339} = .8730.$$

Now $$q_1 + r_1 = 1191.18,$$
$$q_2 + x_2 r_2 = 181.94 + 842.38 = 1024.32.$$

The difference between these is 166.86 H.U.: it is the main part of E, identical with AU' in Table 16 A, where the value for the same conditions, by independent and less precise computation, was found to be 167.0.

This result does not include the last term of Eq. (116): according to Table III., $w_1 = .018$, $w_2 = .0167$; it is fair enough to use a rough mean, giving the upper value a little more influence, say .0175; then

$$\frac{144}{778}(p_1 - p_2)w_m = 0.185 \times 135 \times .0175 = .44 \text{ H.U.}$$

Adding this to the 166.86 found above, we get

$$E = 167.30 \text{ H.U.}$$

The second quantity E_1 given in Table V. is the effective work of the Carnot cycle, the same as AU in Tables 16 A and 16 B: it is computed by Eq. (78). For the case above, $t_1 = 358.16$, $t_2 = 213.03$, $T_1 = 818.16$, $t_1 - t_2 = 145.13$, $r_1 = 860.62$; then

$$E_1 = \frac{145.13}{818.16} 860.62 = 152.66 \text{ H.U.}$$

Finally, E_0, equal to $E - E_1$, is the effective work of adiabatic expansion of water originally at p_1 and t_1, represented by the area AED in Fig. 45. On general principles, it could be more precisely calculated by making $x_1 = 0$ in Eq. (116), especially for small ranges of pressure, because E and E_1 are then large quantities with a relatively small difference between them; but usually the method of subtraction indicated above will be found more convenient and sufficiently exact.

Below the E's are entered in Table V. the limiting values of the final steam-fraction, x_2 for $x_1 = 1$, x_{20} for $x_1 = 0$; also the corresponding specific volumes at p_2.

(*d*) STEAM OF ANY INITIAL QUALITY.—The reason for tabulating all the quantities below E appears when we consider the general adiabatic equation of relation, (70),

$$x = \frac{a_1 - a_2 + x_1 b_1}{b_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

and note that just as x_1 varies between the limits 0 and 1, so x varies between x_{20} and x_2, and v varies between v_{20} and v_2. That is, in Fig 45, the variable intercept E′C′, between the two adiabatic curves AE and BC, is divided in constant ratio by the intermediate adiabatic FG.

Suppose that, for $p_1 = 150$ and $p_2 = 15$, $x_1 = .60$: then the total energy, represented by AFGD in Fig. 45, we get either by adding 60 per cent. of E_1 to E_0, or by subtracting 40 per cent. of E_1 from E; giving either

$$\begin{aligned} E' &= .6 \times 152.66 + 14.64 \\ &= 91.60 + 14.64 = 106.24 \end{aligned}$$

or

$$\begin{aligned} E' &= 167.30 - .4 \times 152.66 \\ &= 167.30 - 61.06 = 106.24. \end{aligned}$$

This table reduces operations with the adiabatics of saturated steam to simple slide-rule arithmetic; while the drawing of adiabatic curves for superheated steam is provided for by Table II.

In getting the two v-limits, v_2 is taken to be the same as $x_2 s_2$; for where s_2 is small, x_2 is near unity, and where x_2 departs from 1.00, s_2 is large and w_2 relatively insignificant: but for v_{20} the exact expression

$$v_{20} = w_2 + x_{20} u_2$$

is used.

(*e*) JET-CONDITIONS DETERMINED.—The general results set forth in Table V. are applied in Table 24 A and Fig. 46 to the particular case of the flow of a steam-jet under adiabatic conditions. The arrangement of this table is similar to that of Table V., and the first quantity E is taken directly from Table V. Then the next, V, is the velocity in feet per second of the jet at p_2, if all the energy

E is used up in accelerating the pound of steam and changed into kinetic energy of the latter; the relation is

$$\frac{1}{g}\times\frac{V^2}{2}=778E,$$

or

$$V=\sqrt{778\times 64.32\times E}=\sqrt{50040E}. \quad . \quad . \quad . \quad (118)$$

Next are given, first, the volume v_2 of the pound of steam at p_2, likewise taken directly from Table V.; and second, the corresponding area of cross-section of a jet discharging 1 lb. per sec., expressed in square inches, and got by the operation

$$a=144\frac{v_2}{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

Finally, the diameter in inches of a round jet of this capacity is also computed and given as d.

In Fig. 46 the values of a from Table 24 A are plotted on a pressure base: each curve is for steam which starts in the dry saturated state at the p_1 marked upon it, and gives the cross-section, at each successive p_2 according to the scale below the diagram, of a perfect jet discharging one pound of steam per second. The group of curves below and to the right of the curve KBL was first drawn, to represent the whole table to the scale marked at the right: above KBL, parts of these curves are again laid out, to the larger scale at the left.

(*f*) The Form of the Steam-jet.—The curve for $p_1=150$ lbs. is most fully given in Fig. 46, running from $p_2=148$ to $p_2=2$ lbs.: all the other curves begin at $p_2=0.9p_1$. We note that the jet consists of three parts; first the rapidly contracting entrance, shown by the curve through A and B toward C; then the choke or neck, smallest at C; last, the flare or spread CDE. At first the velocity increases faster than the specific volume, so that the area of the section diminishes; over quite a range of pressure-drop the two increase together at nearly equal rates; and then, as the value of V becomes very large, so that increase in V is small relative to increase of V^2, the volume grows much faster than the velocity, and the cross-section increases rapidly. The limiting

TABLE 24 A. STEAM-JET TABLE.

p_2	$p_1=$	250	220	195	170	150	135	120
195	E	21.06	10.22					
	V	1027	715					
	v_2	2.306	2.326					
	a	.324	.468					
	d	.642	.772					
170	E	32.34	21.63	11.50				
	V	1272	1041	759				
	v_2	2.600	2.622	2.644				
	a	.294	.363	.502				
	d	.612	.680	.800				
150	E	42.55	31.93	21.86	10.45			
	V	1459	1264	1046	723			
	v_2	2.899	2.924	2.948	2.975			
	a	.286	.333	.406	.593			
	d	.604	.651	.719	.869			
135	E	50.96	40.42	30.43	19.13	8.75		
	V	1597	1422	1234	978	662		
	v_2	3.179	3.206	3.232	3.262	3.289		
	a	.287	.325	.377	.480	.716		
	d	.604	.643	.693	.782	.954		
120	E	60.29	49.85	39.94	28.72	18.40	9.70	
	V	1737	1579	1414	1199	960	697	
	v_2	3.524	3.553	3.582	3.614	3.645	3.671	
	a	.292	.324	.365	.434	.547	.758	
	d	.610	.642	.682	.743	.834	.983	
105	E	70.65	60.27	50.40	39.29	29.09	20.50	10.89
	V	1880	1737	1588	1402	1207	1013	738
	v_2	3.961	3.994	4.025	4.062	4.095	4.124	4.156
	a	.303	.331	.365	.417	.489	.587	.811
	d	.622	.649	.682	.729	.789	.864	1.016
90	E	82.39	72.10	62.37	51.34	41.20	32.69	23.21
	V	2030	1899	1767	1603	1436	1279	1078
	v_2	4.533	4.570	4.606	4.646	4.685	4.717	4.754
	a	.322	.346	.375	.417	.470	.531	.634
	d	.640	.664	.692	.729	.773	.822	.900
75	E	96.04	85.78	76.25	65.34	55.28	46.89	37.48
	V	2192	2072	1953	1808	1663	1532	1370
	v_2	5.321	5.364	5.405	5.452	5.496	5.534	5.577
	a	.350	.373	.398	.434	.476	.520	.586
	d	.667	.689	.712	.744	.778	.814	.864

$p_1=$		250
p_2	E	10.90
	V	739
220	v_2	2.075
	a	.405
	d	.718

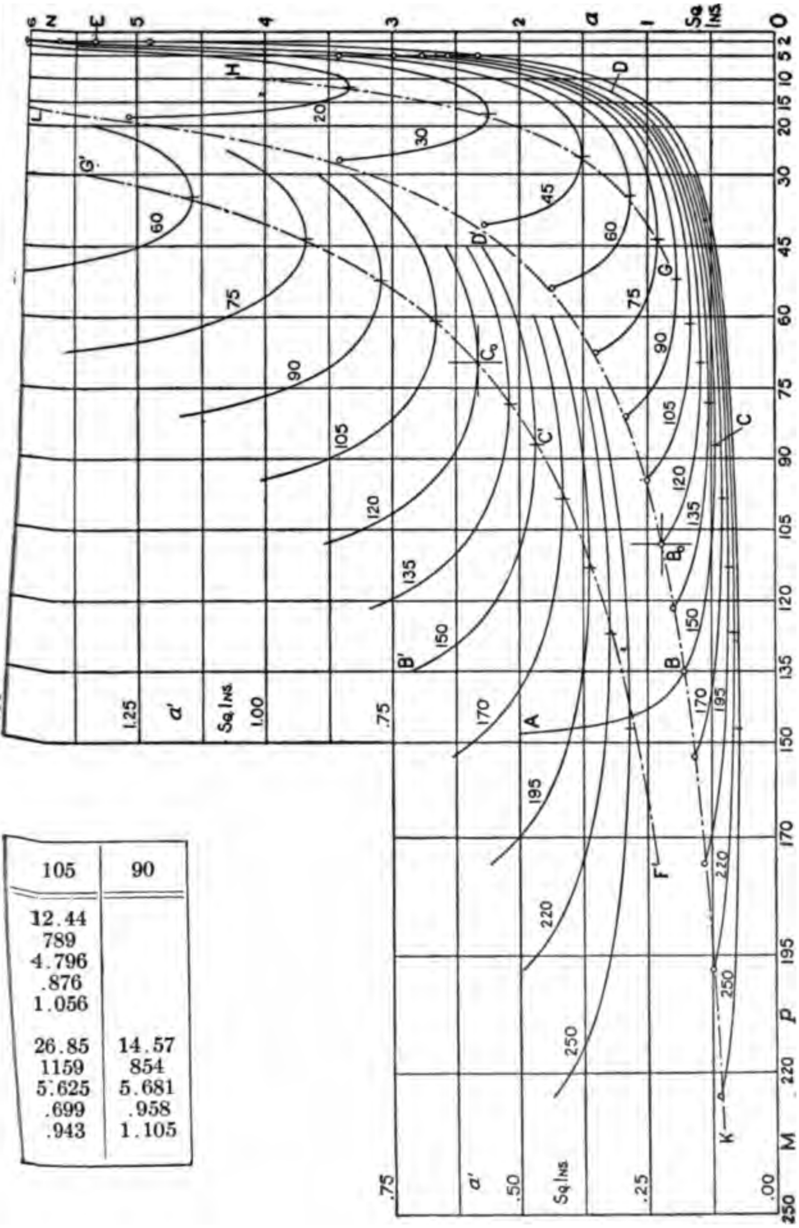


FIG. 46.—Steam-jet Curves.

105	90
12.44	
789	
4.796	
.876	
1.056	
26.85	14.57
1159	854
5.625	5.681
.699	.958
.943	1.105

TABLE 24 A—*Continued.*

[illegible]		250	220	195	170	150	[illegible]	[illegible]
[illegible]	E	112.25	102.23	92.71	81.96	72.08	[illegible]	[illegible]
	V	2370	2262	2154	2025	1890	[illegible]	[illegible]
40	[illegible]	6.471	6.524	6.574	6.630	6.683	[illegible]	[illegible]
	[illegible]	.393	.415	.439	.471	.507	[illegible]	[illegible]
	[illegible]	.708	.727	.746	.775	.803	[illegible]	[illegible]
	E	132.68	122.78	113.42	102.95	93.13	[illegible]	[illegible]
	V	2577	2479	2383	2270	2179	[illegible]	[illegible]
45	[illegible]	8.342	8.406	8.469	8.539	8.600	[illegible]	[illegible]
	[illegible]	.466	.488	.512	.542	.574	[illegible]	[illegible]
	[illegible]	.770	.789	.807	.831	.855	[illegible]	[illegible]
	E	160.25	150.59	141.45	131.08	121.58	[illegible]	[illegible]
	[illegible]	2832	2745	2661	2561	2467	[illegible]	[illegible]
50	[illegible]	11.94	12.03	12.11	12.21	12.31	[illegible]	[illegible]
	[illegible]	[illegible]	.631	.656	.687	.718	[illegible]	[illegible]
	[illegible]	.878	.896	.914	.935	.956	[illegible]	[illegible]
	[illegible]	[illegible]	177.10	168.16	157.99	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	2977	2901	2812	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	17.23	17.35	17.49	17.62	[illegible]	[illegible]
	[illegible]	[illegible]	.834	.862	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	1.000	1.017	1.068	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 24 A—*Continued.*

105	90	75	60	45	30	20	$=p_1$	p_2
44.00	31.88	17.61					E	
1484	1263	939					V	
6.836	6.905	6.986					v_2	60
.663	.787	1.072					a	
.919	1.001	1.168					d	
65.53	53.62	39.50	22.32				E	
1811	1638	1406	1057				V	
8.799	8.885	8.988	9.115				v_2	45
.699	.781	.921	1.242				a	
.943	.997	1.083	1.257				d	
94.62	82.97	69.20	52.43	30.64			E	
2176	2038	1861	1620	1238			V	
12.57	12.69	12.84	13.01	13.25			v_2	30
.832	.897	.993	1.157	1.540			a	
1.029	1.069	1.125	1.214	1.389			d	
122.31	110.94	97.46	81.08	59.83	29.92		E	
2474	2356	2208	2014	1731	1223		V	
17.99	18.16	18.36	18.61	18.93	19.40		v_2	20
1.047	1.110	1.197	1.330	1.575	2.284		a	
1.155	1.189	1.235	1.301	1.416	1.705		d	
141.27	130.09	116.82	100.02	79.75	50.35	20.97	E	
2659	2551	2418	2237	1998	1587	1024	V	
23.23	23.44	23.70	24.03	24.42	25.02	25.63	v_2	15
1.258	1.323	1.411	1.547	1.760	2.269	3.602	a	
1.266	1.313	1.356	1.403	1.497	1.533	2.142	d	
166.77	155.81	142.82	127.00	106.55	77.77	49.01	E	
2889	2792	2674	2521	2309	1972	1566	V	
33.29	33.59	33.94	34.38	34.96	35.79	36.64	v_2	10
1.659	1.732	1.828	1.964	2.180	2.613	3.370	a	
1.454	1.485	1.526	1.581	1.666	1.824	2.071	d	
207.99	197.38	184.81	169.53	149.79	122.04	94.35	E	
3227	3143	3041	2913	2738	2471	2173	V	
61.89	62.42	63.06	63.84	64.88	66.37	67.90	v_2	5
2.763	2.860	2.986	3.156	3.412	3.868	4.500	a	
1.876	1.908	1.950	2.005	2.084	2.219	2.394	d	
258.01	247.81	235.74	221.04	202.14	175.92	149.08	E	
3593	3521	3435	3326	3180	2967	2719	V	
141.4	142.5	143.9	145.6	147.9	151.0	154.5	v_2	2
5.666	5.828	6.034	6.305	6.697	7.335	8.147	a	
2.686	2.724	2.772	2.833	2.920	3.056	3.221	d	

ordinates, one for the pressure p_1, the other for zero-pressure, are asymptotes to the curve, the latter rising to infinity for zero-velocity at the start and for infinite volume at perfect vacuum.

The study of the ideal form of the steam-jet has two important practical applications: from the entrance-curve ABC we deduce the rate of flow through an orifice under any conditions; while the discharge-curve CDE furnishes data for the determination of the proper shape of the nozzle which will deliver the jet of maximum attainable energy—this energy to be applied to the doing of useful work, especially in the steam-turbine.

§ 25. Steam-jet Curves.

(a) RATE OF FLOW.—From the shape of the entrance-curve ABC, Fig. 46, we see that, for a given initial pressure, the rate of flow through a given orifice will increase with the drop in pressure, at first rapidly, then slowly. This rate will reach its maximum when the lower pressure is at the value corresponding to the least section, at C; and any further reduction in the pressure on the discharge side will have no effect upon the rate of flow. This fact—that, when steam flows from a high pressure to a low, the pressure in the orifice will automatically adjust itself to the value which will give the maximum discharge—is embodied in Napier's experimental formula,

$$W=\frac{ap_1}{70}, \quad \dots \dots \dots \quad (120)$$

where a is the cross-section of the orifice in square inches, p_1 the absolute initial pressure in pounds per square inch, and W the discharge in pounds per second; this formula applying so long as p_2 is not greater than about $0.6p_1$.

(b) COMPARISON WITH NAPIER'S FORMULA.—In Fig. 46, the choke-point C, as at first located by a purely graphical determination of the lowest point of the curve, was found to be at from 0.57 to 0.59 of p_1, with 0.58 as the indicated mean value of the ratio. The C-points, as marked by short cross-lines, were then placed at exactly $0.58p_1$ on all the curves. Now if Napier's

formula is absolutely correct, we should have, letting a_0 represent the least section of the jet,

$$a_0p_1=70. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

By actual measurement of a_0 the following results were obtained —a few small discrepancies in the lower values, within the limit of graphical accuracy, being smoothed out:

TABLE 25 A. DATA FOR COMPARISON WITH NAPIER'S FORMULA.

p_1	a_0	a_0p_1	p_1	a_0	a_0p_1
250	.286	71.5	105	.664	69.7
220	.324	71.3	90	.771	69.4
195	.364	71.0	75	.920	69.0
170	.416	70.7	60	1.142	68.5
150	.470	70.5	45	1.510	68.0
135	.520	70.2	30	2.247	67.4
120	.584	70.0	20	3.340	66.8

An equation of relation found from these values of a_0p_1 is

$$a_0p_1^{.973}=61.55. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

In Fig. 46 an equilateral hyperbola is drawn through the C-point for 120 lbs., marked C_0: this curve $F'C'C_0G'$ is continued across the small-scale low-pressure curves as GH. The axes of the hyperbola are OM and ON, and it represents the assumption $a_0p_1=C$. Note that it drops below the C-points for high pressures and rises above them for low pressures, as the numerical values in Table 25 A would indicate.

(c) DEDUCTION FROM THE EXPONENTIAL EQUATIONS.—A result similar to that just set forth can be got by working from Equations (71) and (61); and this alternate method will now be developed, because it shows another way of approaching the problem of the behavior of the steam-jet, and leads to some useful results.

For adiabatic expansion we have

$$p_1v_1^n=p_2v_2^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

where $n=1.135$ for $x_1=1.00$: while the steam-volume formula for the dry-saturated condition is

$$p_1v_1^m=483, \quad m=1.065. \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Now for the work represented by the diagram ABCD in Fig. 45 we have—see § 7 (g)—

$$U=144\left[p_1v_1+\frac{p_1v_1-p_2v_2}{n-1}-p_2v_2\right]$$

$$=144\frac{n}{n-1}p_1v_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$=1210.7p_1v_1\left[1-\left(\frac{p_2}{p_1}\right)^{.1189}\right]. \quad . \quad . \quad . \quad . \quad . \quad (123)$$

From (61),

$$p_1v_1=483^{\frac{1}{m}}p_1^{\frac{m-1}{m}}=331.2p_1^{.06103}. \quad . \quad . \quad . \quad . \quad (124)$$

Then (123) becomes

$$U=1210.7\times 331.2p_1^{.06103}\left[1-\left(\frac{p_2}{p_1}\right)^{.1189}\right] \quad . \quad . \quad (123')$$

This is the energy stored in the jet, in foot-pounds; and to get the velocity V we proceed as in (118), finding

$$V=\sqrt{64.32U}=16060p_1^{.0305}\left[1-\left(\frac{p_2}{p_1}\right)^{.1189}\right]^{\frac{1}{2}} \quad . \quad . \quad (125)$$

Now the specific volume of the expanded steam is

$$v_2=v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}=\frac{331.2}{p_1^{.9390}}\left(\frac{p_1}{p_2}\right)^{.8810}=331.2p_1^{-.939}\left(\frac{p_2}{p_1}\right)^{-.881}. \quad (126)$$

Then for the area a we get

$$a=144(v_2\div V)$$

$$=\frac{144\times 331.2}{16060}p_1^{-.9695}\left(\frac{p_2}{p_1}\right)^{-.8810}\left[1-\left(\frac{p_2}{p_1}\right)^{.1189}\right]^{-.5} \quad . \quad . \quad (127)$$

Whence

$$ap_1^{.97}=2.970\frac{\left(\frac{p_1}{p_2}\right)^{.881}}{\sqrt{1-\left(\frac{p_2}{p_1}\right)^{.119}}} \quad . \quad . \quad . \quad . \quad . \quad (128)$$

The discrepancy between the value of the exponent of p_1 in this formula and that found by trial for Eq. (222) is due to the

fact that neither of the primary equations for this last discussion is in quite exact agreement with actual conditions.

(*d*) Discussion of Equation (128).—To calculate by this formula a set of values of a such as that given in Table 24 A and plotted in Fig. 46 would involve at least as much numerical work as the method there used, with the disadvantage that a number of important intermediate quantities would be left undetermined. An attempt to investigate for the minimum value of $(ap_1^{.97})$ by the standard method of equating to zero the first derivative in terms of the pressure-ratio would involve one in a mathematical maze. But this equation serves one very useful purpose in that it brings out clearly the fact, suggested by Fig. 46, that the a-curves, if considered on a basis of pressure-ratio rather than of absolute pressure, are similar in form. In other words, for a given ratio of p_2 to p_1, the ratio of a to a_0 is the same for all the curves. This is made apparent in Fig. 46 by drawing the hyperbola KBL through the initial point of the 120-lb. curve at B_0—the initial points of all these curves corresponding to $p_2=0.9p_1$, as before stated. It appears that the relation of this hyperbola to the B-points is similar to that of the curve F′C′G′-GH to the C-points.

(*e*) Curves of Discharge.—Having then the values of a_0, given in Table 25 A or found by (122), we need only determine and record the ratio of a to a_0 for the full range of pressure-ratio in order to be able to compute the area of the pound-second orifice for any conditions, or, conversely, the discharge per square inch. This idea of using pressure-ratio as a base is first applied in Fig. 47, where the curves of Fig. 46 are redrawn on this system. This figure is intended to serve as a graphical table, from which can be read either the area of cross-section or the diameter of a pound-second jet of initially dry steam. The practically useful part of the diagram is that from the left margin to the choke-line CC: the part to the right of this line is added merely to show the character of the beginning of the expansion of the jet. In laying out the curves, values of a are measured from the lower base-line, values of d from the top line, as indicated by the scales. The line PP is drawn to show how the pressure drops, the full height of the diagram representing the initial pressure.

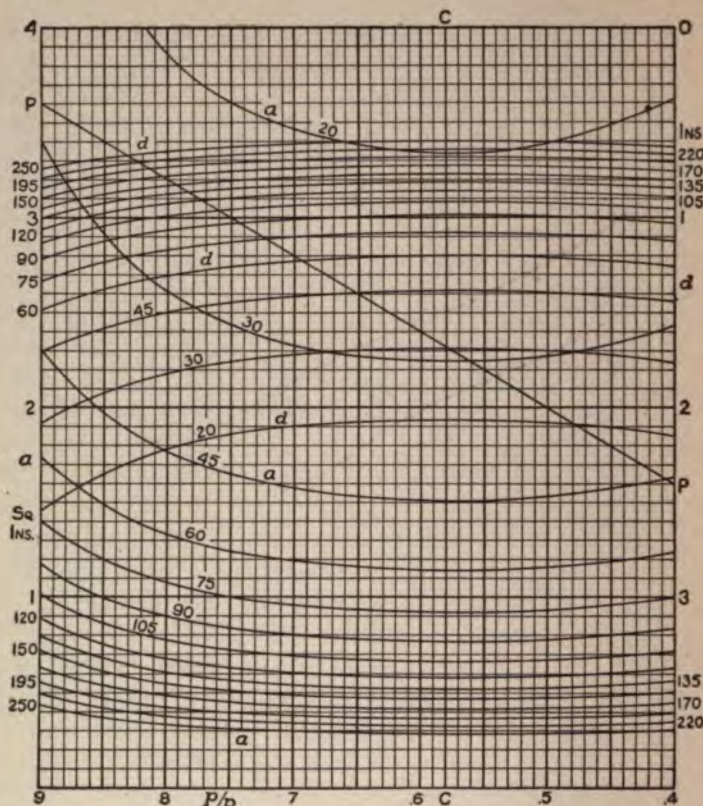


FIG. 47.—Curves of Discharge.

As a test of the similarity of these curves, the following results were measured and computed from the original of Fig. 47, for $p^* = 0.9p_1$:

TABLE 25 B. DATA FROM FIG. 47.

$p_1 =$ 250	195	150	120	90	60	30	20
$a_0 =$ .286	.364	.470	.584	.771	1.142	2.247	3.340
$a =$ .433	.557	.720	.895	1.177	1.736	3.31	5.07
$\frac{a}{a_0} =$ 1.57	1.53	1.53	1.53	1.525	1.52	1.473	1.52

* Dropping the subscript, we shall henceforth use the plain symbol p for the variable lower pressure, p_1 still standing for the initial pressure.

It appears that these curves, while accurate enough for most purposes, are not to be depended upon for very precise results—the one for 30 lbs. showing an error of about 3 per cent.: but that the ratio of areas is substantially the same for a given pressure-ratio, for all the curves, seems to be quite well established.

Example 1.—If steam flows from a vessel at 150 lbs. pressure into one at 120 lbs., through a properly shaped orifice of 1 sq. in. area, how much will be discharged in one minute?

Here $p_1=150$, $p=120$, $p/p_1=.8$.

From the 150-lb. curve at 0.8 we read $a=.55$ sq. in., for 1 lb per sec.; then 1 sq. in. will discharge $1\div.55=1.82$ lbs. per sec., or 109.2 lbs. per min.

Note.—The question as to what is the proper shape of orifice for the securing of this full theoretical discharge will be discussed in § 27.

(*f*) Expansion of the Jet.—The representation of the curves of Fig. 46 in the manner of Fig. 47 is completed in Fig. 48. Here, however, the pressure-ratio is not laid out to a uniform scale, as that would unduly separate the ordinates at the left, while crowding those at the right. A better shape for the curves is got by making the abscissa proportional to the square root of the reciprocal of the ratio $p:p_1$, that is, to $\sqrt{p_1\div p}$. The resulting variable scale of pressure-drop is given at the top of the diagram, and the manner in which the pressure varies is shown by the curve PP. Points on the curves corresponding to different absolute terminal pressures, from 15 lbs. to 2 lbs., are marked by small circles and joined by curves showing their relations.

An interesting coincidence made evident by the shape of the *d*-curves is, that the kind of pressure-change assumed in spacing the ordinates of this figure is what will occur in a conical nozzle.

Drawn on section-paper with small divisions, and to a large scale, these curves would be useful for determining, with the fullest accuracy needed in practice, the dimensions of the flaring nozzle which is essential to the production of a perfect steam-jet: that is, of a jet in which is realized the full utilization of energy described in § 25 (*a*). Thus in Fig. 44, the conditions being the same as those of Fig. 45, where $p_1=150$ and $p_2=15$, the diameter of the tube D is to the diameter of the throat C in the ratio of the numerical values 1.20″ and .77″ scaled from the *d*-curve for 150

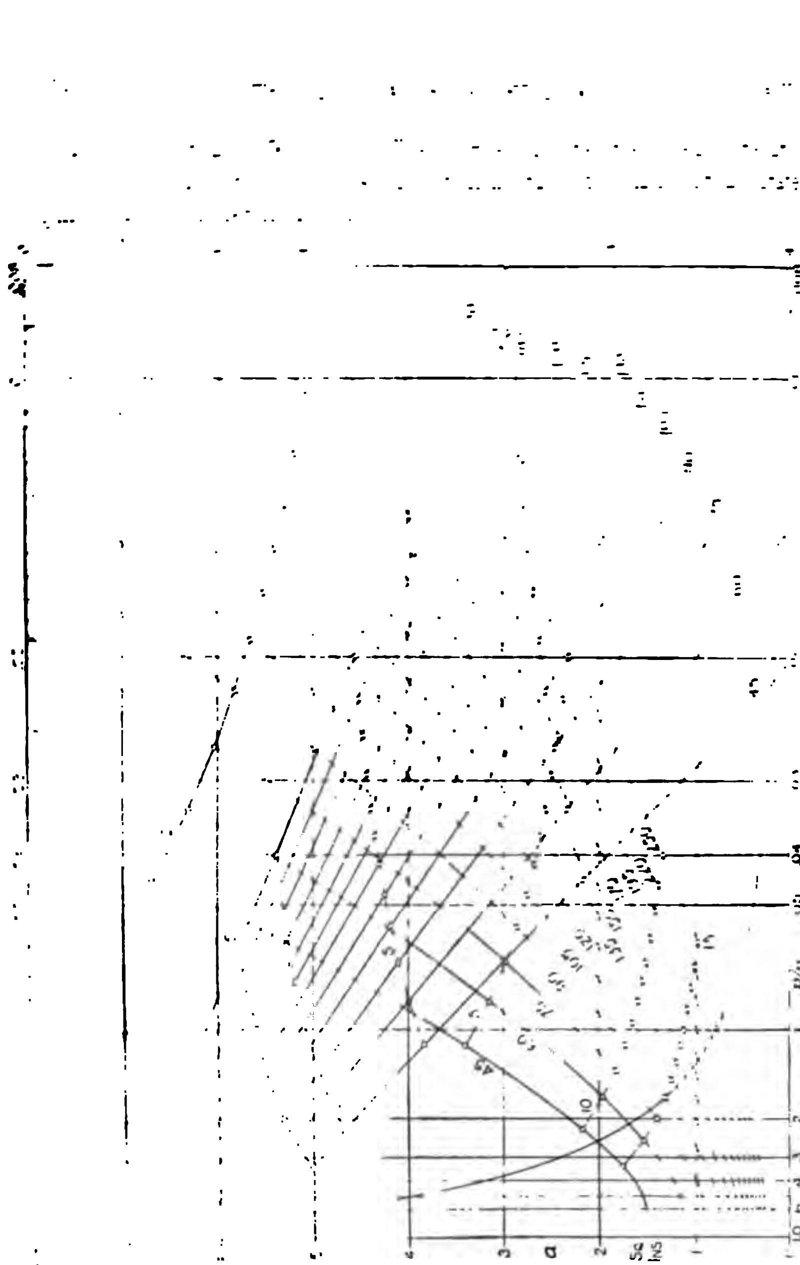


FIG. 48.—Curves of Jet Expansion

. . . up a discussion of the shape of the confining . . . outside influences which affect the energy-

transformation in the development of the steam-jet, and before considering the various ways in which the energy of the jet is applied to the doing of useful work, we will put into a form still more convenient for general use the results of the computations embodied in Figs. 46, 47, and 48; combining the methods of § 24 (*e*) and of § 25 (*c*) as may seem advisable.

§ 26. A General Determination of the Adiabatic Jet.

(*a*) Scope of the Discussion.—In Figs. 49, 50, and 51, and in the accompanying tables are presented the following:

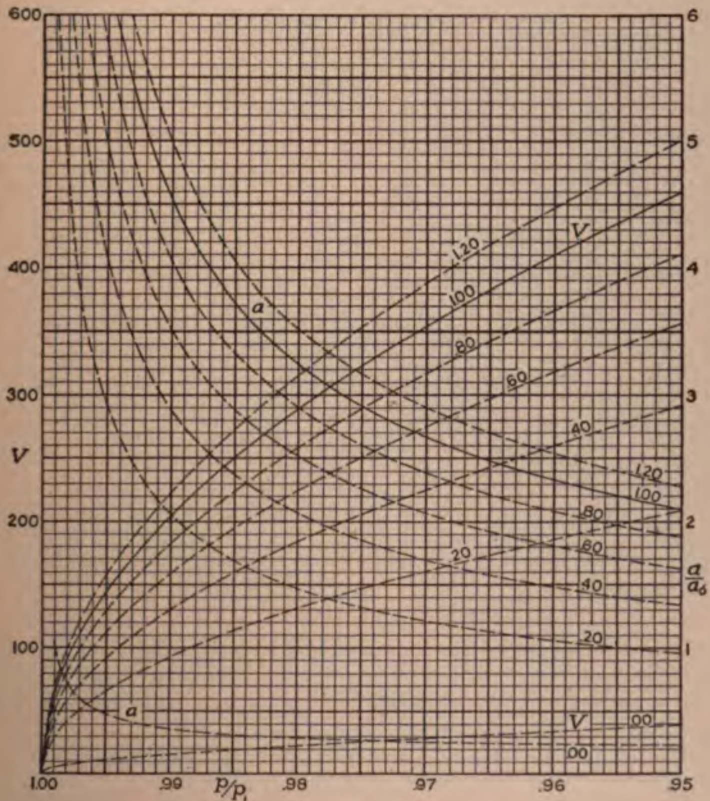
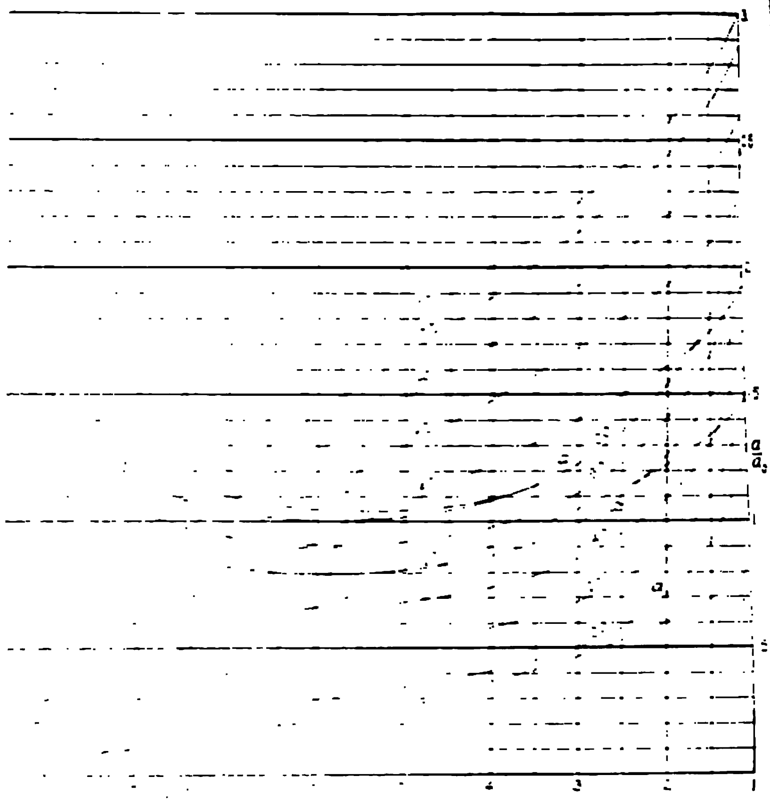


Fig. 49.—Curves for Small Pressure-drop.

... area of the jet ... steam ... shown ... numerical



... of superheat – these ... quantitative.

... assume ideal condi- ... actions: con-

... terms to Fig. 47, the

base-line representing pressure-ratio to a uniform scale. Figs. 49 and 51 show the extremes of the operation, spread out laterally, Fig. 50 the wide intermediate range: the several ordinate-scales are chosen with a view to the space available and the range of

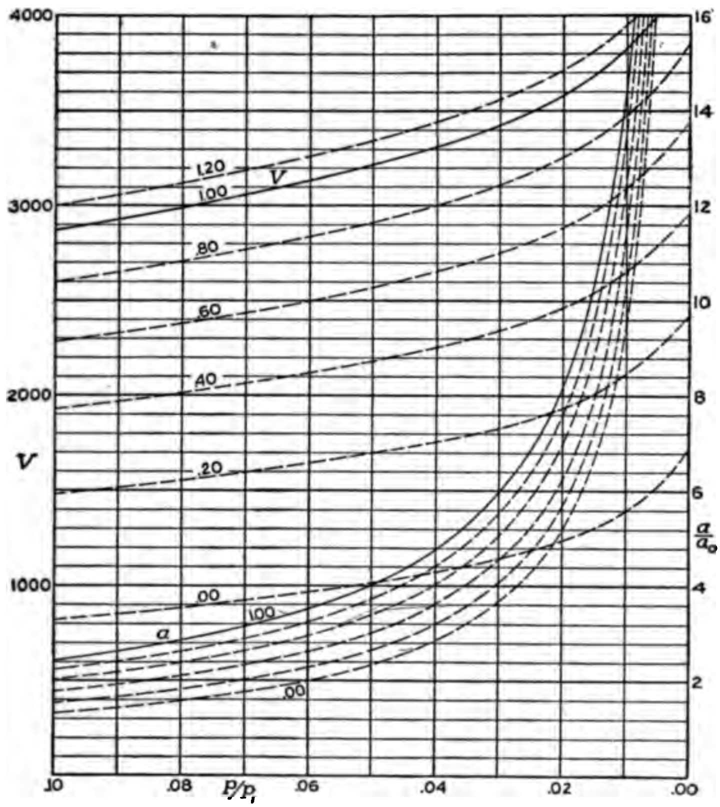


FIG. 51.—Terminal Conditions.

each group of curves, and are marked on the figure, that for V at the left, that for a/a_0 at the right. Note that the areas of section are given, not in absolute measure, but as compared with the choke-area of the dry-steam curve, the same a_0 as in the preceding section.

These diagrams are all drawn from values computed for $p_1 = 120$

TABLE 26 A. STEAM-JET QUANTITIES FOR $p_1 = 120$, $x_1 = 1.00$.

1 Ratio of Pressures. R_p	2 Energy of Jet, Foot-pounds. U	3 Velocity of Jet, Feet per sec. V	4 Ratio of Areas. a/a_0	5 Steam-weight Divisor. K	1 Ratio of Pressures. R_p	2 Energy of Jet, Foot-pounds. U	3 Velocity of Jet, Feet per sec. V	4 Ratio of Areas. a/a_0	5 Steam-weight Divisor. K
.999	64	64	14.31	1002.	.84	11,034	843	1.267	88.73
.998	128	91	10.06	701.	.83	11,784	871	1.237	86.61
.997	192	111	8.26	579.	.82	12,542	898	1.209	84.68
.996	256	129	7.11	498.	.81	13,307	925	1.184	82.93
					.80	14,080	952	1.162	81.38
.995	321	144	6.38	447.	.79	14,861	978	1.143	80.04
.994	385	157	5.86	410.	.78	15,650	1004	1.126	78.85
.993	449	170	5.44	381.	.77	16,448	1029	1.111	77.81
.992	514	182	5.08	356.	.76	17,255	1054	1.098	76.90
.991	578	193	4.78	335.	.75	18,071	1079	1.087	76.12
.990	643	203	4.54	318.	.74	18,896	1103	1.077	75.42
.988	772	223	4.14	291.	.73	19,730	1127	1.068	74.77
.986	901	241	3.85	270.	.72	20,573	1150	1.059	74.17
.984	1030	257	3.62	253.5	.71	21,425	1173	1.051	73.62
.982	1160	273	3.42	239.5	.70	22,286	1197	1.044	73.11
.980	1290	288	3.24	227.1	.69	23,157	1220	1.037	72.63
.978	1420	302	3.09	216.4	.68	24,038	1243	1.031	72.19
.976	1550	316	2.96	207.3	.67	24,930	1266	1.025	71.79
.974	1681	328	2.85	199.6	.66	25,833	1289	1.020	71.44
.972	1811	341	2.75	192.6	.65	26,748	1312	1.016	71.14
.970	1942	353	2.66	186.1	.64	27,675	1335	1.012	70.86
.965	2270	382	2.48	173.9	.63	28,615	1358	1.008	70.61
.960	2600	409	2.33	163.4	.62	29,569	1380	1.005	70.40
.955	2931	434	2.20	154.3	.61	30,538	1402	1.003	70.25
.950	3264	458	2.09	146.6	.60	31,522	1424	1.002	70.18
.94	3935	503	1.923	134.7	.59	32,522	1446	1.001	70.11
.93	4613	545	1.795	125.7	.58	33,539	1469	1.000	70.04
.92	5299	584	1.690	118.4	.57	34,574	1491	1.000	70.04
.91	5992	620	1.604	112.3	.56	35,627	1514	1.001	70.11
.90	6692	655	1.533	107.4	.55	36,699	1536	1.002	70.18
.89	7398	689	1.473	103.2	.54	37,791	1559	1.003	70.25
.88	8111	722	1.421	99.53	.53	38,903	1581	1.005	70.40
.87	8831	754	1.376	96.37	.52	40,036	1604	1.008	70.61
.86	9558	785	1.336	93.57	.51	41,191	1627	1.012	70.86
.85	10,292	814	1.300	91.05	.50	42,368	1650	1.016	71.14

TABLE 26 A—*Continued.*

R_p	U	V	a/a_0	K	R_p	U	V	a/a_0	K
.49	43,570	1674	1.020	71.44	.24	83,950	2324	1.386	97.10
.48	44,790	1697	1.025	71.77	.23	86,170	2355	1.423	99.60
.47	46,040	1720	1.030	72.13	.22	88,480	2386	1.462	102.4
.46	47,310	1744	1.036	72.53	.21	90,870	2418	1.503	105.3
.45	48,610	1767	1.042	72.97	.20	93,360	2451	1.546	108.3
.44	49,940	1791	1.049	73.45	.19	95,950	2484	1.592	111.5
.43	51,290	1815	1.056	73.97	.18	98,673	2519	1.640	114.9
.42	52,670	1840	1.064	74.53	.17	101,540	2556	1.691	118.4
.41	54,080	1865	1.073	75.13	.16	104,570	2594	1.744	122.1
.40	55,520	1890	1.082	75.78	.15	107,800	2634	1.803	126.3
.39	56,990	1915	1.092	76.48	.14	111,250	2676	1.873	131.2
.38	58,500	1940	1.103	77.25	.13	114,980	2720	1.960	137.3
.37	60,040	1965	1.115	78.09	.12	119,020	2767	2.072	145.1
.36	61,610	1991	1.128	79.01	.11	123,440	2818	2.220	155.4
.35	63,230	2017	1.142	79.99	.10	128,280	2873	2.420	168.8
.34	64,880	2042	1.157	81.04	.09	133,570	2931	2.63	184.2
.33	66,570	2058	1.173	82.16	.08	139,410	2994	2.87	201.0
.32	68,300	2094	1.190	83.36	.07	145,840	3063	3.14	219.9
.31	70,080	2121	1.208	84.64	.06	153,120	3139	3.52	246.5
.30	71,900	2149	1.228	86.00	.05	161,500	3223	4.01	280.9
.29	73,770	2178	1.249	87.46	.04	171,200	3318	4.77	334.1
.28	75,690	2206	1.271	89.04	.03	182,900	3430	5.98	418.8
.27	77,670	2235	1.296	90.77	.02	200,000	3587	8.17	653.9
.26	79,700	2264	1.323	92.68	.01	228,900	3837	14.53	1018.
.25	81,790	2294	1.354	94.79	.00	281,600	4256	∞	∞

lbs. abs., the pressure for which, according to Table 25 A, Napier's formula, agrees exactly with the theoretical deduction; and which is taken as a good representative mean value.

The curves are distinguished by marking them with the particular value of x_1 belonging to each. The initially superheated curve is drawn for a degree of superheat which will make the initial volume exceed by 20 per cent. that for dry saturated steam, which corresponds to $x_1 = 1.00$; and by a sort of analogy, limited strictly to volumes, as distinguished from heat-quantities, this state is indicated by letting x_1 be 1.20 and marking the curve with this value.

(*b*) THE CASE OF STEAM INITIALLY DRY.—In Figs. 46, 47, and 48, and the accompanying text, this case is covered by working out fully the curves for a large number of initial pressures.

By interpolation in Table 24 A or on the diagrams, values can be got for intermediate conditions. In Figs. 49, 50, and 51, and especially in Tables 26 A and 26 B, this same matter is put into more convenient shape for general numerical application.

(c) IN TABLE 26 A the following quantities are given:

Col. 1. The "independent variable," the abscissa of the diagrams, is the ratio of the changing lower pressure p to the fixed initial pressure p_1, so that $R_p = p \div p_1$.

Col. 2. The quantity U is the same thing as E in Tables V. and 24 A, but the energy is here reduced to F.P. instead of being expressed in H.U.

Col. 3. V is the velocity of the steam corresponding to the kinetic energy U.

Items 2 and 3 belong to the particular initial pressure for which the table is worked out.

Col. 4. gives the ratio of the cross-section at any pressure to the least area of section, at the choke. Assuming that the curves are similar for different initial pressures, this column is of general application.

Col. 5. The divisor K is analogous to the constant 70 in Napier's formula, the equation for the rate of flow past any particular cross-section of the jet (that at which the pressure has any value p) being, in the same terms as (120),

$$W = \frac{ap}{K}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (129)$$

These values of K belong to this particular pressure; they are got by multiplying the special choke-area $a_0 = 70.04$ by the general ratios of areas given in Col. 4.

Cols. 3 and 4 are represented by the curves marked 1.00 in Figs. 49 to 51: all the quantities are here reduced to close numerical expression, and the smallness of the divisions facilitates interpolation. The divisions are especially close over the first part of the range, which covers the conditions of the flow of steam in pipes, where the pressure-drop is small.

(d) TABLE 26 B, through which the full determination of 26 A is applied to any case, may be described—and its use illustrated—as follows:

TABLE 26 B. CONSTANTS FOR DIFFERENT INITIAL PRESSURES.

1 p_1	2 R_U	3 R_V	4 R_a	5 a_0	6 K_0	7 R_K
250	1.0458	1.0226	.4896	.2858	71.45	1.0201
240	1.0432	1.0214	.5094	.2974	71.38	1.0191
230	1.0405	1.0201	.5310	.3099	71.28	1.0177
220	1.0377	1.0187	.5545	.3236	71.19	1.0164
210	1.0347	1.0172	.5801	.3386	71.11	1.0153
200	1.0317	1.0155	.6083	.3551	71.02	1.0140
190	1.0284	1.0141	.6395	.3733	70.92	1.0126
180	1.0250	1.0124	.6740	.3934	70.81	1.0110
170	1.0215	1.0107	.7125	.4159	70.70	1.0094
160	1.0177	1.0088	.7558	.4412	70.58	1.0078
150	1.0137	1.0068	.8050	.4698	70.47	1.0061
140	1.0095	1.0047	.8607	.5024	70.34	1.0043
130	1.0049	1.0024	.9250	.5400	70.20	1.0023
120	1.0000	1.0000	1.000	.5837	70.04	1.0000
110	.9947	.9974	1.088	.6353	69.88	.9976
100	.9889	.9945	1.194	.6969	69.69	.9951
95	.9858	.9929	1.255	.7327	69.61	.9939
90	.9826	.9913	1.323	.7722	69.51	.9924
85	.9792	.9895	1.399	.8164	69.39	.9908
80	.9756	.9877	1.484	.8660	69.28	.9892
75	.9717	.9858	1.580	.9221	69.16	.9874
70	.9676	.9837	1.690	.9862	69.03	.9856
65	.9633	.9815	1.816	1.0596	68.89	.9836
60	.9586	.9791	1.963	1.1455	68.74	.9814
55	.9535	.9765	2.136	1.2469	68.58	.9792
50	.9480	.9736	2.344	1.368	68.41	.9767
45	.9419	.9705	2.597	1.516	68.22	.9740
40	.9351	.9670	2.912	1.700	68.00	.9709
35	.9276	.9631	3.316	1.936	67.76	.9674
30	.9189	.9586	3.853	2.249	67.47	.9633
25	.9087	.9533	4.601	2.686	67.15	.9587
20	.8964	.9468	5.717	3.337	66.74	.9529
15	.8808	.9385	7.563	4.415	66.23	.9456
10	.8593	.9270	11.221	6.550	65.50	.9352
5	.8237	.9076	22.028	12.857	64.29	.9179

Col. 1. p_1 is the initial pressure in pounds per square inch absolute, from which the expansion is to begin, with $x_1=1.00$, for any case.

Col. 2. R_U is the ratio of the jet-energies, comparing the energy due to expansion from any p_1 to $R_p \times p_1$ with that due to

$=.584 \times .608 = .355$ (that is, a_0 for 120 multiplied by R_a for 200). Now for $R_p = 0.1$ we get from Table 26 A, Col. 4,

$$a = 2.410\,a_0 = 2.410 \times .355 = .856 \text{ sq. in.};$$

and for one-quarter of a pound per second the area would then be .214 sq. in. Note that the value of a, .86 sq. in., could be got roughly by interpolation on the 0.1 line in Fig. 48.

Col. 6. K_0 is the Napier divisor, corresponding with a_0p_1 in Table 25 A, and found by evaluating this product a_0p_1, using Cols. 1 and 5. Compare Cols. 5 and 6 of this Table with the similar quantities in Table 25 A.

Col. 7. R_K is the ratio of each K_0 to that for 120, found by direct division. It can be used for modifying Col. 5 of Table 26 B so as to fit any case, since all other similarly located K's are in the same ratio as the K_0's.

EXAMPLE 3.—How much steam will be discharged through an orifice of 1 sq. in. area from a chamber at 90 lbs. into one at 75 lbs.?

Here $R_p = .833$, and from Table 26 A, Col. 5, K is found by interpolation to be 87.3. This would be for $p_1 = 120$; for $p_1 = 90$ we must multiply it by $R_K = .9923$, getting 86.6 as the corrected value. Then by substitution in the general formula (129),

$$W = a\frac{p_1}{K} = 1.0\frac{90}{86.6} = 1.039 \text{ lbs. per sec.}$$

This R_K, with the K-values in Col. 5 of Table 26 A, furnishes the simplest solution to the problem stated in this example.

By means of these two tables all questions as to the dimensions of the ideal steam-jet with steam initially dry-saturated can be answered with a degree of accuracy quite sufficient for any practical purposes.

(e) MIXTURES OF STEAM AND WATER.—The curves on Figs. 49 to 51, for values of x_1 from .80 down to .00, are laid out from computations made according to the methods of § 24 (d). As an example of these methods, take the case where $x_1 = .60$, p/p_1 or $R_p = .9$, and find values of U, V, v, a, etc.:

From an adiabatic table like Table V., but with a different set of values of p_2, the limiting values of E and v are, for $p_2 = 108$ lbs.,

$$E = 8.603, \qquad E_0 = .079, \qquad E_1 = 8.524 \text{ H.U.}$$
$$v_2 = 4.055, \qquad v_{20} = .055 \text{ cu. ft.}$$

Now

$$E_4 = E_1 - \Delta E_2 = [illegible] = 5.193 \text{ H.U.},$$

or

$$E = [illegible] = 4040 \text{ F.P.}$$

From this

$$V = [illegible] = [illegible] \text{ ft. per sec.}$$

Again,

$$v = [illegible] = 2.455 \text{ cu. ft.}$$

and

$$a = 144\frac{v}{V} = 144\frac{2.455}{[illegible]} = .693 \text{ sq. ins.}$$

For $x_1 = 1.00$, $a_1 = .584$ [illegible] so that [illegible] $= .693 \div .584 = 1.186$; on the diagram, which was drawn from slide-rule computations, the corresponding value is about 1.19.

The steam-expansion diagrams for this whole series of curves are given in Fig. 52, similar in general terms to Fig. 45. These diagrams make graphic the changes in the energy of the jet and in the specific volume of the steam, caused by variation in the initial steam-fraction x_1—the intermediate adiabatics dividing the space between curve 1.00 and curve .0 into five equal parts. It will be noted that, in order to show both ends of the expansion with reasonable clearness, it was necessary to adopt a device just [illegible] reverse of that used when combining indicator-cards from a [illegible] engine—enlarging the pressure-scale and shortening up [illegible] in the lower part of the diagram. In this particular [illegible] two changes are in the same ratio, pressures being five [illegible] one-fifth, as large in II. as in I.: as a result, the [illegible] unit of area is the same for both parts of the diagram, [illegible] directly comparable. This figure brings out clearly [illegible] of the expansion into the vacuum-range, below [illegible] atmospheric pressure, especially when the expansion- [illegible] clear down to the line of back-pressure, instead of [illegible] short as in the ordinary engine.

[illegible] point to be noted on Fig. 51 is the way that [illegible] meet the line of zero pressure, at the right edge of the [illegible] the terminal points are not determined accurately here,

the curves being simply carried forward from the lowest computed value, at $p=1$ lb. or $R_p=.00833$. This brings out clearly the fact that, even though v goes to infinity for $p=$zero, Fig. 52, the area of the expansion diagram has a finite limit.

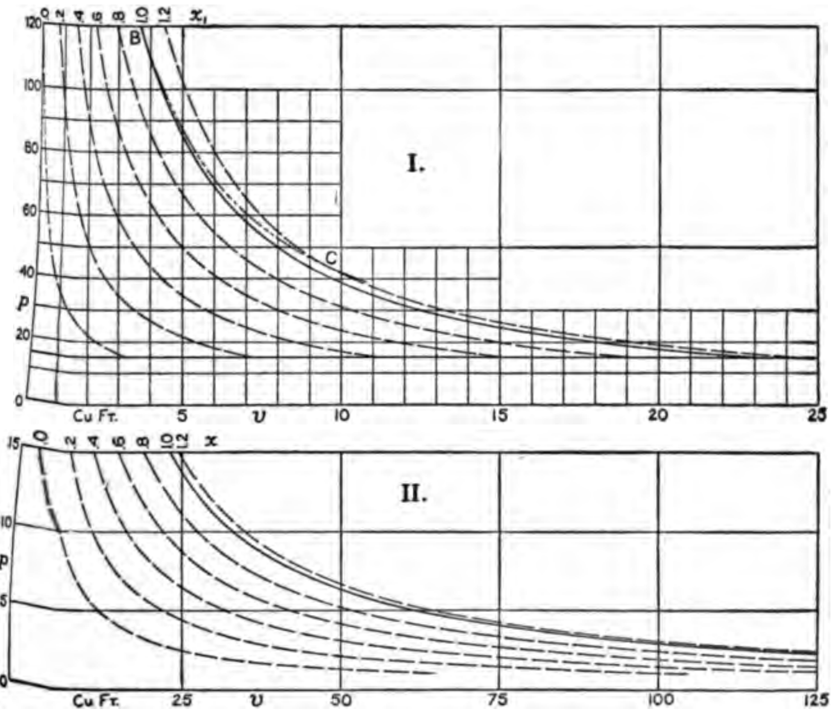


FIG. 52.—Pressure-volume Diagrams for Figs. 49 to 51.

(*f*) SUPERHEATED STEAM.—The effect of initial superheat is illustrated by the curves marked 1.2, as explained in (*a*). Computations for the upper part of this curve were made by the exponential method, using Eq. (72),

$$pv^{1.333}=C.$$

With this index $n=1.333$, the expansion-curve in Fig. 52 will drop much more rapidly than the curve of constant steam-weight, for which $n=1.065$. The latter is drawn in as BC, for $x_1=1.00$,

and the adiabatic 1.2 crosses it at C, at about 45 lbs. pressure—which means that the steam passes, at this point, from the superheated to the saturated state. From here on the index will be 1.135, according to Eq. (71).

In making the calculations for curve 1.2, the change of index from 1.333 to 1.135 was not made abruptly, but a shading-off effect was introduced, n being gradually decreased in successive 5-lb. intervals as the adiabatic approached BC. The manner of this variation is a matter of judgment, as the behavior of superheated steam near the saturation-curve is not known precisely.

A noticeable fact is the small relative value of the superheat, which value decreases as the pressure falls. The amount of heat put into the steam in order to produce a given change of volume during superheating at constant pressure is so much less than that involved in an equal volume-change during evaporation, that it represents a comparatively small part of the internal energy upon which draft is made during a subsequent adiabatic expansion.

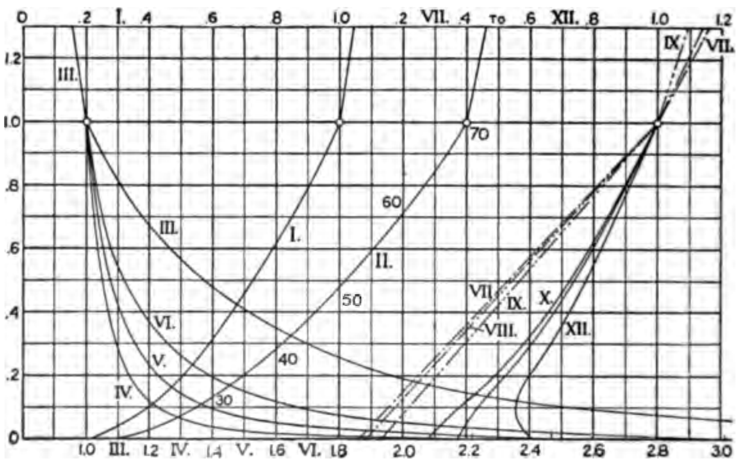


FIG. 53.—Curves of Comparison.

(*g*) COMPARISON OF CURVES FOR DIFFERENT INITIAL STATES.—Glancing over the curves on Figs. 49 to 51, we note a general similarity in form, which becomes greater as the pressure drops

As x_1 decreases, the curves depart from those for $x_1=1.00$, at an increasing rate. An approximate locus of the choke-point is sketched on Fig. 50, at CC, showing how this point is reached earlier in the expansion as the initial fraction of water is greater. As to the a-curve for 1.2, soon after passing its choke-point it reaches a place where the increase of V and that of v over the corresponding values for $x_1=1.00$ are in the same proportion; and from there on these two curves agree, the differences shown by the calculated values being too small to appear on the diagram.

In Fig. 53 and in Table 26 C, a comparison between these curves is expressed in quantitative terms: corresponding curves and columns in the table are designated by the same number. In the figure, x_1 is the independent variable, measured vertically; and the abscissas have various meanings, being measured according to the several scales marked along the top and bottom of the diagram. Described in detail, the matters here set forth are as follows:

I. This a_0 is the choke-area for each condition, in terms of that for $x_1=1.00$. It will be noted that most of the quantities given are, like this one, ratios rather than absolute quantities in any particular measure.

II. K_0 is the number by which a_0p_1 (a_0 in square inches, here) is to be divided in order to get the weight discharged per square inch of choke-area. The question as to how nearly constant this K_0 is, for low values of x_1, as p_1 varies, has not been gone into: it would be a matter of interest, though of no practical importance in this connection, to fix the constants of formulas analogous to (122) and (129), for hot water.

III. W, the reciprocal of a_0 in I., is the relative weight discharged through a certain area of choke from the various states. It increases quite rapidly with the proportion of water: but a comparison of the rate of efflux of hot water with that of cold water under the same pressure-conditions would show that the latter is much greater; the reason appears when we note how very small a portion of the pressure-drop is available to accelerate the jet of hot water before the choke is reached—see the a-curve for $x_1=.00$ on Fig. 49.

TABLE 26 C. THE EFFECT, UPON THE STEAM-JET,

	I.	II.	III.	IV.	V.	VI.
x_1	a_0	K_0	W	R_a		
				0.2	0.1	.01
1.2	1.030	72.1	.971			
1.1	1.150	71.1	.984			
1.0	1.000	70.0	1.000	1.000	1.000	1.000
.9	.954	66.8	1.049	1.005	1.010	1.016
.8	.905	63.4	1.106	1.015	1.024	1.033
.7	.849	59.4	1.178	1.024	1.039	1.054
.6	.790	55.3	1.267	1.036	1.055	1.080
.5	.727	50.9	1.376	1.053	1.078	1.122
.4	.660	46.1	1.515	1.07	1.11	1.18
.3	.589	40.9	1.697	1.10	1.16	1.26
.2	.509	35.0	1.963	1.14	1.23	1.39
.1	.396	27.4	2.525	1.23	1.39	1.68
.0	.217	15.2	4.61	2.22	2.47	3.02

IV., V., VI. These curves compare the ratios of expansion of the jet, each in terms of its own choke-area. The numbers 0.2, 0.1, .01, at the heads of the columns, are values of R_p, the ratio of pressure-drop which forms the base in Figs. 49 to 51, three particular cases being investigated. As an illustration, let a be the area at $p=0.1p_1$: then for curve 1.00, $a/a_0=2.420$, from Table 26 A; for curve .20, $a'=1.515\,a_0$, from Fig. 50; while $a_0'=.509\,a_0$ as above; therefore $a'/a_0'=1.515\div.509=2.976$, and the ratio of the two jet-expansion ratios is $R_a=2.976\div2.420=1.23$. What we show by these curves is how much the rate of area-growth of initially wet jets exceeds that of the jet from dry steam. It will be noticed that the difference in relative proportions increases as the terminal pressure is lower.

VII., VIII., IX. For the same limits as the preceding set of three curves, these show how the energy of any jet compares with that of the 1.00 jet, when expanded through the same range. The "curves" are, of course, straight lines, the energy varying exactly as the steam-fraction: the base, the energy U of the dry jet, is a different quantity for each column.

OF VARIANT INITIAL COMPOSITION. TABLE 26 C.

	VII.	VIII.	IX.	X.	XI.	XII.
x_1	R_U			$R_U \times W$		
	0.2	0.1	.01	0.2	0.1	.01
1.2	1.104	1.092	1.063	1.071	1.060	1.032
1.1						
1.0	1.000	1.000	1.000	1.000	1.000	1.000
.9	.906	.908	.913	.950	.953	.958
.8	.812	.816	.826	.898	.902	.913
.7	.718	.724	.740	.845	.852	.871
.6	.624	.632	.653	.790	.800	.827
.5	.530	.540	.566	.729	.743	.779
.4	.436	.449	.480	.660	.681	.728
.3	.342	.357	.393	.580	.606	.667
.2	.248	.265	.306	.487	.520	.601
.1	.154	.173	.219	.389	.437	.553
.0	.060	.081	.132	.276	.374	.608

X., XI., XII. The preceding comparison is for jets of the same rate of flow, measured in weight/time. Multiplying this R_U by the relative weight W, we get a ratio of energies per unit of area of cross-section of throat of jet.

On the superheated side, the one determination made is not enough to fix the shape of the curves: those that are extended above $x_1 = 1.00$ are simply carried up as straight lines, and have only a roughly illustrative value.

(*h*) NOTES ON METHODS OF COMPUTATION.—Before leaving this part of the subject, it may be well to call attention to a few special points in connection with the computations preliminary to the laying out of Figs. 49 to 51.

In making up the adiabatic table for $p_1 = 120$, it was found that the methods of § 24 (*c*) could not be altogether depended upon for accurate results in the upper part of the table, above $p = .8p_1$. Thus, down to $p = 108$ lbs., it was necessary to modify slightly the values of E, got by (116), in order to have them increase smoothly; although this correction, ranging in value from .01 to .04 H.U., was not greater than the possible inaccuracy of computation with 5-place logarithms.

The correction just described is of practical insignificance; but in the finding of E_0, which, while itself a small quantity, must nevertheless be quite accurately determined if we are to measure closely the energy expended in the first part of the expansion of hot water from high pressure, greater difficulty was encountered. The subtraction of E_1 from E, used farther down in the table, as in Table V., was not satisfactory; and in trying to get E_0 by Eq. (116), making $x_1=0$, it was found that, down to $p=108$ lbs., $(q_2+x_2r_2)$ came out equal to q_1, instead of being less by an appreciable and increasing amount. Resort was then had to the simple mechanical method of dividing the upper part of the diagram, like Fig. 45, into horizontal strips of the width $\Delta p=1$ lb., finding the mean of the initial and final volumes during this pressure-drop of 1 lb., and getting the area in foot-pounds by $\Delta U_0=144\Delta p\times v_m=144v_m$; or in heat-units by $\Delta E=.185v_m$: then by adding these partial values cumulatively, successive E_0's were obtained. This method was used down to $p=96$ lbs., where it was found to come into agreement with $E_0=E-E_1$. The v's were computed in the regular manner, from $v_{20}=w_{20}+x_{20}s_2$.

Fig. 24 shows that Zeuner's exponental adiabatic formula, Eq. (71), fails for values of x_1 below .6. In connection with the difficulty above described, an effort was made to find out whether a different index would give suitable curves; but it soon became apparent that adiabatics for mixtures in which the water-weight predominates cannot be represented by an equation of the form $pv^n=C$.

§ 27. Form and Influence of the Nozzle.

(a) GENERAL STATEMENT.—The preceding determination of the form of the steam-jet, made in § 24 and elaborated in the next two sections, while it is fully worked out along certain lines, is yet not quite complete. Based upon the fundamental assumption that there is no loss or deflection of available energy in the formation of the jet, definite relations have been found between internal pressure on one hand and velocity and sectional area on the other; but all questions as to the manner of the pressure-drop, with reference to distances along the center-line of the jet, have

been left for settlement to the confining surface, that is, to the nozzle. It has been assumed that the shape of the nozzle, and the consequent play of guiding and of internal forces, is such as will accompany and help to produce the desired perfect transformation of energy into the kinetic form.

Now it is evident that within the jet certain definite, limited forces act upon a limited mass of steam; so that there must be a limit to any force-effect, as, for instance, to the rapidity with which the jet can change its area of cross-section. As a preliminary, then, to the study of the influence of different shapes of the confining body and of different conditions of discharge, we will now take up an investigation into the force-action within the jet; and especially will try to determine the shape which the jet tends to assume under the action of its own internal forces alone, the externally applied guiding forces being eliminated as far as possible. This discussion will be partly only in general terms, but will lead also to some quantitative results.

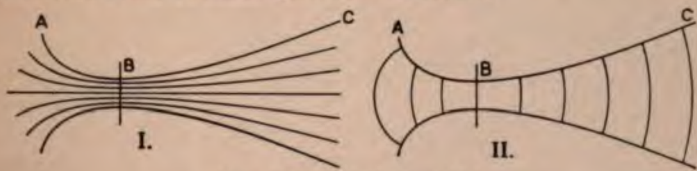


FIG. 54.—The Natural Jet.
I. Stream Lines. II. Loci of Uniform Condition.

(b) THE CONDITIONS WITHIN THE JET, as to motion and as to force, are partly illustrated in Fig. 54. I. shows the lines of flow, along which the several small streams or filaments of the jet tend to travel: these converge to the choke at B, then diverge. The shape of any one of these stream-lines—that is, the path of any minute portion of the vapor—is the resultant effect of a linear velocity along the line of travel, of a direct acceleration along this line, which is due to the expansion of the steam and increases the velocity, and of a transverse acceleration, at right angles to the velocity and continually deflecting it outward. The limit of curvature is reached when the centrifugal force of the stream, on account of the curved path, just equals the expansive pressure which pushes it away from the center-line.

It is now apparent that a certain condition as to pressure, velocity, and energy of the steam will exist, not in a plane section of the jet, but over a curved surface. On a longitudinal section of the jet these surfaces of uniform condition will trace the curves shown in Fig. 54 II. The area a worked out for Figs. 46 to 51 is really the area of this curved surface: which surface is at all points perpendicular to the lines of flow. When, however, the nozzle diverges but slowly, then the error caused by taking a as a plane cross-section is insignificant.

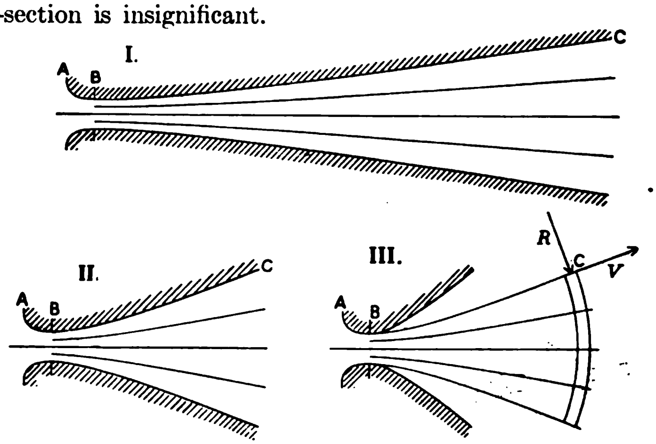


FIG. 55.—Different Shapes of Nozzle.

(c) THE FREE JET.—The problem of finding the limiting form of the jet is set forth graphically in Fig. 55. Here I. represents a nozzle of such slow expansion that the transverse acceleration of the outer streams calls for only a very small force. But suppose that the nozzle, keeping the same transverse dimensions and a generally similar form, is shortened up; as the curvature of the profile BC increases, the accelerating force required to make the outer streams travel along this path likewise increases, leaving a decreasing proportion of the internal pressure free to be exerted by the jet upon the nozzle. At the limit, the jet will just touch the guide-surface, without any pressure being felt between them: and this condition is supposed to be represented in II. If the nozzle is still further shortened, as in III., the jet will not follow it; but will, if not modified by such actions as the "picking up" of the surrounding atmosphere, keep the shape shown in II.

The drag of the surrounding atmosphere—whether of air or of steam, at rest (comparatively)—is eliminated by imagining the discharge to be into a perfect vacuum, of unlimited absorbing power. And the problem to be solved may therefore be stated as follows:

Into what shape will the steam-jet naturally fall if the discharge is from the orifice into a perfect vacuum?—by "orifice" is meant the entrance-portion AB of the complete nozzles shown in Figs. 54 and 55. Or, from the other point of view, what form is to be given to the flaring nozzle, beyond the choke, in order that it may just touch the jet, without pressure between them?

(*d*) THE INTERNAL FORCE-ACTIONS are illustrated in Fig. 56, which shows an element of the jet included between two of the curved section-surfaces of Fig. 54 II., separated by the distance *ds*. This may be thought of as a portion of Fig. 55 II., enlarged. There are two force-actions to be considered, one along the stream-lines, or linear, the other at right angles to these lines, or transverse.

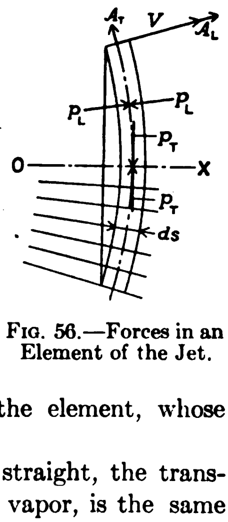


FIG. 56.—Forces in an Element of the Jet.

In regard to the first, we note that the linear pressure p_L is uniform all over the surface of the element: but it decreases by the amount dp in the passage from one surface to the other, through the distance ds. This unbalanced pressure dp, upon the area a, is what accelerates lineally the mass of the element, whose volume is ads.

At the center, where the stream-lines are straight, the transverse pressure p_T, or the internal stress in the vapor, is the same as p_L. But passing from the center outward, we see that each stream-line is a little more curved than the one within it, so that there must be a continual increase in the centripetal accelerating force (the center toward which the acceleration is effected being the instantaneous center of curvature of the stream-line). Then in the free jet, p_T drops, in a manner yet to be determined, from the full value p_L or p_1 at the center to zero at the circumference of the section. The meaning of the decrease of p_T is, that more

... produce acceleration or ... the accelerating force; the rem-...

... = 135.

... the transverse acce-... in order to get a value ... as in any case of ... we shall have found a relation ... the radius of curvature of ...

... of Fig. ... and ... manner of change of V as ... and the time-effects ... transverse and at ... new value which will be the ...

... THE STREAM-LINES.—The ... this action is reasoned out ...

... the same throughout the ... because the linear pres-...

... side by side without ... their successive cross-... That is ... by imagi-... shape and ... be carried ... their shapes must ... transverse acceleration ... to a maximum ... result of this acceleration ... as the jet advances and the rate ... directly as the proportional distance of the ... from the center of the jet.

To find the rate of pressure-drop which corresponds with

the preceding conditions, consider Fig. 57, which shows another view of the thin cross-sectional division of Fig. 56. An annular element of this slice, of the width dr at the mean radius r, will have the volume $2\pi r\,dr\,ds$, and the mass

$$M=\frac{2\pi r}{vg}dr\,ds; \quad . \quad . \quad (134)$$

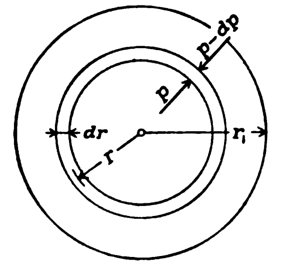


FIG. 57.—A Sectional Element of the Jet.

where v is the specific volume of the steam in the jet at p_1. Now the force acting to accelerate this mass is the pressure-difference dp, acting outward over the linear area $2\pi r\,ds$ of the ring, or

$$F=2\pi r\,dp\,ds. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

If the acceleration a, equivalent to force divided by mass, is to vary directly as the distance from the center, we have for its determining equation

$$a=\frac{F}{M}=vg\frac{dp}{dr}=br; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

where b is an undertermined coefficient.

Transforming (136), and integrating between the limits 0 and r, or p_1 and p, we get

$$\int_{p_1}^{p}-vg\,dp=\int_0^r br\,dr,$$

the first member of the equation having the minus sign because acceleration increases as pressure decreases. From this,

$$vg(p_1-p) \quad \text{or} \quad vgf=\tfrac{1}{2}br^2; \quad . \quad . \quad . \quad . \quad . \quad (137)$$

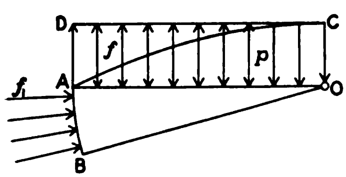


FIG. 58.—Diagram of Pressure-variation.

which shows that the curve of pressure-drop, laid out on a radial base-line OA in Fig. 58, will be a parabola.

Now when $f=p_1$, $r=r_1$; and from (137),

$$b=2vg\frac{p_1}{r_1^{\,2}}. \quad . \quad . \quad (138)$$

Substituting back in (136), we get the general value

$$a = 2vgr\frac{p_1}{r_1^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

and for the acceleration of the outer stream-line, where $r = r_1$,

$$a_1 = 2vg\frac{p_1}{r_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

It is interesting to note that this is the same acceleration as would be produced if the whole mass of the element were concentrated in a ring at the outer circumference, and a force equal to f_1 or p_1 per unit of area were to act upon it. The total force would be, omitting the thickness-factor ds, since it would appear in both expressions,

$$F = 2\pi r_1 p_1.$$

The volume is πr_1^2, and the mass

$$M = \pi r_1^2 / vg.$$

Then

$$a_1 = \frac{F}{M} = 2vg\frac{p_1}{r_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (140')$$

It will be seen that the preceding discussion, beginning with Fig. 57, is only approximate, in that it uses the plane-section element. Either of the above methods of deriving a value of the acceleration could be applied to the true curved element; but it is obvious that the exact mathematics of the subject would be very complex. Since any actual nozzle will be far within the limit of spread, a good enough determination of this limit can be made by approximate methods, which will now be applied.

(*f*) PROFILE OF THE FREE JET.—The primary equation upon which a determination of the shape of this curve must be based is

$$\frac{V^2}{R} = 2vg\frac{p}{r}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

the subscripts being dropped from p_1 and r_1 of (140); from which we have

$$R = \frac{r}{2g}\frac{V^2}{pv}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

To work along mathematical lines and establish an equation for the jet-profile, we should have to follow the methods of § 25 (*c*), using the exponential formulas. From Eq. (123), keeping n in

literal form, and dropping the factor 144 because p must be in pounds per square foot to give a consistent value in (135), we get

$$U=\frac{V^2}{2g}=\frac{n}{n-1}p_1v_1\left[1-\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}\right];$$

and the substitution of this value of V^2 in (142) gives

$$R=r\times\frac{n}{n-1}\,\frac{p_1v_1}{pv}\left[1-\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}\right]. \quad . \quad . \quad . \quad . \quad (143)$$

From $p_1v_1^n=pv^n$, it is easy to get

$$\frac{p_1v_1}{pv}=\left(\frac{p_1}{p}\right)^{\frac{n-1}{n}}.$$

Whence

$$R=r\times\frac{n}{n-1}\,\frac{1-\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}}{\left(\frac{p}{p_1}\right)^{\frac{n-1}{n}}}; \quad . \quad . \quad . \quad . \quad . \quad (144)$$

or, for $n=1.135$, and using R_p for p/p_1,

$$R=r\times 8.41\frac{1-R_p^{.1189}}{R_p^{.1189}}. \quad . \quad . \quad . \quad . \quad . \quad (145)$$

TABLE 27 A. DATA FOR JET-PROFILE.

R_p	r	m	R	R_p	r	m	R
.95	1.446	.051	.074	.35	1.070	1.118	1.196
.90	1.239	.106	.131	.30	1.110	1.285	1.425
.85	1.140	.164	.187	.25	1.164	1.508	1.755
.80	1.080	.226	.244	.20	1.244	1.775	2.208
.75	1.043	.293	.306	.16	1.321	2.075	2.74
.70	1.022	.364	.372	.13	1.400	2.308	3.23
.65	1.008	.442	.446	.10	1.553	2.648	4.11
.60	1.001	.526	.527	.08	1.696	2.942	4.99
				.06	1.877	3.34	6.27
.575	1.000	.572	.572	.05	2.000	3.60	7.20
				.04	2.184	3.92	8.55
.55	1.001	.620	.621	.03	2.445	4.35	10.64
.50	1.008	.722	.728	.02	2.86	4.98	14.25
.45	1.021	.838	.855	.01	3.81	6.13	23.30
.40	1.041	.967	1.007				

To get a complete mathematical solution of the problem, it would be necessary to find an expression for r in terms of p or R_p—which is easily enough derived, but is rather complex, as might be gathered from (127). Having then expressions for the radius of curvature R and for the ordinate r or y of the curve, both in terms of p, it might be possible to deduce the equation:

but the process appears forbiddingly complicated, and an approximate, graphical solution which suggests itself, based on Eq. (145) and our previous determination of r, is the only thing practically available.

(*g*) CONSTRUCTION FOR PROFILE.—The numerical data for Fig. 59 are given in Table 27 A: r is the radius of the jet-section, in terms of the radius of the choke, r_0, as unity, and is got by taking the square root of the ratio a/a_0 in Table 26 A, Col. 4; m is all after r in (145), so that $R=r\times m$; and R is the radius of curvature, expressed in the same terms as r, or by its ratio to r_0.

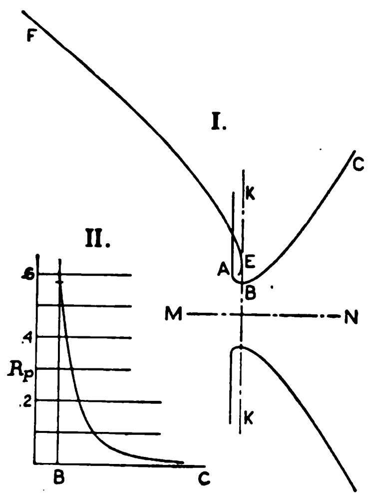


FIG. 59.—Profile of Free Jet.

The curve in Fig. 59 I. is started at B, on the choke-radius, and is laid out in both directions, a short piece at a time: r is taken as the vertical ordinate from the center-line MN to the curve. The value of R for each interval corresponds with the r at the beginning of the interval: and the evolute EF, the locus of the center of curvature, is made a fair curve. It is evident from the reasoning leading to Eq. (140′) that the true value of r would, in length, lie between the perpendicular to the center-line MN, as here used, and a normal to the curve measured to this same center-line; being rather nearer the latter. So that in this method there is a major error which makes the nozzle flare too rapidly: in other words, the curved cross-section according to Fig. 56, being larger than the plane section for a given point of the curve, would correspond to a lower pressure and to a longer radius R; so that the true curve would depart from the center-line less rapidly than does the one here drawn.

With some graphical jugglery, the true curve could be closely approximated—one such that, if it were developed along a num-

ber of equally spaced stream-lines, then the curve through points on these lines at the same distance from the B-section, or the profile of a section of uniform condition, would be at right angles to the stream-lines, meeting the requirement stated under Fig. 54. But Fig. 59 I. serves well enough the useful purpose of showing that the free jet has a very wide angle of flare; and any nozzle intended for actual use will be kept far within this limit.

At II. is shown a curve of pressure-drop, on the developed profile BC as base. Before reduction, the diameter of the throat in this figure was one inch; and the drop of pressure from $.57p_1$ to $.01p_1$ took place along about two inches of jet-profile.

It is to be noted that, according to Eq. (144), nozzles of different size or capacity, but for the same ratio of pressures, will have geometrically similar profiles.

(*h*) Profile with Partial Reduction.—By this rather indefinite title is meant a jet-profile made up of inner stream-lines of the free jet, or a sort of core of the free jet, in which the transverse pressure lost in passing from the center to the rim of any cross-section is a constant fraction of the pressure at the center, or where the transverse acceleration of the outer stream is a constant fraction of that of the outer stream of a free jet. Thus in Fig. 60 the radius R is taken ten times as great as that given in Table 27 A, or the acceleration a is one-tenth of a_1, comparing (139) and (140). This means that the jet is the core of a free jet of ten times its diameter (at the choke); and the loss of transverse pressure is only one per cent., as appears when we substitute the value of b in (137) and get

Fig. 60.—Profile for $f=.01p$.

$$f=\frac{r^2}{r_1^2}p_1: \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

which is the equation of the curve AC in Fig. 58.

This profile is drawn by the same method as Fig. 59 I., and

the inherent error is much smaller with the slower expansion of the jet here shown. Of course, the expansion of the steam in the jet, or its drop in pressure along the axis, is just as rapid as in the large, enveloping, free jet.

It must be understood that variation in transverse pressure is entirely a question of *curvature* of the profile, not of its inclination. After any direction of stream-travel has been established, it can be continued along a straight line, as in a conical nozzle, without the action of any free (unbalanced) guiding force.

(*i*) Design of the Nozzle.—The first deduction to be made from Fig. 59 is in regard to the shape of the entrance. Noting that the value of R for R_p equal to 0.6 is about 0.5, by Table 27 A, we infer that the use of a radius half that of the hole, in "rounding the corner" of a hole drilled through a plate, ought to be enough to prevent the contraction of the jet sketched at Fig. 61 I., and secure the full discharge computed by Eq. (120), or by the more exact method of (129) with K_0.

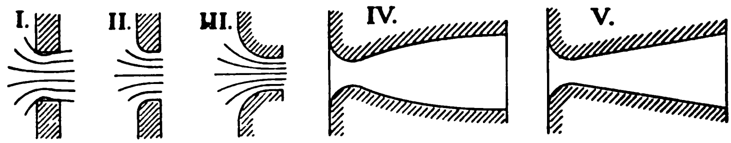


Fig. 61.—Nozzle Details.

An orifice of this shape is shown at II.; but while the jet will not leave the surface of the nozzle under this condition, it seems reasonable to believe that there will be a considerable amount of wasteful vortex-action in the initial formation of the jet; and that a larger radius, such as that in III., where R is $2r_0$ or the diameter of the orifice, can be better depended upon for complete energy-transformation. Practically, an approach-curve of uniform radius is preferable to the variable curve derived in Figs. 59 and 60.

The proper shape for a nozzle which is to deliver its jet in parallel streams—the form sketched in Fig. 44—is shown in Fig. 61 IV.: just beyond the throat the flare may be quite rapid; but the reverse curve which gives the outer streams a transverse acceleration inward should be of very long radius, even longer than ... for by Eq. (146). If the streams are too rapidly deflected

inward, there will be a strong crowding of the steam against the surface of the nozzle, with a resulting variation of pressure throughout the cross-section which will have a tendency to produce cross-currents or eddies. Further, the nozzle would have to be continued in cylindrical form for some distance beyond the end of the curve in order to insure complete rectification of the stream-lines.

The conical nozzle, Fig. 61 V., with a narrow angle of flare, is what is generally used in the steam-turbine; for the moving blades of the wheel receive the jet right at the mouth of the nozzle, so that the tendency of the stream-lines to separate has very little chance to become effective. In some cases—most usually, in fact—the nozzle has a rectangular cross-section, and is wedge-shaped; this differs from the cone as to the manner of area-change, but is similar in that it delivers a diverging jet. The curve of pressure-drop along the axis, for the conical jet, is illustrated in Fig. 48. The determination of the ratio of the diameter at the mouth to that at the throat is made by the methods of the preceding sections.

(*j*) The Limit of the Simple Quantitative Treatment of the flow of steam is reached when we have found the shape of a nozzle in which the transformation of pressure-work into kinetic energy can take place with the least possible disturbance by secondary, modifying actions. Down to the mouth of the properly formed nozzle, the effects of friction, of the shock of meeting of bodies of steam at different velocities, and of eddy-currents are relatively insignificant: beyond there, these disturbing influences become so much stronger that, while the principal action may be more or less approximately determined by, or predicted from, general considerations, it is only by experiment under actual conditions that correct or close results can be got. A general presentation of the character and of the effects of these disturbing influences will be given in the next section.

Failure to realize in the nozzle the full, possible kinetic energy of the jet will have a twofold effect; on the one hand, at a particular pressure, the velocity of the current will be less, on the other hand, the specific volume will be greater, than in perfect action—

the latter because more heat remains in the steam; consequently the section-area will be greater, which means that the particular pressure will exist farther along the axis, or that the drop in pressure will be retarded.

§ 28. Conditions Beyond the Nozzle.

(*a*) DISCHARGE FROM THE ORIFICE.—When the jet escapes directly from the orifice, instead of being expanded in a flaring nozzle down to the discharge pressure, the conditions are somewhat as sketched in Fig. 62. The surrounding atmosphere effectually prevents any such rapid lateral expansion as that shown by Fig. 59. Instead there is a sort of pick-up action, whereby streams of air (or of steam, if the discharge is into a steam-vessel) are drawn into and mingled with the streams of the jet. In this action energy is dissipated in two ways: to some degree there is a shock-effect, wherein the impact of bodies of fluid with different velocities sets up, within these bodies and the surrounding medium, a wave-motion analogous to, if not identical with, the usual acoustic vibration which gives the sensation of sound; besides this, and chiefly, there is a rapid change from the direct, forward movement of the whole jet to a whirling, churning motion of the divided streams; and the mechanical, kinetic energy, at first merely changed in respect to the direction of motion, is presently re-converted into heat as these eddy-currents come to rest in the body of the surrounding atmosphere.

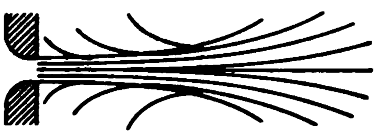


FIG. 62.—Discharge from Orifice.

Reference to Table 26 A will show that the velocity of efflux from an orifice will be about 1400 ft. per sec., varying but little with the initial pressure—this when the pressure on the discharge side is not above that in the choke of the jet. With direct discharge, as in Fig. 62, a part of the expansion-work yet available at the choke-pressure p_0, above p_2, will be applied to increasing the velocity of the jet, or at least of the central part of it; but the effect of most of this work will be at once merged into the churning action at the surface of the jet, where the dissipation of available

energy into the non-utilizable form of heat at low temperature begins immediately.

This same action of entraining the surrounding fluid and of the formation of eddy-currents would take place if the jet were reduced to the pressure of the discharge, and given a higher velocity, by expansion in a flaring nozzle; but with some difference in the quantitative relations, as appears when we turn to Table 24 A and note the velocities, first for discharge at 15 lbs. abs., and then at 2 lbs. abs.

(b) WIRE-DRAWING.—When the whole energy of the jet is wasted in this way, without any attempt to apply it mechanically, as when a throttling or reducing valve is used for lowering the pressure of a current of steam, we have what is commonly called "wire-drawing"—a name which originated in connection with the action of engine-valves in throttling currents of steam, as described in § 18 and § 19 (c). In this action as a whole, and barring radiation, no heat is lost from the steam: and since the heat-content of a certain weight of saturated steam decreases with the temperature (and pressure), according to Eq. (57), § 10, it follows that the excess of heat, in amount .305 B.T.U. per pound and per degree, is set free from the duty of keeping the steam in the saturated condition. This heat can have two effects: if the steam was dry as well as saturated at the start, it will be superheated above the saturation temperature t_0 corresponding to the lower pressure; but if a small amount of moisture was mixed with the steam, then the hot water will be evaporated. Of course, these two effects are usually mingled, in varying proportions. If the steam was originally superheated through a certain range above t_1, throttling will make the range above t_0 greater.

With steam initially saturated, the quantitative relations in this process are as follows:

If we have steam at p_1 and t_1, with $(1-x_1)$ of moisture, the heat-content above water at 32° F., according to Eq. (54), is

$$Q_1 = q_1 + x_1 r_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (147)$$

If, at p_0, the temperature is that for saturation, or t_0, then the original fraction of moisture was so great that the heat set free

… evaporate it all, or to do more than just evapo-
… temperature t_2 is above t_0, there was only
… the steam has been superheated. The heat-

… $x_0 r_0$ . . . . (148)

… $r_1 + .48(t_2 - t_0)$ . . . . (149)

… definite-value formulas, by means of (57)
… ase

… $366 + .305(t_1 - 212) - (1 - x_1)r_1$ . . . . (150)
… $366 + .305(t_0 - 212) - (1 - x_0)r_0$ . . . . (151)

$$1 - x_1 \frac{r_1}{r_0} = \frac{.305(t_1 - t_0)}{r_0} \quad . \quad . \quad . \quad . \quad . \quad (152)$$

…

… $366 + .305(t_0 - 212) + .48(t_2 - t_0)$; . . . (153)

… (150), whence

$$\ldots \frac{.305(t_1 - t_0) + .48(t_2 - t_0)}{r_1} \quad . \quad . \quad . \quad . \quad (154)$$

… Steam-pipes are usually proportioned so … velocity of 5000 to 6000 ft. per min. At the … boiler a certain amount of pressure must … velocity, and in overcoming the resist… Referring to Fig. 49 or Table 26 A, we see … on account of a velocity of 8… .0025, and it is generally accepted … cause a drop about half of that … one-third of one per cent. … total loss on these accounts … expenditure of available … of friction of the walls.

The formula for friction-effect is similar to that for the flow of water in pipes: it is partly rational, partly altogether empirical. Put into the most convenient shape for application to steam, it is

$$p=\frac{V^2}{28{,}000}d\frac{L}{D}; \quad \ldots \ldots \ldots \quad (155)$$

where

$p=p_1-p_2$=loss of pressure from the entrance at p_1 to the exit at p_2, in pounds per square inch;

V=velocity of current, in feet per second;

d=specific weight of steam, in pounds per cubic foot, at p_1;

L=length of pipe, in feet;

D=diameter of pipe, in inches.

The rational portion of the formula is derived as follows:

A pipe of diameter D (here in feet) and length L will have an inner surface $S=\pi DL$ square feet: if the resistance to the motion of the fluid over this surface can be expressed as a force f per sq. ft., which seems reasonable, then the work done in moving the whole column of steam in the pipe through a distance of one foot is fS ft.-lbs.; and the work per second, with velocity V, is $W=fSV$ ft.-lbs. To overcome this resistance, we have the unbalanced pressure $144p$, on the area $A=\pi D^2/4$, acting through the distance V ft. in one second, and doing the work $W=144pAV$: equating this with fSV and substituting the pipe-dimensions, we get

$$\pi fDL=36\pi pD^2;$$

whence

$$p\propto f\frac{L}{D}. \quad \ldots \ldots \ldots \quad (156)$$

It is the value of the resistance f that is purely empirical: it has been found by experience to vary as the square of the velocity and as the density of the fluid: but if the loss of pressure is expressed in "head," or in height of steam-column having a weight equivalent to p pounds per square inch on its base, then d goes with p into the expression for the head H, and we get the usual hydraulic formula.

Of course, all the numerical constants involved are combined

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between a smaller and less expensive pipe, with large pressure-drop, and a larger and costlier pipe, with greater radiating surface. The problem is essentially one to be solved by trial, and the common rule given at the beginning of this article—make V equal 5000 to 6000 ft. per min.—is this practical solution. Thermally, since the heat of formation rises very slowly with the pressure, it might pay better to run the boiler at a pressure well above that desired at the engine, and force the steam through a small pipe at high speed; but this advantage is balanced by the greater weight and cost of a stronger boiler. To reduce the loss by bends, it is now usual in the best practice to make long, easy curves, either with special cast-iron fittings, or by bending the pipe: the latter method has the additional advantage of giving greater flexibility for expansion and contraction under change of temperature.

EXAMPLE 1.—A steam-pipe is 6 ins. in diameter, 75 ft. long from boiler to engine, and has six elbows and one globe-valve; the pressure at the boiler is 100 lbs. abs., and the velocity of flow is 80 ft. per sec.: what pressure is likely to be lost from boiler to steam-chest?

If $V=80$, the entrance-loss will be about $100\times.003=.3$ lb.

Six elbows + one valve = 7.5 elbows; each is equal to 20 ft. of pipe, and all together equal 150 ft.: adding this to the actual 75 ft., we get $L=225$: from the Steam-table, $d=.2275$: then in (155),

$$p=\frac{6400\times.2275\times225}{28000\times6}=1.95 \text{ lbs.}$$

And the total loss would be 2.25 lbs.

(*d*) THE SEPARATION OF WATER.—An important fact in connection with flow in pipes is that if the current is a mixture of steam and water, there is a strong tendency for the two fluids to separate, simply on account of difference in density. If the water is small in amount, say not more than three per cent. by weight, it will probably remain well diffused, if the current is rapid and an occasional bend keeps it stirred up. In a long, straight, horizontal pipe, even a very small proportion of water is likely to settle to the bottom of the current.

At a bend, the lighter streams of steam change direction much more readily than the denser particles of water: and this fact is the basis of the action of the steam-separator, which, besides making

the current turn one or two sharp corners so as to throw out the water, provides a quiet resting-place where this water can collect, without being picked up again by the steam, as it must be in a pipe. A good separator will take out all but a very minute fraction of the moisture in a current of steam.

(*e*) THE EXTENDED NOZZLE.—In (*a*) we considered the two cases of discharge from an orifice and from a properly designed nozzle—one which has just the degree of expansion needed to reduce the pressure in the jet at its mouth to that in the discharge-chamber. Suppose now that the nozzle, conical in form as the simplest case, is extended beyond the proper length: then the action of the jet will be somewhat as indicated in Fig. 63, by the pressure-curve at II. The steam expands in the nozzle to a pressure well below that of the discharge, then rises to the latter in the manner shown by the curve. This rise in pressure means that the steam is compressed, and the current retarded, with loss of mechanical energy through re-conversion into heat: but the action is too complex and contains too many uncertain elements for mathematical treatment.

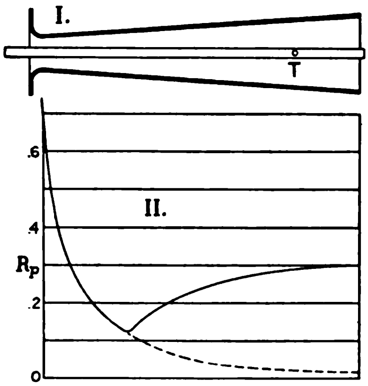

FIG. 63.—The Extended Nozzle.

The arrangement of apparatus for experiment upon steam-jets is indicated in Fig. 63 I., the essential element being the exploring-tube T, which has a small hole drilled across it, is closed at the right-hand end, and at the left passes out through a stuffing-box and is fastened to a screw-device whereby the hole can be brought to any definite point along the axis of the nozzle. The outer end of the tube is connected to an accurate pressure-gage; and from the positions of the tube and the readings of the gage, curves like that in II. can be plotted.

Fig. 64 is a diagram reproduced from Stodola's "Steam Turbines," and shows the results of a series of tests conducted along

the lines just indicated. The terminal pressure was regulated by a valve between the discharge-chamber and the condenser. Several of the curves show a very abrupt rise, after the minimum

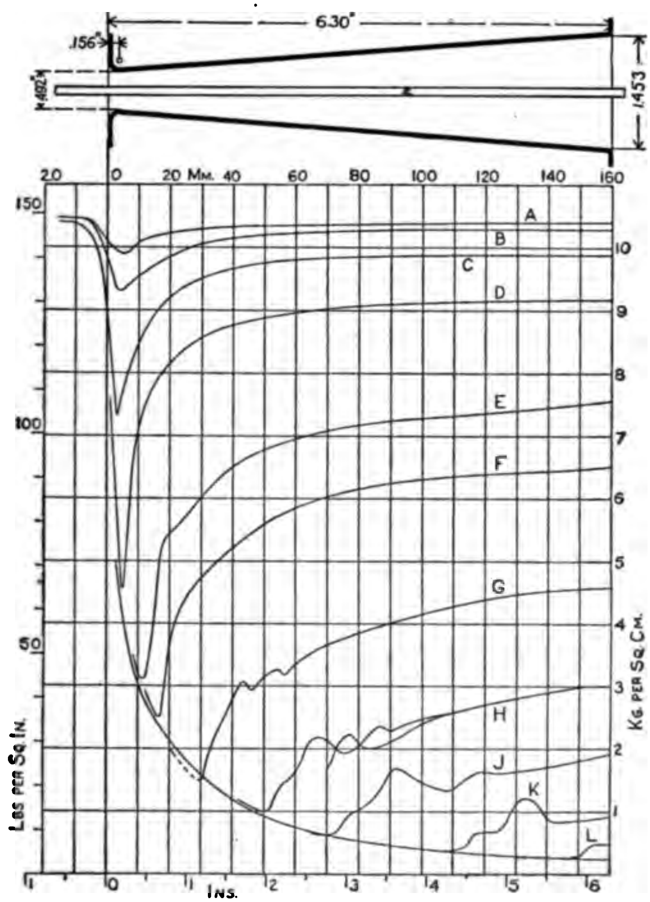

FIG. 64.—Curves of Pressure-variation.

pressure has been reached, as if caused by the shock of meeting of two bodies of steam at different velocities; and at the lower pressures there appears to be an incipient formation of acoustic vibrations. There is no self-suggestive law of relation between

the initial, the lowest, and the final pressure: and it is probable that a large number of experiments, under various conditions would have to be made before such a law could be formulated. Stodola gives other diagrams of this sort, for different forms of nozzle; and develops a considerable amount of theory of the irregular actions beyond the simple nozzle: and the reader is referred to his work for a fuller treatment of the whole subject of the complex motions of steam-currents.

(*f*) JET-ACTION REVERSED.—In the retardation with rise of pressure shown in Figs. 63 and 64, there is a re-conversion of mechanical energy from the kinetic form to that of static pressure energy: this is effected through a compression of the steam, which is adiabatic so far as concerns the reception of heat from without but is modified by the heat-equivalent of all the mechanical energy lost. In simple theory, barring friction, radiation, and eddy-action, it ought to be possible to reverse completely the expansion-process which forms the jet, and, by gradually retarding the current in a suitably formed nozzle, to restore the steam to its original state. Practically, only a certain fraction of restoration can be realized.

This means that in order to compress by a jet, or to lift fluid from a lower to a higher pressure by transforming energy of velocity into energy of pressure, a considerably greater velocity must be given to the jet at the low pressure than it would gain in an expansion or drop from the higher to the lower pressure. Just what will be the efficiency of the operation in any case can be found only by trial; or might be predicted by close analysis of the proportions and of the performance of apparatus which has been developed by experience into successful working. But there seems to be very little precise information generally available on this subject.

It is in the steam-blower and the injector, in the rotary fan and the centrifugal pump, that this principle of action is applied.

§ 29. Applications of the Steam-jet.

(a) THE THROTTLING CALORIMETER.—This name is given to an instrument based on the principles set forth in § 28 (b), whereby the fraction of moisture in a sample of steam can be determined. One form of this "calorimeter" is outlined in Fig. 65. The sample is drawn from the steam-main S through the perforated pipe P and the valve V (which is wide open when the instrument is working), into the chamber C, where the temperature t_1 is read: it then passes through the small orifice at O into the low-pressure chamber L, here open to the air, and the temperature is read at t_2. In this particular case, $t_0=212°$, and (154) becomes

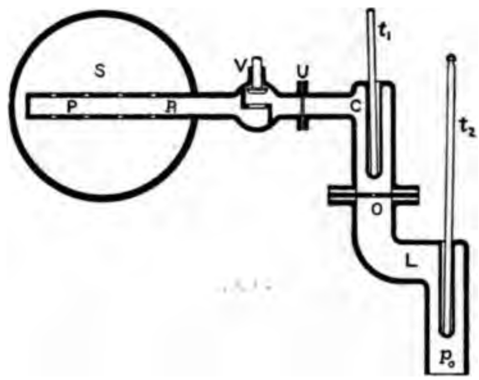

FIG. 65.—Throttling Calorimeter.

$$m_1 = \frac{.305(t_1-212)-.48(t_2-212)}{r_1}. \quad . \quad . \quad . \quad (158)$$

The limit of effective working of the throttling calorimeter, for the case of exhaust into the atmosphere, is given in Table 29 A; where, for different values of p_1, the following quantities are tabulated:

p_1 = initial pressure, pounds per square inch above atmosphere (by gage);

$h_1 = .305(t_1-212)$ = available excess heat in one pound of dry saturated steam at p_1, above similar steam at 212°;

$m_{10} = h_1 \div r_1$ = fraction of initial moisture which can be just all evaporated by h_1, or which will make $m_0=(1-x_0)=0$ in (152);

$t_s = h_1 \div 0.48 =$ number of degrees of superheat after throttling to atmospheric pressure, if the steam was originally dry, or if m_1 was zero;

$t_{20} = t_s + 212° =$ temperature of superheated steam at $p_0 = 14.7$ abs., for $m_1 = 0$.

It will be noted that both terms in the numerator of (154) or (158), namely $.305(t_1 - t_0)$ and $.48(t_2 - t_0)$, are quantities of heat, expressed in B.T.U.

TABLE 29 A. LIMIT OF THROTTLING-EFFECT.

p_1	h_1	m_{10}	t_s	t_{20}
250	58.90	.0714	122.7	334.7
220	55.97	.0673	116.6	328.6
200	53.56	.0638	111.6	323.6
180	51.03	.0604	106.3	318.3
165	49.01	.0577	102.1	314.1
150	46.85	.0548	97.6	309.6
135	44.53	.0517	92.8	304.8
120	42.03	.0485	87.6	299.6
105	39.28	.0450	81.8	293.8
90	36.26	.0412	75.5	287.5
80	34.07	.0385	71.0	283.0
70	31.66	.0355	66.0	278.0
60	29.01	.0323	60.4	272.4
50	26.08	.0288	54.3	266.3
40	22.72	.0249	47.3	259.3

(b) CORRECTION FOR RADIATION.—The calorimeter is, of course, well covered with non-conducting material; but it has considerable surface in comparison with the small amount of steam passing through it, so that it must be calibrated for radiation-effect. To find what correction must be made in the indications of the instrument, it is tested with dry steam; which may be drawn from the steam-pipe near the boiler at a time when the current is flowing so slowly that there is sure to be no priming or can be got from a good steam-separator, suitably arranged. Any moisture shown under this condition is to be charged to the instrument, and deducted from the gross result got in a working-test. A modification of the method of (158), very convenient of application in this connection, is as follows:

Let $h_1 = .305(t_1-212)$ as above, and $h_2 = .48(t_2-212)$; then if $m_1=0$, t_2 will have the value t_{20} given in Table 29 A; and the effect of initial moisture in the steam is to diminish h_2 from its maximum value $h_{20}=h_1$, or to lower the discharge temperature from t_{20} to t_2. What the calorimeter actually measures is this heat h_2, in the discharged steam over and above the saturation-content. As the initial moisture increases, h_2 drops farther below h_{20}; and m_1 can be got by the relation

$$m_1 = \frac{h_{20}-h_2}{r_1} = \frac{.48(t_{20}-t_2)}{r_1}. \qquad (159)$$

Now the effect of radiation is to make either the heat-deficiency $h=(h_{20}-h_2)$, or the temperature-drop $t=(t_{20}-t_2)$, greater than it ought to be: and by the calibration-test above described this extra-effect due to radiation, either h_r in B.T.U. or t_r in degrees, can be found. If t_n and h_n were the particular values of t_2 and h_2 in the radiation-test (with dry steam), then

$$t_r = (t_{20}-t_n) \quad \text{and} \quad h_r = (h_{20}-h_n); \qquad (160)$$

and when these have been found and recorded, we correct (159) by subtraction, getting

$$m_1 = \frac{h_1-h_2-h_r}{r_1} = \frac{.48(t_{20}-t_2-t_r)}{r_1}. \qquad (161)$$

It is even more direct to use t_n, equal to $(t_{20}-t_r)$, as found by experiment, in the formula

$$m_1 = \frac{.48(t_n-t_2)}{r_1}. \qquad (162)$$

This actual lower temperature with initially dry steam, t_n, is sometimes called the normal temperature of the calorimeter. It is more convenient, however, to get the value of t_r or h_r by calculation; because this correction will be applicable over a wide range of steam-pressure, since the initial temperature, upon which the rate of radiation depends, does not greatly change with the pressure.

(c) RANGE AND ACCURACY OF THE CALORIMETER.—Table 29 A shows that as p_1 is lower the effective range of the throttling calorimeter is less. But if the instrument is connected so as to discharge into a vacuum condenser, thereby lowering t_0, it can be used on quite wet steam of low pressure, as even, perhaps, on the exhaust from a non-condensing engine. But in this arrangement the pressure p_0, which determines t_0, must be very accurately measured, because the temperature varies so rapidly with the pressure when the latter is low, as shown by curve I. on Fig. 21 And with the usual condition of atmospheric discharge, reading of the barometer is essential to high accuracy; as is also, in any case, the calibration of the thermometers. It is to be noted however, that the upper thermometer can be replaced by a pressure-gage, if desired: and that if the method of (161) is used with the same thermometer for t_2 in both radiation test and steam test, then the absolute correctness of this thermometer is of little importance, as only the difference between readings is involved.

Ordinary "chemical" thermometers, 12 to 15 inches long with a graduation up to about 400° F., and with only about 3 inches of their length immersed in the oil or mercury in the thermometer-cup, will have a considerable error in their readings on account of the relatively low temperature of the exposed stem and of the thread of mercury inside it. For usual steam-temperatures, the reading will be from 2° to 5° low, increasing with the difference between the temperature measured and that of the air, and with the length of mercury column exposed above the cup. So that thermometers for use with steam should be calibrated under actual conditions, in regular cups immersed in steam; an accurate pressure-gage, with the t-column of the Steam-table, furnishing a satisfactory standard.

Accuracy in the determination of moisture by throttling depends upon the correctness of the constant 0.48, the specific heat of superheated steam under constant pressure. This was found by Regnault for atmospheric pressure, from experiments far less precise than those which he made on saturated steam. The inherent difficulties in the determination are great, but experiments made in the last few years, while not yielding very consistent

results, or establishing any law even approximately, show that the specific heat increases with the temperature. It appears to start at 212° F. with a value of about 0.40, and increase pretty rapidly at first, then more slowly at high temperatures. For the range in the calorimeter, 0.48 seems to be a very proper value, the best results varying from 0.4 to 0.5. Fuller information in regard to this specific heat will be found in the Appendix.

EXAMPLE 1.—In the calibration test of a throttling calorimeter with open exhaust, the mean observations were $p_1=105.6$ lbs. by gage, $t_1=337.6°$, $t_2=287.5°$.

Using the temperatures as read,

$$h_1=.305(337.6-212)=38.31 \text{ B.T.U.},$$
$$t_s=38.31\div .48=79.7°, \text{ and } t_{20}=291.7°.$$

The t_2 here read is also t_n: then

$$t_r=291.7-287.5=4.2°,$$
$$h_r=4.2\times .48=2.02 \text{ B.T.U.}$$

According to the pressure-gage, t_1 should be 341.2, so that the upper thermometer reads 3.6° low; this would change h_1 to 39.41 and t_{20} to 294.1°: but t_2 would have almost as great an error on account of incomplete immersion as t_1, so that the value of t_r from corrected temperatures would be nearly the same as that from the actual readings.

EXAMPLE 2.—In a steam test with the above calorimeter, the readings were $t_1=325.3°$, $t_2=232.7°$; then, using these actual temperatures,

$$h_1=.305\times 113.3=34.56,$$
$$h_2=.48\times 20.7=9.94,$$
$$r_1=884.5;$$

and by (161),

$$m_1=\frac{34.56-9.94-2.02}{884.5}=\frac{22.6}{884.5}=.0256.$$

The error caused by using uncorrected temperatures will be greater in this case than in the preceding, because t_2 is lower, and the thermometer will be nearer the true temperature. But it appears that, for all practical purposes, it is sufficiently accurate to use the actual readings of the thermometers, provided that the latter are reasonably correct and show up practically the same in a comparison test, either against each other or against a good standard thermometer with the same degree of immersion.

(*d*) THE SEPARATOR CALORIMETER.—The principle of the steam-separator is applied in another instrument for determining the quality of steam, which may be used as an alternative to, or in combination with, the throttling calorimeter. In the form shown in Fig. 66, it is intended to be coupled in on the high-pressure side of the throttler of Fig. 65, at the union U, when the steam is too wet for the first instrument—which is shown by t_2 dropping almost to t_0, say within five degrees. This combination constitutes the older form of the Barrus Universal Calorimeter; the separator is intended to take out nearly all the moisture from the sample, sending almost dry steam over to the throttler.

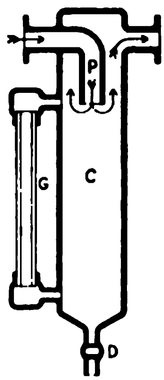

FIG. 66. Separator Calorimeter.

The drain-valve D is regulated so as to keep the water in the chamber C at a certain level, well below the mouth of the inlet-pipe P; and the water escapes into a vessel partly filled with cool water, or through a cooling-coil, and is weighed.

If s_1 = weight of steam discharged in a given time, and s_2 = weight of water separated, then, very simply,

$$m_1 = \frac{s_2}{s_1 + s_2} = \frac{s_2}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (163)$$

The need of measuring s_1 by condensing and weighing for sure results is a point of practical inconvenience: which can, however, be eliminated by standardizing the orifice through which the steam escapes, by means of a series of calibration tests.

The radiation test for this calorimeter consists in supplying dry steam and seeing how fast water collects; if this rate is expressed as s_r pounds per hour, and the other quantities put in terms of the same period, the corrected result will be

$$m_1 = \frac{s_2 - s_r}{s_1 + s_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (164)$$

It has been found by experience that the steam delivered by the separator is so nearly dry that this instrument can be used alone as an independent and self-contained apparatus. And in a

later form of the combined calorimeter, the separator is placed on the discharge side of the throttler, to take out any water not evaporated on account of wire-drawing down to atmospheric pressure, but coming into action only when the steam is quite wet.

When both parts of the apparatus shown in Figs. 65 and 66 are used together, the separator giving the moisture m_a according to (163) or (164), and the throttler showing m_b from (159) or (161), the total fraction m_1 is the sum of these two indications: the small error due to the fact that m_a is a fraction of s_1+s_2, while m_b is based on s_1 alone, is entirely negligible.

(*e*) PICK-UP APPARATUS.—Under this title are comprised the steam-blower and the injector—a type of apparatus for doing work by means of the expansive energy of steam which is, mechanically, the simplest known, in that it has really no machine parts at all. Nevertheless it comes under the general definition of the steam-engine, and will therefore be briefly described and discussed.

The principle of action has already been partially enunciated, in § 28 (*f*). The jet, more or less perfectly formed, escapes into a chamber filled and supplied with the substance to be moved (air or water); there it picks up or entrains a certain amount of this substance, forming a mixed jet of greater weight but much smaller velocity; and then this resultant jet is discharged through a suitable retarding nozzle against a pressure higher than that at the mixing point.

All devices of this class are very inefficient as machines, judged by the criterion of the ratio of work got out to work put in. There are large wastes of kinetic energy in the mixing operation, and in the retardation of the current against rising pressure there are further losses of effect. Besides, to insure delivery, the discharged jet must have a very considerable residual velocity and energy, which is necessarily dissipated into heat as it comes to rest.

(*f*) STEAM-BLOWERS, used for producing draft for boiler-furnaces, are of two types. The first is the exhaust-jet of the locomotive, in which the whole body of steam used by the engine mixes with the products of combustion, drawing them into an enlarged jet which is expelled up the smoke-stack. These gases, coming from the boiler-tubes into the smoke-box, are at a temperature much above that of the exhaust steam: consequently the steam is not

condensed, but is rather dried and superheated, and we have the case of two gases mixing. An example from recent practice is outlined in Fig. 67, a good deal of non-essential detail being slurred over or omitted altogether in the drawing; especially the spark-arresting screens, which serve also to break the force of the current from the tubes into the space above the exhaust-nozzle N, and to

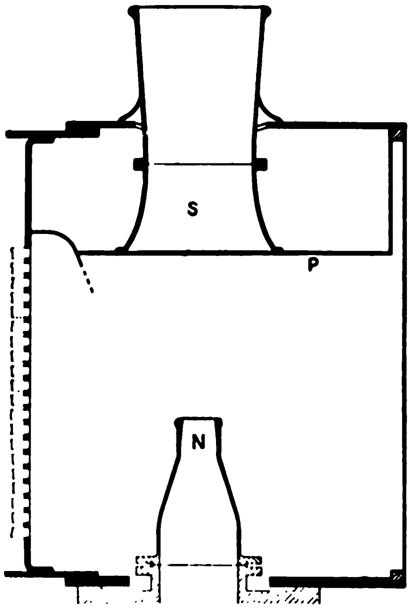

FIG. 67. Locomotive Smoke-box.

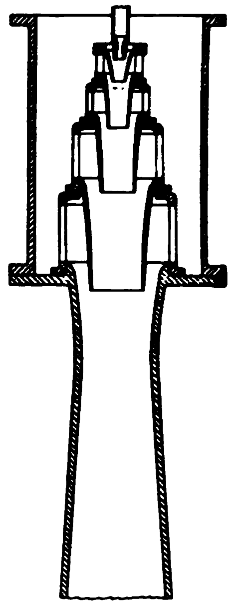

FIG. 68. Forced-draft Blower.

deflect the greater part of it downward, so that the jet draws mostly from below. This is from a large locomotive, where the stack has to be let down into the smoke-box: and a dead space is left above the horizontal partition P.

The second type of blower is illustrated by the example in Fig. 68, where a small jet of high-pressure steam is used for moving a stream of air relatively much larger than that in the locomotive (as compared with the weight of the steam), but against a much smaller resistance. Further, air of ordinary temperature is drawn into this blower, so that the steam will be partly condensed in the formation of the mixed jet—the action approaching that of the

injector in this respect. Note how the entering air is split up into several concentric divisions, with the intention of reducing the eddy-loss during the mixing operation.

The following example, worked out with assumed values of the data, will show the method of computing the efficiency of this sort of apparatus in actual operation, and will give some idea of its probable value for the first type:

EXAMPLE 3.—In a locomotive boiler, 1 lb. of coal will evaporate about 6 lbs. of water, and the gaseous products from 1 lb. of coal will weigh about 18 lbs., so that there will be 3 lbs. of gas to 1 lb. of exhaust steam. We will assume that the mixture of hot gas and superheated steam will have a temperature of 400° F.; and that the gas alone has the same density as pure air. For air at 400°, disregarding the small variation from atmospheric pressure in the smoke-box, the specific volume would be, by Eq. (8),

$$v = 12.4 \times \frac{860}{492} = 21.7 \text{ cu. ft.}$$

And for steam, which measures 26.6 cu. ft. per lb. when saturated at 212°, we get the increase due to superheating up to 400° to be, from (63),

$$p(v-s) = .65 \times 188,$$

$$v-s = \frac{.65 \times 188}{14.7} = 8.3 \text{ cu. ft.},$$

whence $v = 26.6 + 8.3 = 34.9$ cu. ft.; which satisfies the frequently given rough relation that steam is five-eighths as heavy as air of the same pressure and temperature.

The total volume discharge through the stack, per pound of steam, is then just 100 cu. ft.; and the resistance overcome is the pressure-difference, or the vacuum in the smoke-box, which we shall take to be equivalent to 4 ins. of water-column. One foot of water-column equals 62.4 lbs. per sq. ft., so that we have here a resistance of 20.8 lbs. per sq. ft.; and the work done in expulsion will be $100 \times 20.8 = 2080$ F.P.

Now the steam was expelled from the cylinder and forced through the nozzle by the back-pressure upon the piston, over and above the atmosphere, together with the available work of the uncompleted expansion, represented by the triangle CHD in Fig. 29; this may easily amount to 4 lbs. per sq. in. through the whole steam-volume; and taking the latter to be 24 cu. ft., to allow for the condensation due to work done in the cylinder, we get

$$4 \times 144 \times 24 = 13,820 \text{ F.P.}$$

as the energy of the steam-jet per pound of steam. Then the mechanical efficiency of the apparatus is

$$E=\frac{2082}{13{,}820}=.151.$$

Referring forward to Eq. (167), which applies equally well to this case, we find the limit of efficiency in the entraining operation, with the above data, to be about 25 per cent., the other 75 per cent. of the jet-energy being necessarily changed into heat.

(*g*) THE INJECTOR.—The simplest type of this apparatus is outlined in Fig. 69, where, as in Figs. 65 and 66, all structural detail is omitted, and only the essential form is shown. Steam from the boiler enters at S, and its admission to the nozzle is controlled by the hand-regulated valve V. To start the injector, this valve is opened a little way, and the steam-jet at first draws out air from the water-chamber W and from the water-pipe, until the vacuum is sufficient to lift the water; as soon as the flow of water is established, steam is turned on full, and the mixed jet, formed by condensation of the steam in the tube T, all reduced

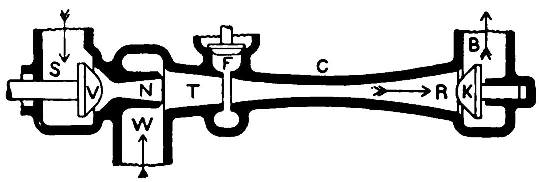

FIG. 69.—The Simple Injector.

to water by the time it gets to the choke C, and then slowed up in the retarder R, develops enough pressure to open the check-valve K and force itself into the boiler. The overflow F permits the escape of the mixed steam and air at the start, and of the first water, which is propelled by a jet of steam too weak for the regular discharge. This overflow is located at a part of the tube where the pressure in the jet is less than that of the atmosphere, in normal running; and the automatic check-valve at F prevents air from getting in to spoil the vacuum.

The rate of delivery of an injector can be varied over a considerable range—in some cases from full capacity to as little as 40 per cent. of this rate. In the simple form, this regulation is made chiefly by choking down the current of water in the suction-pipe;

something can be done by diminishing the supply of steam; but if this is cut down very much, the jet may become too weak for expulsion, and the injector will "kick back."

(*h*) THE COMPOUND INJECTOR.—The injector with only one tube is not very sure in its action if the supply-water must be lifted through any considerable height: to overcome this difficulty, two working jets are used, one for suction, the other for forcing. The general arrangement and the form and dimensions of the working parts of the injector outlined in Fig. 70 are copied from an actual design, the "Metropolitan": but besides the omission of details of construction and the mere indication of such parts as stuffing-boxes, some of the essential parts are transposed from their regular positions, so as to make a clearer illustrative drawing.

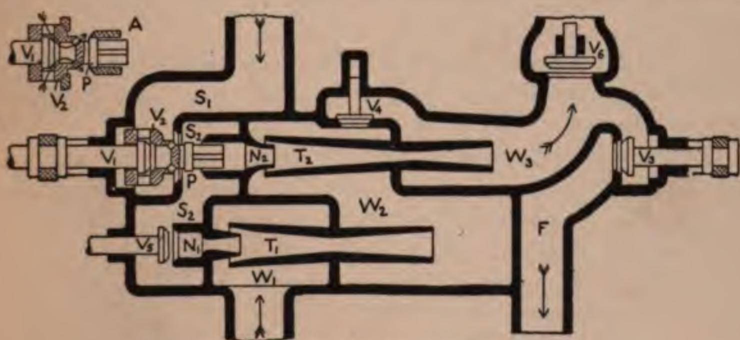

FIG. 70.—The Double-tube Injector.

The stems of the two valves V_1 and V_3 are rigidly connected by external side-bars, so that they must move together, and are controlled by a suitable lever-handle at the left. When the injector is idle these are pushed over to the right so that both steam-valves, V_1 and V_2, will be closed and the overflow V_3 wide open. To start, the handle is drawn back just a little, bringing the lifting-valve V_1 into the position shown at A, and admitting a small amount of steam through the chamber S_2 to the lifting-nozzle N_1, so as to establish the suction. The piston P keeps the nozzle N_2 shut off, not only during this preliminary admission of steam, but also until V_2 has been given quite a movement. In fact, V_1 is not essential to the working of the apparatus, but is rather designed

to equalize the pressure on the two sides of V_2 before this larger valve is moved, so that there will be no great resistance offered to the moving of the handle. In order to insure prompt filling of the whole injector with water at the start, the overflow is put on the discharge-chamber W_3; and the passage through the forcing-tube T_2 is supplemented by the check-valve V_4.

As soon as a good stream of water appears at the overflow, the handle is pulled all the way back, giving full admission past the main valve V_2 to the steam-chamber, and closing the overflow: in the drawing, the injector is not quite wide open, and there would be some waste of water through the overflow-valve. The small valve V_5, at the lifting-nozzle, is not intended ever to be closed, but is adjusted by hand so as to regulate the rate of delivery by varying the amount of water supplied to the forcing-tube.

(*i*) THEORY OF THE INJECTOR.—The mechanics of the mixing or entraining operation, in which the greater part of the wasted energy disappears, is as follows:

If a small body moving at a high speed impinges upon a larger body moving slowly, but in the same direction, the sum of the momenta, or of the several mass×velocity products, will be the same before, during, and after the impact. If the bodies are non-elastic, they will move together after impact, and there will have been a loss of kinetic energy: for if

$$M_1V_1+M_2V_2=(M_1+M_2)V, \quad . \quad . \quad . \quad . \quad (165)$$

then will

$$M_1V_1^2+M_2V_2^2>(M_1+M_2)V^2.$$

To prove this, change $E_2=(M_1+M_2)V^2$ to $\dfrac{(M_1V_1+M_2V_2)^2}{M_1+M_2}$, then subtract this from $E_1=M_1V_1^2+M_2V_2^2$: after reduction, we find the difference to be

$$E_1-E_2=\frac{M_1M_2}{M_1+M_2}(V_1^2-V_2^2). \quad . \quad . \quad . \quad . \quad (166)$$

With elastic solid bodies, there is a rebound after impact, in which most of the energy that has been used up in compression is restored; with fluids, mingling together, any such action is impossible, and Eq. (166) therefore applies to the operation under con-

sideration. We let M_1 stand for the steam and M_2 for the substance moved, which might be called the "load"; usually the initial velocity and energy of the latter is so small as to be practically negligible; and dropping $V_2{}^2$ from (166) we get

$$E_1 - E_2 = \frac{M_2}{M_1 + M_2} M_1 V_1{}^2 = \frac{M_2}{M_1 + M_2} E_1. \quad . \quad . \quad (167)$$

In the injector, a usual working proportion is about 12 lbs. of water to 1 lb. of steam; and under this condition, the wasted energy will be $\frac{12}{13}$ and the effective energy only $\frac{1}{13}$ of that in the steam-jet. The latter may have a velocity of from 2400 to 3000 ft. per sec., and a kinetic energy of 90,000 to 140,000 F.P. per pound of steam (see Table 26 A), varying with both the boiler-pressure and that at the mouth of the steam-nozzle. Disregarding V_2 in (165), we find, approximately,

$$V = \frac{V_1}{\sqrt{13}} = 665 \text{ to } 830 \text{ ft. per sec.};$$

and similarly

$$E_2 = \frac{E_1}{13} = 7000 \text{ to } 11000 \text{ F.P.}$$

All the wasted energy, $E_1 - E_2$, is changed back into heat.

The question as to the pressure against which the injector will discharge is answered by first finding the velocity due to overflow under the effective boiler-pressure, the difference between the absolute boiler-pressure p_1 (plus resistance of pipe and valves) and the suction-pressure at the injector. Velocity is got most conveniently by the hydraulic formula

$$V = \sqrt{2gh}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

To find the effective head h, we divide the boiler-pressure above atmosphere by $.427 = 61.5 \div 144$ (using 61.5 instead of 62.4 for the weight of 1 cu. ft. of water in order to take account of the temperature of water as usually discharged from an injector); and add to this the suction-head, measured up to the injector, and considered negative if the supply is drawn from an elevated tank.

Suppose, for instance, that the boiler-pressure is 180 lbs. by gage, and that the water is lifted 10 ft.: then $180 \div .427 = 422$ ft.; adding the 10 ft. and substituting in (168), we get

$$V = \sqrt{64.32 \times 432} = 166.6 \text{ ft. per sec.}$$

The water-jet must have a velocity considerably greater than this, because of the losses of energy in the retarding operation: but it appears that there is a wide margin between the two limits of velocity—that is, between the greatest velocity that could result from the mixing operation, and the least velocity that could produce the desired pressure of discharge.

In making a test of the working of an injector, it is necessary to measure only the water drawn in and to read the temperature of the supply and of the discharge, besides knowing the pressure and the quality of the steam. The energy that is effectively applied to the work of pumping is relatively so small that it can be neglected in the heat-equation by which we determine the amount of steam. For this equation, we assume that all the heat given off by 1 lb. of steam above the discharge temperature t_2 is taken up by w pounds of water in being raised from its original temperature t_0 to t_2. The total heat of the steam, above 32°, being found as $Q = H. - m_1 r_1$, we have

$$Q_1 - (t_2 - 32) = w(t_2 - t_0). \quad . \quad . \quad . \quad . \quad . \quad . \quad (169)$$

An example will make clear the relations involved and the method of getting results.

EXAMPLE 4.—An injector supplied with steam at 94.5 lbs. by gage, with 2.6 per cent. of moisture, draws water at the rate of 3240 lbs. per hour, showing the temperatures $t_0 = 72.6°$, $t_2 = 156.3°$: the supply is 16 ft. below the injector, and the discharge pressure is 99.2 lbs. per sq. in. by gage. How much steam does it use, and what is its effective thermodynamic performance?

For this steam, at 109.2 lbs. abs.,

$$Q_1 = 1183.8 - .026 \times 878.4$$
$$= 1183.8 - 23.0 = 1160.8 \text{ H.U.}$$

Then by (169)

$$w = \frac{1160.8 - 124.3}{156.3 - 72.6} = \frac{1036.5}{83.7} = 12.38.$$

Dividing the total water-weight $W = 3240$ by this ratio, we get the steam per hour to be

$$S = 3240 \div 12.38 = 262 \text{ lbs.}$$

For the mechanical performance, we have the effective work of forcing $3240 + 260 = 3600$ lbs. of water, at 61.0 lbs. per cu. ft., against a total pressure of $94.5 + (16 \div 2.31) = 101.4$ lbs. per sq. in. Solving in

terms of work per pound of steam rather than by the hour, and by the pressure volume method, we get

$$PV = 144 \times 101.4 \times \frac{13.38}{61.0} = 5802 \times .220$$
$$= 1276 \text{ F.P. or } 1.64 \text{ H.U.}$$

So that out of a total heat-supply of $1160.8 - 40.6 = 1120.2$ B.T.U. (estimated above the temperature of the cold feed-water), only 1.64 B.T.U. is effectively transformed; and the absolute efficiency has the ridiculously low value

$$E = \frac{1.64}{1120} = .00146.$$

(*j*) Range of the Injector.—In the matter of discharge-pressure, it is possible, by suitably proportioning the steam-nozzle and the inlet to the mixing-tube, to have this pressure far above that of the steam, even using the exhaust from a non-condensing engine to feed the boiler. In this case the water pumped per pound of steam will be relatively small, because the energy of the steam-jet is small; consequently the supply-water must be cold, so that the steam will be condensed without making the final temperature too high. Further, the fact that this temperature must be high precludes the lifting of the water by suction, because the attainable vacuum in the injector will be small.

Ordinary high-pressure injectors are usually proportioned so as to deliver against a pressure from 20 to 40 per cent. above that of the steam—that is, in regular working there is a large excess of energy in the water-jet over just what is necessary for delivery.

It is frequently desirable to use feed-water of fairly high initial temperature, as from the hot-well of a condensing plant. The upper limit of suction-temperature in a well-proportioned injector will be about 130° F.—which, in Example 4, would make t_3 about 212°—so that feed at from 90° to 110° is entirely practicable; but of course this water must be supplied to the injector, if of the simple type, at or above its level.

(*k*) The Steam-turbine.—Of this important and increasingly prominent type of steam-motor, only the simplest fundamental principles will be given at this point, further treatment of the subject being reserved for a later chapter.

The working parts of a small De Laval turbine, which, as to the elements involved in the application of the jet, is of the simplest

form built, are shown in Fig. 71. In I. a cylindrical section through the blades at mid-length is developed or flattened to the plane of the drawing; while II. is a view from the left side of I. The true form of the blades or buckets, as also the manner of holding them

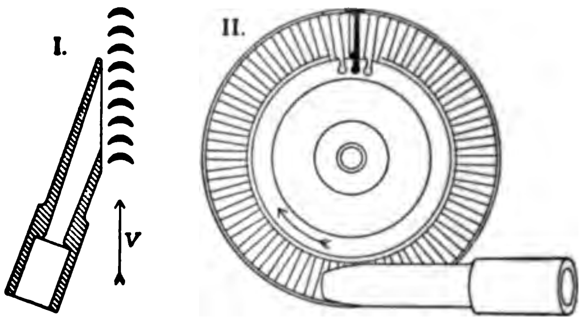

FIG. 71.—The De Laval Turbine.

in the wheel, is shown by the detail at the top of II.; otherwise their form is merely indicated.

The manner of action of this turbine can best be explained with the help of the velocity diagram in Fig. 72. Here AB represents the full velocity of the steam, at the mouth of the nozzle and along its axis. This steam strikes a row of "buckets" which are moving with the velocity V_T; and its velocity relative to these buckets is got by combining with the absolute velocity V of the steam that of the nozzle relative to the moving blades, which is equal to V_T reversed: this gives V_1 as the velocity with which the steam enters the wheel.

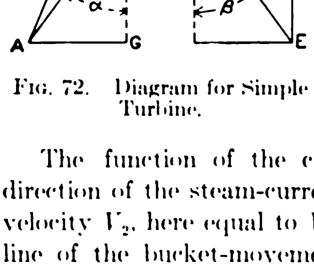

FIG. 72. Diagram for Simple Turbine.

The function of the curved turbine-blade is to change the direction of the steam-current, so that it escapes with the relative velocity V_2, here equal to V_1 and making the same angle with the line of the bucket-movement. The pressure of the steam upon this curved guide, due to the inertia with which it resists lateral acceleration, is the working-force in the turbine.

Combining with V_2 the absolute velocity V_T, at EF, we g

DF $=V_0$ as the absolute velocity of exit of the steam. The drop in kinetic energy of the steam due to change from V to V_0 represents the effective performance of the turbine. In the figure, V_0 is about 40 per cent. of V, so that the residual energy is about 16 per cent.

The jet-angle ABC being 20°, it is obvious that the minimum value of V_0 is DH, equal to $V \sin 20°$ or $.342V$. To secure this minimum, V_T must be half of the projection BG, half of $V \cos 20°$, or $.470V$. Since it is highly desirable that V_T be kept as small as possible, and since the angle HDF can have a considerable value (even up to 30°) without making DF much greater than DH, it appears that good results will be got by making V_T equal to $0.4V$ or $\frac{3}{8}V$. With steam-velocities ranging from 2500 to 3000 ft. per sec., the velocity of the buckets should be from 1000 to 1500 ft. per sec. In actual De Laval practice, the range is from 525 ft. per sec. with a 4-inch wheel at 30,000 R.P.M. to 1400 ft. per sec. with a 30-inch wheel at 11,000 R.P.M. To bring these rotary speeds down to something practically applicable, a pair of gears with the ratio 10 to 1 is interposed between the turbine-shaft and the power-shaft.

(*l*) Compound Turbines.—In order to reduce the linear speed of the turbine-buckets without diminishing the effectiveness of the utilization of steam-velocity, two general lines of procedure are available. The first is to divide the reduction of velocity into a number of successive steps after the manner of Fig. 73; in the second, the expansion of the steam is divided into stages like those in a multiple-expansion engine, a complete turbine element, simple or complex, being employed for the utilization of the velocity due to each pressure-
' n In this work, the words
l compound will be
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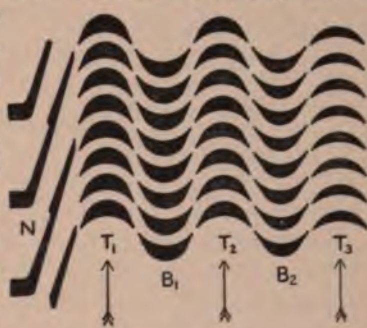


Fig. 73.—Element of Curtis Turbine.

purpose as to the mechanical arrangement
ment.

The arrangement of a complex turbine is shown in Fig. 73, where the view given bears the same relation to the axis of rotation as does that in Fig. 71 I. The nozzles and the guide-vanes B_1, B_2 are fixed to the frame or casing of the machine, and the three rows of buckets are carried on the rim of a wheel.

The velocity diagram for the first line of buckets is ABCDE in Fig. 74, similar to Fig. 72: the first line of vanes reverses CE to EF, here assumed symmetrical with CE. The rest of the process, together with the manner of getting circular-arc profiles which will be tangent to the several velocities, is self-evident.

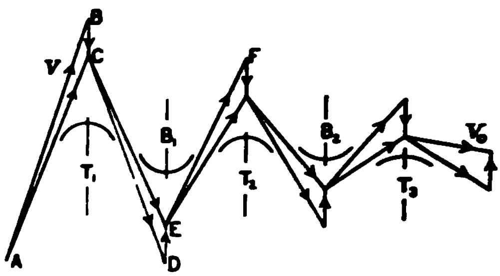


FIG. 74.—Multiple-step Velocity Diagram.

The question as to the form of profile necessary for symmetrical velocity reversal—that is, for making equal any pair of angles as α and β on Fig. 72—will be taken up later. It will be noted that the steam-current in Fig. 74 will increase in cross-section as its velocity is lowered: room is given for this increase partly by making the vanes and buckets thinner, partly by increasing the length of the open spaces, in a direction at right angles to the plane of the drawing, or along the radius of the wheel.

The thermodynamic principles underlying the working of the steam-turbine have been quite fully developed in this chapter; the questions involved in the use of the steam-jet as a driving agent belong chiefly to the domain of mechanics, and will therefore be taken up more at length after the mechanical presentation of the common engine has been given.

CHAPTER VI.

THE ENTROPY-TEMPERATURE DIAGRAM.

§ 30. General Ideas as to Entropy.

(*a*) ENTROPY is a quantitative property of heat, already defined mathematically in § 13 (*c*), and extensively used in our computation upon the adiabatics of steam. According to that definition, when a portion of heat Q is imparted to a body at the temperature T, the body acquires entropy N measured by the equation

$$N=\frac{Q}{T}. \qquad (170)$$

If the operation takes place with varying temperature, so that only the infinitesimal dQ is imparted at any particular T, we have

$$dN=\frac{dQ}{T}; \qquad (171)$$

whence, during the passage from T_1 to T_2, the entropy acquired is

$$N=\int_{T_1}^{T_2}\frac{dQ}{T}. \qquad (172)$$

Without attempting, at this point, to give any other idea of entropy than that of a mere mathematical ratio (or summation of ratios), we will now proceed to apply it to the analysis of simple thermodynamic operations.

(*b*) THE ENTROPY-TEMPERATURE DIAGRAM.—Just as work is represented graphically by the product of its two factors, force and distance or pressure and volume—that is, by an area—so also may heat imparted be diagrammed as the product of two factors, one the absolute temperature, the other the entropy. This appears at once from the equations above; and the sequence

... s which ... suggests itself as a history of the development of the ... oncept of entropy is, first a desire thus to represent heat as a compound quantity, then the natural adoption of temperature as one factor and the determination and naming of the other factor.

In this system, the two simplest operations—the ultimate operations, in that, while others can be resolved into them, these cannot be reduced to anything simpler—are the isothermal and the adiabatic, the elements of the Carnot cycle. The first is change of

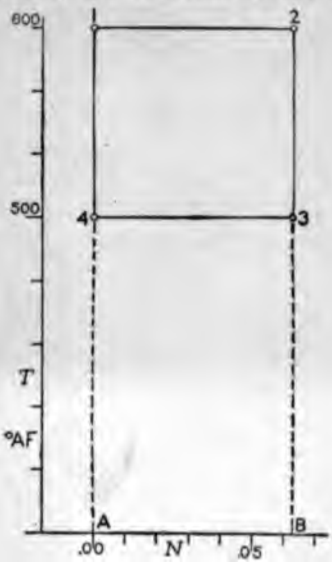


FIG. 75.—Carnot Cycle.

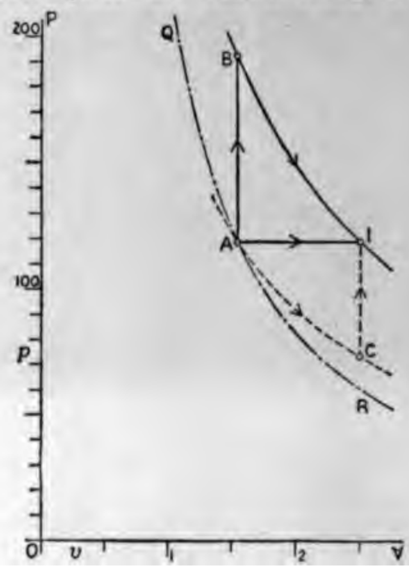


FIG. 76.—Constant Pressure and Constant Volume.

entropy at constant temperature, the second is change of temperature at constant entropy: and the NT diagram for this cycle, with the same designating symbols as on Fig. 20, is the rectangle shown in Fig. 75.

Equation (170) applies to this figure; from 1 to 2 the entropy $N=AB$ is acquired; the heat received is $Q_1=NT_1=$area 12BA; and the heat rejected in Phase III. is $Q_2=NT_2=$area 34AB; finally, the heat converted into work is

$$AU=N(T_1-T_2)=\frac{Q_1}{T_1}(T_1-T_2); \quad . \quad . \quad . \quad . \quad (173)$$

the same entropy being lost from 3 to 4 that was acquired from 1 to 2.

This figure is drawn for the same conditions as Fig. 20. The amount of heat received, Q_1, can be found only from the ratio of isothermal expansion, as in § 8, Ex. 1: then the entropy follows by plain division, the value in this case being $N = 50.36 \div 800 = .63$. It will be noted that the axis of T, or the line of zero-entropy, is here placed right against the left side of the diagram, showing that change in N, rather than something like an absolute value, is the important variant. The difficulties in the way of a measurement of entropy above some reference state, and also the representation of certain other processes besides the isothermal and adiabatic operations, will now be illustrated.

(c) THE TWO SIMPLE OPERATIONS in which the variation of temperature is in constant ratio with the heat received are represented according to the two systems in Figs. 76 and 77. The point 1 on Fig. 76 corresponds with the same point on Fig. 20: and the matter to be discussed is, the different ways of bringing the pound of air to this starting-point of the Carnot cycle there illustrated.

As an initial reference state, the temperature 32° F. naturally suggests itself, and in Fig. 76 the isothermal line AC is drawn for this temperature, while B1 is a similar curve for 800° AF. Two of the possible ways along which the gas can pass from the temperature of 492° AF. to the point 1 are, by heating at constant pressure from the point A where the pressure is p_1, and by heating at constant volume from the point C where the volume is v_1. Again, taking A as a single, definite starting-point, besides the direct change along A1, we could have an isothermal expansion along AC, then rise at constant volume along C1: or rise at constant volume from A to B followed by isothermal expansion from B to 1. The three operations lead to the same final result; and these are, of course, only particular cases out of the infinite number of possible paths from A to 1.

The representation of th[illegible] processes on the NT system is given in Fig. 77, where the [illegible] corresponds to Fig. 76. Letting c stand, in general, [illegible]ant specific heat, we have, in Eqs. (171) and (172[illegible]

$$dN = \frac{dQ}{T} = c\frac{dT}{T}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (174)$$

whence

$$N = c\int_{T_1}^{T_2} \frac{dT}{T} = c \log_e \frac{T_2}{T_1} = 2.3026\, c \log \frac{T_2}{T_1}, \quad . \quad . \quad (175)$$

the last expression being in terms of common logarithms.

For heating at constant pressure, $c = 0.2375$, and (175) becomes

$$N = 0.54685 \log \frac{T_2}{T_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (176)$$

while for constant volume the coefficient is

$$.1690 \times 2.3026 = 0.38914. \quad . \quad . \quad . \quad . \quad . \quad . \quad (177)$$

In passing from A to 1 at constant pressure the heat imparted is

$$Q = .2375 \times (800 - 492) = 73.15 \text{ H.U.};$$

and the entropy gained by the gas is

$$\begin{aligned} N &= 0.5469 \times (\log 800 - \log 492) \\ &= 0.5469 \times 0.21112 = .1153. \end{aligned}$$

This entropy is OM in Fig. 77, and the heat Q is represented by the area A1MO.

Of course, values of N for a number of different T's must be worked out in order to get a series of points for the curve A1; and a similar calculation is necessary for curve AB, or C1, which shows the change at constant volume. The corresponding diagrams AB1C, on Fig. 76 and Fig. 77, give an interesting comparison of the two systems of representing thermodynamic processes.

(*d*) The Measure of Entropy.—What we get by evaluating (170) or (172) is the gain in entropy of the body receiving heat, on account of certain changes in temperature: but there is nothing to indicate at what distance from the OT-axis the measurement of this entropy shall be begun. In other words, there [illegible] definite zero-line for entropy, and [illegible] cannot be meas[illegible] abso-lute terms, as can pressure, volum[illegible]d temperature. [illegible] adiabatic curve through any c[illegible]ion-point, as QB [illegible]

on Fig. 76, will be a vertical reference-line on the NT diagram; but any such line can be used as the co-ordinate axis of temperature.

That there is no absolute zero of entropy, analogous to that of temperature, appears when we note that a logarithmic curve like 1A or 1C in Fig. 77 will go out to minus infinity for $T=0$: so that the total entropy of a substance, due to an imaginary heat-impartation at constant rate from 0° AF. up to T, would be infinite.

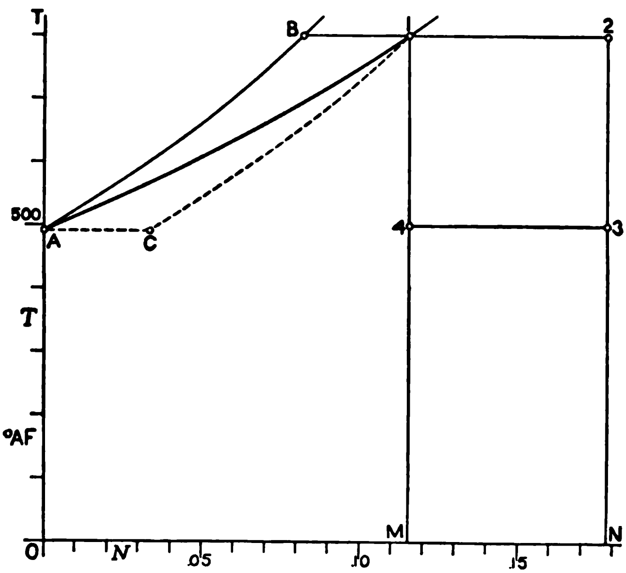


FIG 77.—Entropy Diagram for Fig. 76.

An important practical fact is that any curve from (175), as AB or C1 in Fig. 77, can be moved sidewise as convenient: so that in any co-ordinate system (with any particular scales of T and N) a curve whose condition is that the heat imparted varies definitely with the temperature need not be laid out every time it is to be used. Once drawn, it can be shifted horizontally, as by a template, so as to pass through any desired point. And this is true for any relation of dQ to T, besides the simple relation in (174).

(e) THE HYDRAULIC ANALOGY, in connection with the ideal

heat-engine, as already used in § 8 (k), is the foundation of a line of reasoning that leads to a pseudo-physical conception of entropy which, while it is highly artificial, may serve to connect this rather elusive idea with familiar mechanical principles.

Suppose that a certain weight of water W is at a certain height h_1, above sea-level; and that, in passing through a suitable motor, the water is lowered to and discharged at a level h_2 above the same zero. At the upper level, the potential energy of the water is Wh_1, the product of the weight-force W by the height, or by the distance through which the water is *capable* of moving under the action of this force; at the lower level the residual energy is Wh_2; and the energy given off by the water in being lowered is $W(h_1-h_2)$. It must be clearly understood that in these expressions for work or energy, W stands for force pure and simple, and not for the quantity of the water-substance, which is what we are likely to mean by "weight." Then back of the two factors of work, force and distance, stands the water-substance itself, serving in this case as a sort of vehicle of mechanical energy.

Next, consider the operation of a thermal engine with the Carnot cycle. Heat is taken in at the upper temperature T_1, part of it is transformed into effective mechanical work, and the rest is rejected as heat at T_2: the respective quantities of heat-energy, received, rejected, and converted, being proportional to T_1, T_2, and (T_1-T_2), all in the same ratio. If now, instead of thinking of heat as a simple quantity, we resolve it into the factors T and N, then T is analogous to the height h, and N to the weight W. From this point of view, some writers have called entropy "heat-weight": but the analogy weakens if we try to make N correspond with the force W, even though the tendency of heat to sink from a higher to a lower level (or temperature) is as real a fact of nature as the force of gravity. A useful conception is got by letting N correspond with W as a measure of quantity: so that N will measure a sort of imaginary energy-vehicle, which is back of the factors and back of the heat itself. In a sense, then, the entropy carries the heat.

(f) HEAT-CONVERSION AT CONSTANT ENTROPY.—A characteristic of the performance of the ideal heat-engine is that the entropy

remains constant as the heat is lowered through the working-range of temperature. And from the operation of this apparatus—which is simply the embodiment of the thermodynamic process, the most perfect form, and the fundamental part of all forms, of the process of converting heat into work—we get the following ideas:

When heat is imparted to the working-substance of a heat-engine, the entropy accompanies or carries the heat into the substance.

When heat is transformed into effective mechanical work (the work of a closed cycle), it leaves its entropy in the working-substance, with the residual heat-energy.

The heat rejected takes with it all the entropy that came in with the heat supplied.

In other words, it is a fundamental fact of thermodynamics that entropy goes into or out of the cycle only with heat as heat, not with heat transformed.

This action parallels exactly that of the water in relation to its potential energy in the hydraulic cycle: but, as stated in § 8 (k), the analogy will not hold for the details of the respective processes.

For all practical purposes, the mathematical idea of entropy is sufficient; and in our further discussions the language belonging to that conception will be chiefly used.

(g) Heat-transfer with Gain of Entropy.—Having a certain amount of heat Q_1 in a source at T_1, and capable of being given off for thermodynamic use at this temperature, we may call the quotient N_1, equal to Q_1 divided by T_1, the initial entropy of the heat—even though it may be, strictly, the entropy of a certain amount of the substance of the "source." If the impartation to the working-substance of the heat-engine is isothermal, or reversible at T_1, then the entropy acquired by this substance is the same as the initial entropy of the heat. But if the heat passes ture-gap, or if the process is not reversible, then first term of the equation

$$=T_m\int\frac{dQ}{T}=Q \quad . \quad . \quad . \quad . \quad . \quad (178)$$

is less than the initial entropy N_1 in

$$N_1 T_1 = Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (179)$$

That is, in the purely thermal operation of the transfer of heat from one body to another, there may be an increase in entropy: and with this goes a loss of thermodynamic potentiality, as will now be made clear.

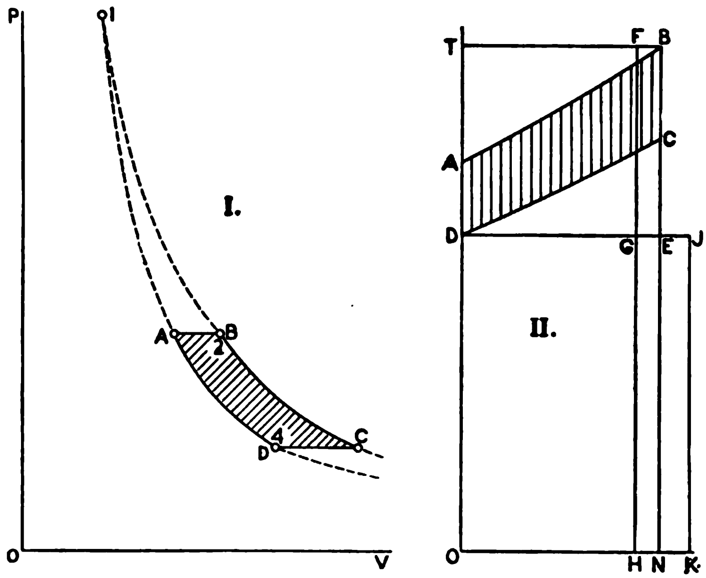


FIG. 78.—Diagrams of a Non-reversible Cycle.

(*h*) A NON-REVERSIBLE CYCLE.—As a concrete example under the statements just made, consider the imperfect cycle in Fig. 78. This consists of two operations at constant pressure, for the reception and rejection of heat, with two adiabatics; it has the same temperature-range and the same general conditions as the Carnot cycle of Figs. 20 and 75; and is formed graphically, in I., by drawing horizontal lines through the points 2 and 4 of Fig. 20. Then the NT diagram is got by moving sidewise the curve $A1$ of Fig. 77, most conveniently by means of a template, until it passes first through 2 and then through 4 of the rectangular diagram. This

graphical determination of the temperature at A and at C is accurate enough for present purposes.

The heat received is $Q_1 = NT_{m1} =$ OABN, according to (178), where N is ON and T_{m1} is the mean height of the curve AB. In the source, at T_1, this same heat Q_1 was represented by the area OTFH, equal to OABN, where OH was got by the relation

$$\text{OH} = N_1 = \frac{Q_1}{T_1} = \frac{\text{OABN}}{\text{OT}}, \quad . \quad . \quad . \quad . \quad . \quad (180)$$

from the measured area OABN.

If this heat were taken into a perfect cycle with the same limits T_1 and T_2, represented by TFGD, then the heat rejected, the unavoidable loss of energy, would be ODGH, equal to N_1T_2; and the effect of the imperfection in the upper part of the cycle is to increase this unavoidable loss to ODEN, by the amount $(N - N_1)T_2$.

Further, on the lower side of the cycle, the heat actually rejected is $Q_2 = NT_{m2} =$ ODCN: here again the heat drops across a temperature-gap to the receiver; and at the temperature T_2 it is represented by ODJK $= N_2T_2 =$ ODCN. Then the addition to the lost heat, on account of imperfect heat-rejection, is NEJK, or $(N_2 - N)T_2$.

This brings us to the conclusion that the avoidable waste in an imperfect cycle is measured by the increase in the entropy of the heat from the source to the receiver.

(*i*) Heat-waste in the Steam-plant.—In any actual apparatus, a certain temperature-difference is essential to the reasonably rapid flow or transfer of heat. In the steam-plant, on the reception side, this difference is very great; thus, with a temperature of steam-formation below 400° F., the highest temperature of the hot gases may be over 2000° F.; so that part of the heat drops across a gap of 1600°; and at the best, the gap is not reduced to less than from 100° to 200° by the time the gases leave the boiler. On the rejection side, the difference is less: from a non-condensing engine, exhaust steam at 212° mixes with air at from 0° to 90°, according to the weather: while in a condenser the difference between the mean temperature of the cooling-water and that of the entering steam is usually from 30° to 60°.

[illegible] show the great increase in the proportion of heat rejected

by the engine, on account of its inability to receive heat at a temperature somewhere near that at which it is generated by the combustion, consider the following case:

Suppose that the furnace could supply heat effectively at 2000° AF., and that the actual limits of the engine are 800° AF. and 500° AF. Then for 1000 H.U. available at 2000°, the entropy is $N_1=0.5$; and worked through a perfect cycle to 500°, only $N_1T_2=0.5\times500=250$ H.U. would be rejected. At 800° AF., the entropy of this same heat would be $N=1.25$; and then the rejected heat cannot be less than $NT_2=1.25\times500=625$ H.U. The increase in the unavoidable waste, from 250 to 625 H.U., is due to the increase in entropy from 0.5 to 1.25, or is $(N-N_1)T_2=0.75\times500=$ 375 H.U.

Of course, an ordinary boiler-furnace could supply only a small part of the heat of combustion of the fuel at a temperature such as 1500° F.: but if the thermodynamic apparatus could use the heat at such temperatures, the regenerative furnace used in metallurgy could supply it without excessive waste up the chimney.

§ 31. Entropy Diagrams for Steam.

(*a*) THE ENTROPY OF FORMATION OF STEAM.—The foundation for all graphical work under this system with steam is a diagram made by plotting Columns 9 and 10 of the Steam-table, as in Fig. 79. The curve PQ shows *a*, laid out from OT, while RS is got by adding *b* to *a*, or measuring off *b* from PQ. Of course, the uniform vertical scale is in degrees of temperature, common or absolute, as marked in the middle of the figure; but the varying scale of corresponding pressures, at the side, is more convenient for use, both with data from the Steam-table and from actual steam diagrams or indicator cards. The line of zero-entropy, OT, is the adiabatic through the initial state, that of water at 32° F.

The complete entropy diagram for the whole operation of steam-formation is made up of the curve PA for the heating of the water and the line AB for the vaporization: then PQ, up to any particular height, is a curve of operation, as is also AB: while RS is only a locus of condition, representing the curve of constant steam-weight.

This diagram does not represent total quantities of heat, because the base-line ON is at 500° AF. instead of at absolute zero. Then each strip, of the width $\Delta N=0.2$ and height 500° below the line ON, stands for 100 B.T.U. For practical use, it is better thus to have the range of ordinary temperatures laid out to a large scale, even though the efficiency and the relative heat quantities are less strikingly apparent.

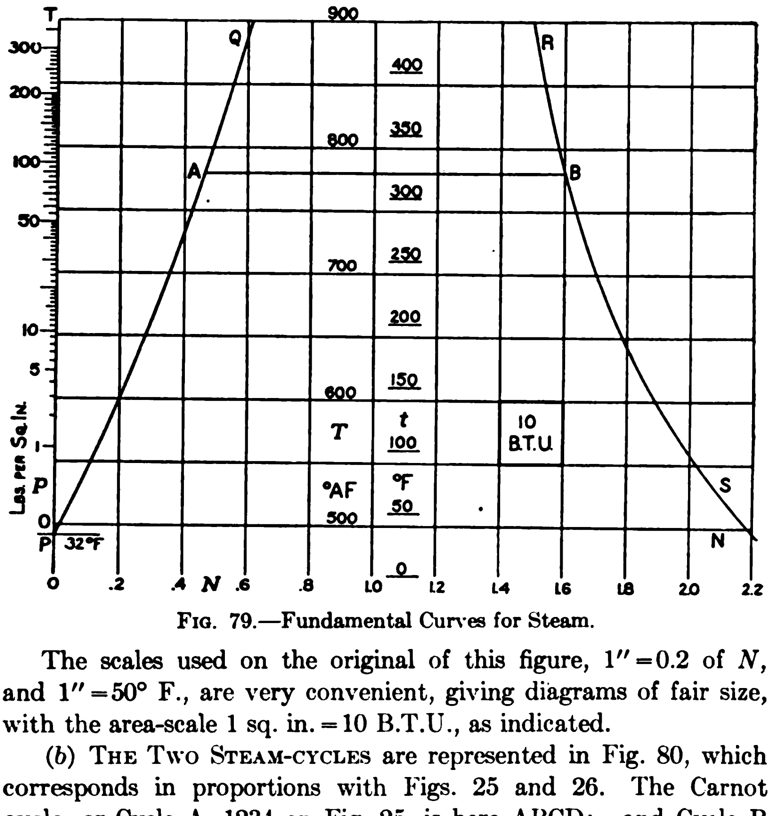


FIG. 79.—Fundamental Curves for Steam.

The scales used on the original of this figure, 1″ = 0.2 of N, and 1″ = 50° F., are very convenient, giving diagrams of fair size, with the area-scale 1 sq. in. = 10 B.T.U., as indicated.

(b) THE TWO STEAM-CYCLES are represented in Fig. 80, which corresponds in proportions with Figs. 25 and 26. The Carnot cycle, or Cycle A, 1234 on Fig. 25, is here ABCD; and Cycle B is ABCE. For the area ABCD, $N_A=.465$, $N_B=1.604$, or $N=1.139$; the respective t's are 320° and 213°, so that $(t_1-t_2)=107$: then $107\times1.139=121.8$ H.U. is the heat transformed, per pound of steam, as calculated also on page 74. The total heat received

is $N \times T_1 = 1.139 \times 780 = 888.4$. For Cycle B, the heat received would be the area under EAB, down to absolute zero.

To show the relatively large effect of the drop in back-pressure

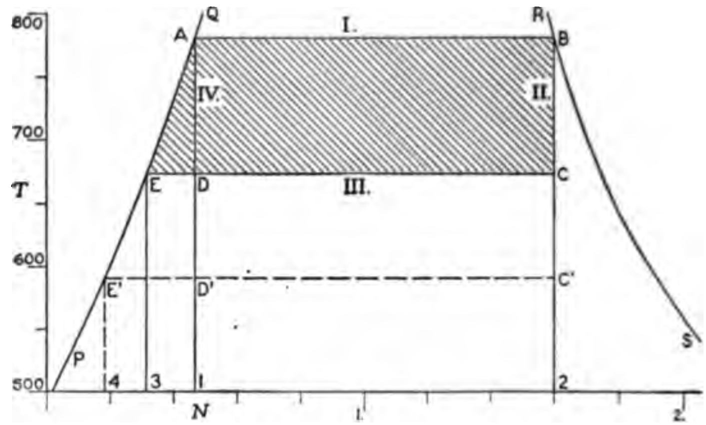


FIG. 80.—Ideal Steam-cycles.

and in exhaust-temperature, due to change from the non-condensing to the condensing condition, the line E'D'C' is drawn for an absolute back-pressure of about 2 lbs.

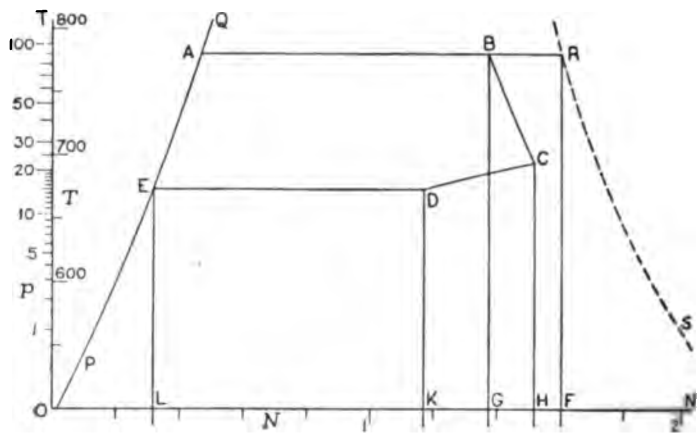


FIG. 81. Entropy Diagram for Fig. 26.

(c) IDEAL STEAM DIAGRAM.—The shaded diagram of Fig 26 is reproduced on the NT system in **Fig. 81.** The ratio AB to

is made equal to AB/A1 on Fig. 26: then BC is the curve of hyperbolic expansion, running well above the adiabatic (to the right of BG, here); and CD is the curve of pressure-drop at end of stroke, laid off as though the steam were condensed at constant volume from C to D. The manner of drawing this will be set forth presently. The adiabatic line RF corresponds with the curve 12 in Fig. 26: and the losses of effect due to imperfect realization of the cycle are represented by the area between BCD and RF, above the level of ED. Note that the full outline of this diagram, including the condensation by the cylinder-walls at constant pressure during admission, is EARBCDE.

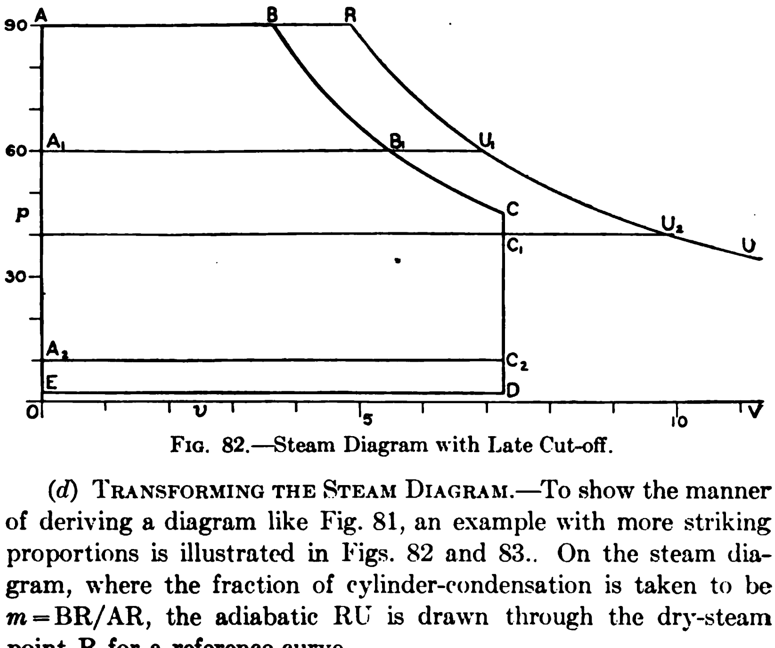


FIG. 82.—Steam Diagram with Late Cut-off.

(*d*) TRANSFORMING THE STEAM DIAGRAM.—To show the manner of deriving a diagram like Fig. 81, an example with more striking proportions is illustrated in Figs. 82 and 83.. On the steam diagram, where the fraction of cylinder-condensation is taken to be m = BR/AR, the adiabatic RU is drawn through the dry-steam point R for a reference-curve.

Then on any isothermal line, as A_1U_1, we have only to reproduce in Fig. 83 the ratio $A_1B_1 : A_1U_1$, in order to get a point B_1 on the *NT* curve BC. Below the release-point C the same method can be used as long as the adiabatic RU is within reach—as for the point C_1. Farther down [illegible] at A_2C_2, we divide the volume A_2C_2,

Fig. 82, by the specific volume s_2 at the pressure OA_2, taken fr the Steam-table: then multiplying the full entropy of evaporati A_2S_1 in Fig. 83, by this ratio, we get the length A_2C_2 which loca the point C_2 on the release curve CD.

Two particular properties of steam—that the isothermals horizontal lines on both systems, and that heat imparted va

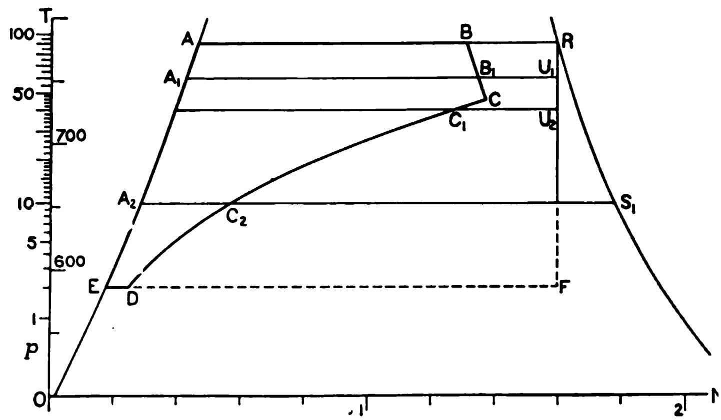


FIG. 83.—Entropy Diagram from Fig. 82.

directly as volume during evaporation—contribute greatly to si plicity and convenience in the construction of the entropy d gram.

So far as the steam left in the cylinder at any instant duri the release is concerned, it is immaterial whether the rest of t steam escapes or is condensed at constant volume: thermod namically, the latter is the simpler operation, and we assume it exist when plotting the curve CD.

The whole loss of effect is now BCDFR, on Fig. 83; and have the result of incomplete expansion represented in much mo convenient form than on a pressure-volume diagram.

The methods just developed are employed when converting indicator diagram to the entropy system: but with actual wor ing conditions several complications are introduced, notably the presence of the clearance-steam along with the working stea in the cylinder, and by the various wire-drawing actions.

(*e*) THE ACTION OF THE CLEARANCE-STEAM, as set forth in § 19 (*d*) and (*e*), must now be more closely scrutinized: and it will appear that the assumption of identical curves of compression and of expansion for this body of steam is not correct.

Under the methods available and employed in engine-testing, there is an inherent uncertainty as to the exact weight of the compressed steam. As a good working hypothesis, however, it seems reasonable to assume that the steam is dry-saturated when compression begins; because a large part of the moisture in the expanded steam must have been swept out as water in the first rush of the exhaust, when the valve opened widely while the piston was near the far end of the cylinder; and during the return-stroke exhaust the heat in the metal has been passing into a decreasing weight of low-pressure steam: so that the residual steam is likely to be fully dried, if not even slightly superheated, when the valve closes to exhaust.

In Fig. 84, let E and C be the points used in calculating the I.S.C.: the two distances FE and DC then represent volumes of saturated steam; but with the difference that the steam at C is dry, while that measured by FE has a considerable proportion of water mixed with it. Finding at G a point on a curve of constant steam-weight through C, we have the I.S.C. represented by GE. To get G, it is most convenient to use the specific steam-weight d from the Steam-table: thus the distance DC, on this figure, was 3.50″; for C, at a pressure of 20 lbs., the weight per cubic foot is .0503; at G, 95 lbs. pressure, it is .2171; then, since the weight is the same at these two points, FG was found to be

$$\frac{3.50\times.0503}{.2171}=.811''.$$

Now let S' be the indicated steam per cycle, as found by GE, or by using E and C in Eq. (98); S the actual weight of working-m, as determined by a condenser-test; and S_0 the clearance-calculated from C or G. At the pressure p_E, S' is repre-GE, and S_0 by FG; and by making

$$\text{GM} : \text{GE} = S : S' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

we represent S by GM. Then the whole weight of steam, S_0+S, in the dry state, would occupy the volume FM.

The curve CJ, here an equilateral hyperbola, if produced upward cuts inside of G, showing the condensation of the compressed steam on account of abstraction of heat by the metal surfaces. This compression is carried clear up to the highest pressure reached in the cylinder; and then expansion of the total steam begins.

As to this expansion, the only workable hypothesis is that the body of steam in the cylinder is homogeneous: then if FE represents the volume of the total steam, FH, got by making

$$\mathrm{FH}:\mathrm{FE}=\mathrm{FG}:\mathrm{FM}=S_0:(S_0+S),$$

will show the volume of the clearance-steam to the same scale, or under the same conditions. A curve of expansion through H, similar to the main expansion-curve ES, and got by dividing any abscissa FE in this same ratio, will complete the representation of the action of the dead steam.

We see, therefore, that this steam goes through the small cycle QJGLHR, with a negative effective work: and that a minor part of the total loss due to cylinder-condensation may be thought of as effectuating itself through the shrinkage in the volume of the clearance-steam, while the greater part results from a similar effect upon the working steam. The idea, in § 19 (a), of separating the two bodies of steam by an imaginary diaphragm—which must, however, be entirely pervious to heat, so as not to interfere with homogeneity of the steam during expansion—may be helpful in connection with the cycle-diagram of the clearance-steam.

(j) WIRE-DRAWING.—The effect of this action, whereby the area AJKBEA on Fig. 84 is lost from the full diagram of expansion of the working-steam, is to change pressure-work first into kinetic energy of the steam-currents, then into heat. So far as results are concerned, this heat might just as well have come into the steam from some outside source. It has a certain thermodynamic value, which is exhibited on the diagram in the fact that the expansion-line ES is a little farther to the right than it would be if this small extra supply of heat had not been received. On this figure, for instance, the total lost area outlined above measured 1.20 sq. ins.

Adopting a volume-scale which would make GM represent the volume of 1 lb. of steam at 95 lbs. pressure, the area-scale was 4250 F.P. or 5.47 B.T.U. per square inch; so that the lost work is 6.6 B.T.U.; and with the latent heat r equal to 885.6, this heat would evaporate .0075 of 1 lb. of steam. This fraction of GM is measured off at EE′, and is so small as to be scarcely visible on the reduced drawing. Other things being equal, however—such as the amount of heat absorbed by the cylinder-walls, down to E—the small shifting of the expansion-curve adds to the diagram a

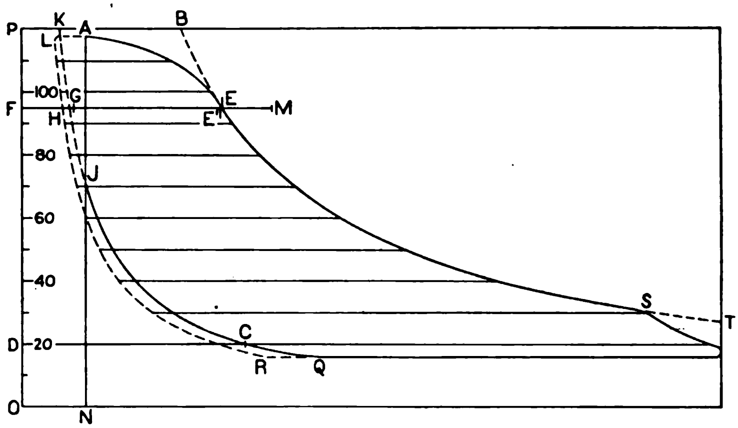


FIG. 84.—Steam Diagram with Large Proportion of Clearance-steam.

little area, which partly compensates, but only to a slight degree, for that lost by wire-drawing. This small action is merged into, and entirely overshadowed by, that of the cylinder-walls.

Wire-drawing through a wider range of pressure, such as that which takes place in the steam-calorimeter, is represented on the entropy system in Fig. 86. But in transforming indicator diagrams, the only practical procedure is to ignore the detail of this process, and represent simply its net result, showing the entropy-equivalent of the realized volume at any pressure, just as is done, for the release-curve, in Figs. 81 and 83.

(*g*) THE TRANSFORMED INDICATOR DIAGRAM.—The full-line diagram ABCDEF in Fig. 85 represents the indicator diagram AESTQJA of Fig. 84: it is derived by finding the ratio of horizontal

intercepts between the end-curves AJQ and AEST to the corresponding specific volumes, in Fig. 84; and then measuring off from AF, in Fig. 85, these fractions of the intercepts between AF and RS. Table 31 A shows the operation in detail.

TABLE 31 A. CALCULATIONS FOR FIG. 85.

Pressure p	Volume Realized v	Specific Volume s	Entropy of Evaporation b	Actual Entropy N
110	2.01	4.02	5.53	2.76
100	2.89	4.39	5.62	3.70
90	3.42	4[illegible]	5.71	4.03
80	4.06	5.[illegible]	5.81	4.35
70	4.84	6.13	5.93	4.68
60	5.65	7.09	6.05	4.82

In this table v is the volume-intercept on Fig. 84, reduced to cubic feet. To get the volume-scale, we have that GM, equal to 3.12″, represents 4.61 cu. ft. of steam; so that 1″ stands for 1.476 cu. ft.: then measurements from the indicator diagram are multiplied by this factor to give the values of v.

The specific volume s is taken directly from the Steam-table. In strict accuracy, u, equal to $(s-w)$, should be used instead of s: but in any work that can be done with actual diagrams the effect of this refinement would be absolutely invisible.

The total entropy of evaporation, b, is here reduced to inches on Fig. 85, so that the result of the calculation,

$$N=\frac{v}{s}\times b, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (182)$$

will also be in inches, and ready to lay off.

At 60 lbs., for instance, the entropy-length for Fig. 85 is

$$\frac{5.65}{7.09}\times 6.05=4.82'';$$

and, working directly upon the figures, it is just about as easy to make the whole computation at once, instead of first reducing v to cubic feet, in the form

$$\frac{3.83''\times 1.476}{7.09 \text{ cu. ft.}}\times 6.05''=4.82''.$$

Graphical constructions, an extension of the principle of Fig. 42 III., can be used for finding the abscissa-length for Fig. 85. But with so many elements involved, the method of measurement and calculation is easier, provided that the arithmetical work is done with a slide-rule, and is quite as accurate.

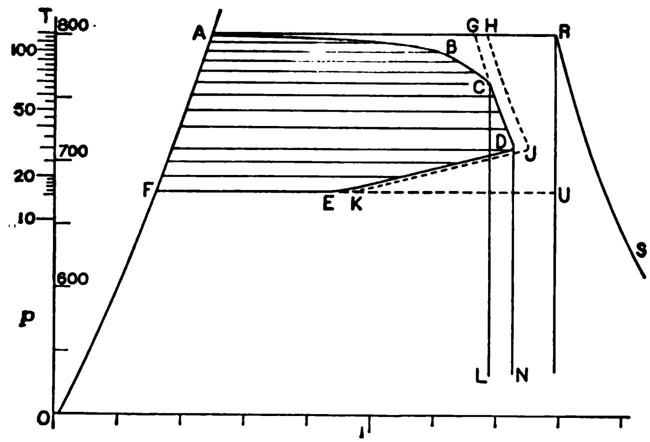


FIG. 85.—Entropy Diagram from Fig. 84.

(*h*) DISCUSSION OF FIG. 85.—In this figure, besides the reproduction of the outline of the indicator diagram, the expansion-curve CD is produced back to G, so as to bound the area AGCBA which represents the loss due to wire-drawing. As suggested at the end of (*f*), the curve ABC does not show in detail the complex thermodynamic operation of wire-drawing; but had better be thought of as simply a reproduction. on this heat-unit system, of the work-diagram in Fig. 84.

By drawing another expansion-curve at a short distance to the left of GCD (corresponding to E'F on Fig. 84), we could show what would take place if there were no wire-drawing: but the area between this curve and CD would be only a small fraction of AGC.

Now the total loss of work due to cylinder-walls and to incomplete expansion is GDEUR: and if we wish to separate the effects of wall-action upon the working-steam and upon the clearance-steam, we draw the curve HJK, by taking intercepts between LHR and BEST on Fig. 84 and going through the operation of

Table 31 A. Then the negative work of the clearance-steam represented by GDEKJH.

One very important advantage of the entropy diagram it represents the quantities of heat involved in any such op as the transfer to and from the cylinder-walls. Thus, d cut-off at C, the walls have taken up the heat represented area between ACL and RU, running down to absolute zero: during expansion the heat LCDX is returned to the steam.

To get a clear idea of the full thermal behavior of the clea steam, we should have to change its cycle-diagram, LJQR Fig. 84, to the entropy system: and could then follow close action of the cylinder-walls during compression. But for the purpose of showing the total loss due to imperfect realizat the steam-engine cycle, the full-line diagram on Fig. 85 is cient. With a compound engine there will be two or mor diagrams, in vertical series, each drawn by the method used h

(*i*) Superheated Steam.—For the operation of superh at constant pressure, and using the specific heat 0.4805 in Eq. we get

$$N = 1.1064 \log \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad .$$

As stated in § 30 (*d*), a single curve can be drawn fo full range of temperature likely to be covered, and then s sidewise as desired. The data for this curve are given in ' 31 B, where the increments of entropy for each 50° of tem ture-rise are calculated.

TABLE 31 B. DATA FOR THE SUPERHEAT CURVE.

T_1	T_2	ΔN	N	T_1	T_2	ΔN	
600	650	.03846	.03846	900	950	.02598	.2
650	700	.03562	.07408	950	1000	.02465	.2
700	750	.03315	.10723	1000	1050	.02345	.2
750	800	.03101	.13824	1050	1100	.02235	.2
800	850	.02911	.16735	1100	1150	.02137	.3
850	900	.02748	.19483	1150	1200	.02044	.3

Besides the successive increments ΔN, the total entropy N a the state at 600° AF., as got by cumulative addition, is giv the fourth column.

The entropy-curve for superheat is drawn at TU in Fig. 86, with TV as the reference-line from which N is measured. Then by means of a template, or by measurement, this curve can be moved horizontally so as to pass through any desired point on the saturation-curve RS.

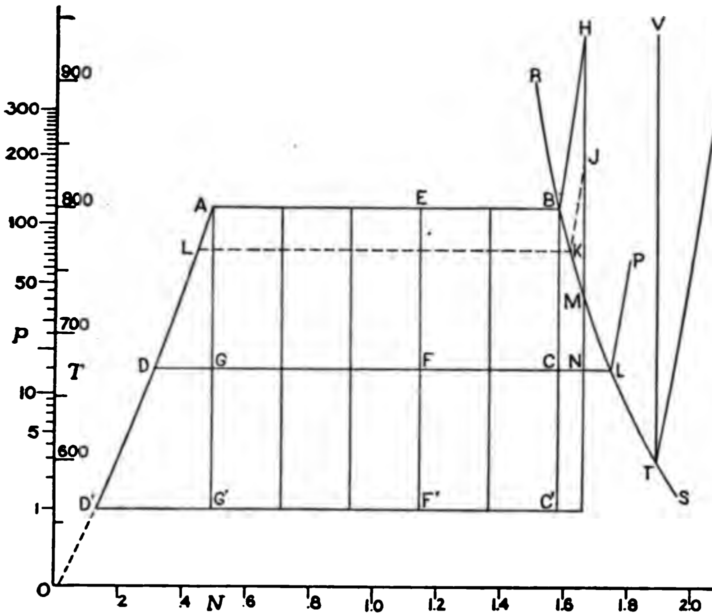


FIG. 86.—Diagram for Superheat and for the Steam-jet.

(j) ENERGY OF THE STEAM-JET.—Fig. 52 is represented in terms of entropy and temperature on Fig. 86, which shows graphically the energy-quantities used in getting the curves on Figs. 49, 50, and 51. AB is the isothermal line of evaporation at 120 lbs. Initially dry steam, expanding in the jet from 120 to 15 lbs., receives the energy ABCD; and with a lower pressure of 1 lb. abs., he energy of the jet is ABC′D′. A series of horizontal lines, ed by the pressure-scale, will give a representation of the suc- values of E in the 120-lb. column of Table 24 A or Table V. vertical lines, spaced at equal intervals between A and B, effects of different initial fractions of moisture; thus

with $x_1=.60$, the effective areas are bounded on the right l adiabatic EF′. This illustrates very clearly the propo: method set forth in § 24 (*d*); areas between AD′ and AG′ sh the energy of a jet of hot water.

For the case of steam initially superheated, the tempe for $v=1.2s$, where $T_S=801.0°$ at 120 lbs. pressure, is fou Eq. (63) to be $T=937.7°$ AF. A superheat curve is passed th B, and limited by T at H: then the vertical HN is the adi line for jet-expansion of the superheated steam. This line c the saturation-curve at M, showing at what point the ste the jet ceases to be superheated and begins to condense. W higher terminal pressure, as 75 lbs., for which LK is drawn temperature of the steam is found by passing the superheat KJ through K to meet HN at J. The relatively small thermal of a superheating action, as compared with evaporation-ef becomes strikingly apparent in this diagram.

(*k*) Diagram for Wire-drawing.—In complete wire-dra the energy put into the steam-jet is all changed back into he the currents come to rest at the lower pressure. The cas steam initially dry at 120 lbs., dropped to atmospheric pre by throttling, is represented by the diagram BCLP. The en of the jet is equal to ABCD: and when this is reconverted heat, it evaporates all the moisture in the steam, along CL, superheats it up to P, to a temperature calculated in Table 2 The area under CLP must be equal to ABCD.

CHAPTER VII.

THE MECHANICS OF THE ENGINE.

§ 32. Forces in the Machine.

(*a*) FORCES ON THE MOVING PARTS OF THE ENGINE.—In taking up a study of the force-actions in the common reciprocating piston-engine, we find a sufficient foundation of practical knowledge of the machine in the description of a simple high-speed engine given in Chapter I.; and the full description of the important variations in general form and of the details of construction will not be undertaken until the mechanical principles, as well as those of the thermodynamic action of the engine, have been set forth.

Starting, naturally, with the steam in the cylinder and the working-pressure which it exerts upon the piston, we note at once that a part of the subject—the total or mean effect of the steam in doing work upon the piston—has already been fully covered in Chapter IV., in the first part of § 20. But what we are now concerned with is not so much the resultant work-effect, expressible in terms of the mean effective pressure, as the actual, variant force-action in the engine: and of the system of forces whose relations we are now to consider, the steam-pressure is merely the primary member, while the final is the working-force which meets the resistance of the externally-applied load. This system is represented in Fig. 101, on an outline of the engine mechanism.

For the present, friction in the machine will not be taken into ount, but we will assume the ideal condition that all the work pon the piston is delivered at the output point. The weights eciprocating parts—that is, the force which gravity exerts n—are likewise neglected, as being relatively insignifi-

Turning now to Fig. 101, we trace out the sequence of force follows:

(b) FORCES ON THE PISTON-SLIDE.—Upon the sliding-pi made up of piston, piston-rod, and cross-head act forces shc first on the main figure, and then, in combination, at A. These a

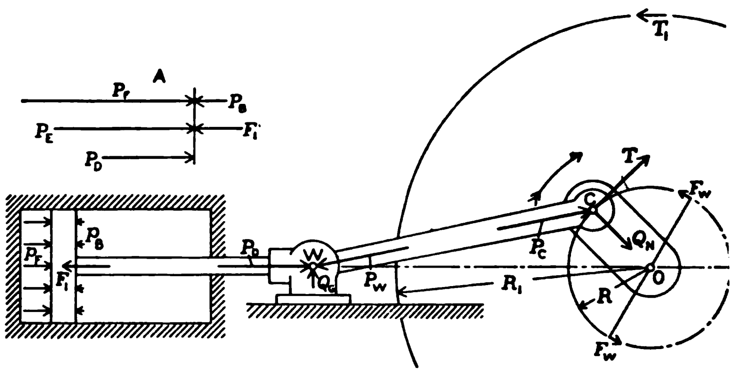


FIG. 101.—Forces on the Moving Parts.

P_F = forward steam-pressure, the full, absolute pressure acti on the forward-moving or driving side of the piston the instant.

P_B = back-pressure, on the exhaust or return-stroke side of t piston. Then

$P_E = P_F - P_B$ is the effective steam-pressure on the piston.

F_1 = inertia-force of slide, its reaction against the accelerati essential to its rapidly changing velocity.

$P_D = P_E - F_1$ is then the final effective driving-force, delivered the wrist-pin W for transmission to the crank.

These forces may be expressed either in pounds per squa inch or as total forces on the piston, bringing in the area-fact A, according to the relation $P = p \times A$. The steam-pressures a usually got from indicator diagrams, and it is then more convenie to keep them in pounds per square inch and bring the other forc to the same terms. Here, however, we are considering tot effects; so that what are really distributed forces, spread out ov the surface of the piston or the mass of the slide, are reduced t and represented by, single, concentrated forces along the axis.

From the beginning to, approximately, the middle of the stroke, the slide is being accelerated, and the inertia F_1 points in the direction shown, against the forward driving-force; but in the second half of the stroke the slide is retarded (or, its acceleration is negative), and F_1 then reverses. At high speeds, this force due to the mass and motion of the working parts modifies very materially the play of the resultant steam-pressure P_E.

Considering just the one position shown, the work being done upon the piston by P_E is not all transmitted to the crank, but part of it is being stored in the slide as kinetic energy. Since, however, the slide has zero velocity at each end of the stroke, the inflow and outflow of energy must balance; and, as a net result, all the effective steam-work will be carried over to the crank, as stated above.

(c) Transmission to the Crank.—The principal forces in equilibrium at the wrist-pin are P_D and P_W, the latter the pressure of the connecting-rod upon the pin; but since these are, in general, not in line, the third force essential to equilibrium is supplied by the guide-reaction Q_G, perpendicular to the slide-bar. In this figure, Q is used to indicate a force which does not move in the direction of its action—that is, one which acts upon a body that does not thus move—so that it cannot do work.

The connecting-rod WC has, of course, an inertia-force of its own, which would have to be determined and taken into account in order to find the exact manner of the force-transmission. This is a difficult operation, and it is usual to adopt the approximation of considering a part of the mass of the rod concentrated at W, where it adds itself to the slide and increases F_1, and the rest at C, where it has a radial, centrifugal inertia-force, with no turning-effect upon the crank. Then the rod can be taken as a weightless link, transmitting force along the center-line from W to C.

Coming now to the crank-pin, we have the force P_C, equal to P_W reversed, exerted upon it by the connecting-rod. Resolving this into components perpendicular to and along the crank-arm OC, T is the turning-force, or force tangential to the crank-circle, while Q_N is the radial component, which simply presses the shaft against the bearings.

This turning-force T, with the lever-arm R, must equa effect the resistance T_1, here taken tangential to the wheel, at radius R_1. In the aggregate the average moment TR must e T_1R_1; but since T varies widely, equilibrium exists only at cer positions of the engine: in all others there is an unbalanced t ing-moment, free to give angular acceleration to the shaft, acting against the moment of inertia of the fly-wheel, represe by the couple F_W-F_W.

Under the assumption that no work is lost through friction, work done by T upon the crank in one revolution must be same as that done upon the piston by the steam-pressure. Du each stroke, the piston moves through the distance $2R$, the cr pin through πR; using the subscript m to mark mean values, have

$$P_{\mathrm{Em}}\times 2R = P_{\mathrm{Dm}}\times 2R = T_{\mathrm{m}}\times \pi R;$$

whence, using P_{m} for either M.E.P.,

$$T_{\mathrm{m}} = \frac{2}{\pi}P_{\mathrm{m}} = .6366P_{\mathrm{m}}. \quad . \quad . \quad . \quad . \quad . \quad (:$$

(*d*) EXACT FORCE-ACTION ON THE CONNECTING-ROD.—Thi illustrated in Fig. 102, the explanation of which is as follows:

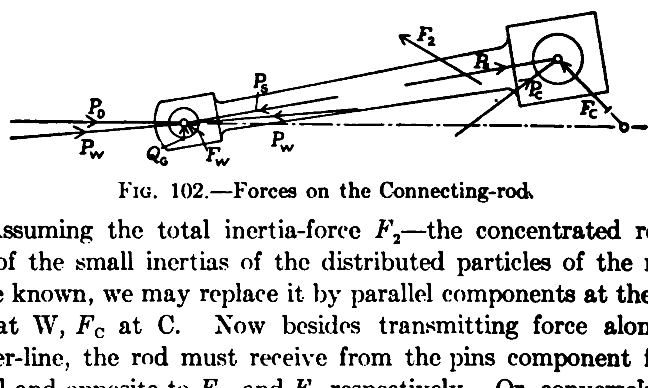


FIG. 102.—Forces on the Connecting-rod.

Assuming the total inertia-force F_2—the concentrated res ant of the small inertias of the distributed particles of the ro to be known, we may replace it by parallel components at the F_W at W, F_C at C. Now besides transmitting force along center-line, the rod must receive from the pins component fo equal and opposite to F_W and F_C respectively. Or, conversely, pressure P_W on the wrist-pin will be the resultant of P_S, along rod, and F_W. By a method which will be developed later, i possible to make the resultant of P_D and Q_G equal that of F_W.

P_S: and then carrying this P_S over to C and combining it with F_C, we get the other pin-pressure.

For the sake of knowing how nearly correct is the approximation described in (*c*), the inertia-effect of the rod is worked out by both methods in § 37 (*e*), with the additional object of determining the best proportion for the division of the rod-mass between W and C. It appears that the method used in Fig. 101 is amply accurate for all practical purposes.

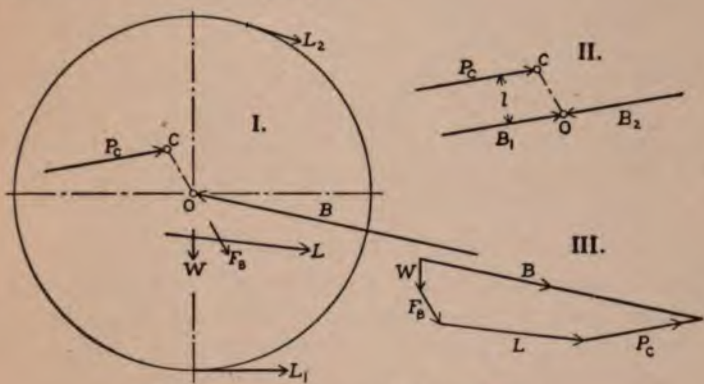


Fig. 103.—Forces on Shaft and Wheel.

(*e*) Equilibrium of the Shaft.—In Fig. 101 the system of forces acting on the shaft is not complete, because the pressure with which the bearings balance the resultant of all the other forces is not included. The complete set of forces is shown in Fig. 103, several being added to those given on Fig. 101. An important mechanical principle which applies in this case is shown at II., and may be stated as follows:

If the force P acting upon a body does not pass through the center or axis O about which the body is compelled to turn, this force will have two effects: it will tend to push the body straight ahead in its own direction, as though a force B_1 equal and parallel to itself were applied at the center, and resisted by the equal and opposite reaction of the bearing; and will also exert a turning-moment, that of the couple made up of P and B_2, tending to turn the body about O.

In order, then, to find the pressure on the bearing, we have only to combine all the forces acting upon the shaft, whether they pass through the center or not. On Fig. 103 these forces are—

P_C = pressure of connecting-rod on crank-pin.

W = weight of whole rotating piece.

L = resultant load force: here the power is supposed to be taken from the engine by a belt, the difference between the two tensions, L_1 on the tight or driving side and L_2 on the slack side, being equal to the T_1 on Fig. 101. Then L is the resultant of the two belt-pulls.

F_B = centrifugal force of the counterweight: this is an eccentric mass, attached to the crank opposite the crank-pin, in order that its centrifugal force may partly balance the inertia-force F_1 of the reciprocating parts, in a manner which will be explained presently.

The resultant of these four forces is found in the force-polygon at III., by laying them out in order and drawing the closing side B: as marked here, its arrow against the others, it is the resultant; as drawn in I. it is the equilibrant, the reaction of the bearing against the shaft.

Of these forces, the one that shows the greatest variation in character is the load L. In many cases—as when the engine is direct-connected to an electric generator, or to a screw-propeller—the resistance is a simple torque, with no tendency to press the shaft upon the bearing in any particular direction. In a locomotive, the resistance is tangential to the driving-wheels. In direct-acting air-compressors and pumps, where the whole engine-mechanism is used merely to regulate the speed and stroke, no power being taken from the shaft, the only resistance to the turning-moment on the crank is the angular inertia of the wheel, again a simple torque.

With the symmetrical arrangement of the center-crank engine in Figs. 2 to 5, it is strictly correct to consider the forces on the shaft as all acting in one vertical plane. But in some cases this is only a representation of resultant effect: with a [illegible] rank engine, for instance, it is necessary to go through quite [illegible] lex

analysis in order to determine the forces on the bearings and the stresses in the shaft.

(f) THE FORCES ON THE ENGINE-BED, on account of steam-pressure and of the inertia of the reciprocating parts, are shown in Fig. 104. The resultant of the two steam-pressures on the cylinder-heads is P_E. The driving-force P_D combines with the guide-reaction to give the crank-pin pressure P_C; and this, transferred to the center of the shaft according to Fig. 103 II., and there resolved back into its components, gives the forces Q_G and P_D as exerted upon the bearing (the bed) by the shaft.

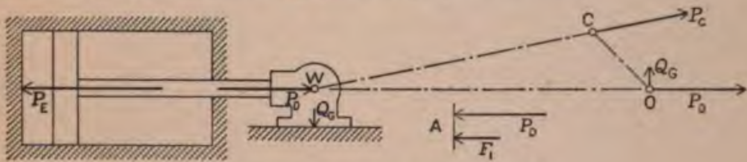


FIG. 104.—Forces on the Engine-bed.

Considering horizontal forces, it appears that the bearing-pressure P_D is less than the steam-reaction P_E, so that the engine-bed is not in static equilibrium. Separating P_E into two parts, as at A, one equal to P_D, the other to F_1, we see that the working-forces proper do form a balanced system within the machine, as shown by the opposing P_D's; but the absorption of a part of P_E-on-the-piston in accelerating the latter, or overcoming the inertia F_1, leaves the corresponding part of P_E-on-the-cylinder as an unbalanced force, free to accelerate the whole body of the engine in the other direction. A close analogy exists between this state of affairs and the recoil action in a gun: there the entire working-pressure of the powder-gases (barring friction) acts to accelerate the projectile, and reacts upon the gun and its mount with a force exactly equal to the inertia of the projectile; similarly the engine, accelerating the slide in one direction, is pushed the other way by a force identical with the inertia-force of the slide. This force, rapidly changing and reversing, tends to produce a shaking effect—which will be a mere tremor, perhaps imperceptible, with massive foundations, but may become a very serious vibration where the foundation mass is relatively small, as in marine engines especially.

The two vertical forces, Q_G at W and at O, Fig. 104, form couple; and since this is the only turning-moment exerted upo the bed, it must, on general principles, be equal to, and in a sen the reaction against, the moment TR produced by the stear pressure upon the crank—an equivalence which will be fully show in connection with the turning-force relations. It is through th couple that the load-torque is felt by the engine-bed.

If the load-force, like L in Fig. 103, is steady and uniform, simply develops a stress in the foundation-bolts, which can easil be provided for.

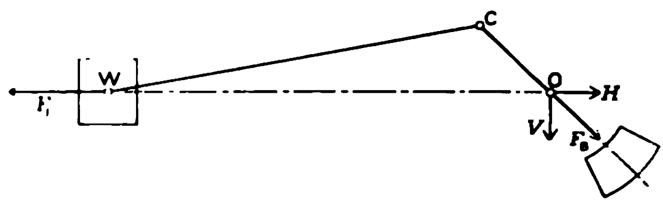


FIG. 105. Effect of the Counterbalance.

(g) COUNTERBALANCING. The method used to diminish th haking effect of the inertia of the reciprocating parts is show in Fig. 105. This consists in placing an eccentric mass on the crank disk, opposite the crank-pin: its centrifugal force F_B will hav components H and V at the center of the shaft; and the forme acts against F_1, reducing the horizontal shaking-force to $(F_1 - H)$. Of course, this introduces the variable free vertical force V; an the proper relative magnitude of $(F_1 - H)$ and V is to be determine by the conditions of the particular case. When no special require-ments are imposed, the logical proceeding is to make them abou equal in their maximum values, which maxima are not simul-taneous, but are separated by about 90° of crank travel.

(h) We have now covered in a general way the whole matte of the action of the forces in the working-mechanism or "main train" of the engine. The next step is to go over the same groun in detail, developing methods of determining the values of al the forces, and studying the manner and the effects of their varia-tion. The steam pressure is determined (and determinable) only by the indicator diagram, whose form we already know; th

leaves the inertia-force as the important unknown quantity; and to find out its value we must make a complete study of the motion of the engine.

§ 33. Kinematics of the Engine Mechanism.

(a) CONSTRAINED MOTION.—In any problem upon the motion of freely moving bodies, it is necessary first to know the forces acting, and then to determine the resulting movements. In a machine, however, the parts can travel only in certain definite paths; and when, further, a major condition can be imposed which will entirely determine the motion of one part, that of the others may be derived by purely kinematic (that is, geometric) methods. In the steam-engine this major condition is that the shaft or crank shall rotate at uniform speed.

This uniform rotation is, of course, secured by the use of a fly-wheel; and in no engine is the uniformity absolute. But in the vast majority of cases the variations in rotary speed, within the revolution, are insignificant; and their effect upon the very large accelerations of the reciprocating parts is negligible.

(b) HARMONIC MOTION.—Before taking up the actual engine-mechanism, consisting of bed, slide, connecting-rod, and crank, we will discuss the simpler mechanism outlined in Fig. 106. In-

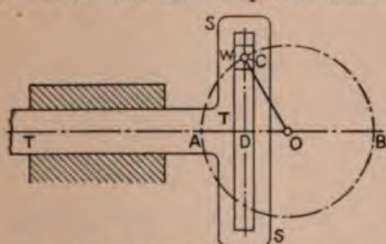


FIG. 106.—The Crossed Slider-crank.

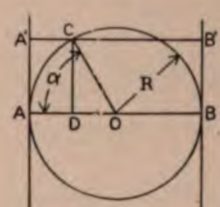


FIG. 107.—Diagram of Piston-displacement.

stead of the connecting-rod, there is a cross-slot SS formed in the slide TT, in which works the small block W surrounding the crank-pin C. The simplest motion that can be gotten from, or determined by, a rotating crank is the harmonic motion of the projection D, of the pin-center C upon the diameter AB, along that diameter.

ond to the motion of the whole slide. Then in Fig. 108, v_0 being laid off from C perpendicular to OC, its horizontal component is

$$v = v_0 \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

We know that when a point travels in a circular path with the velocity v_0, its acceleration, radially inward, is $a_0 = v_0^2/R$, where v

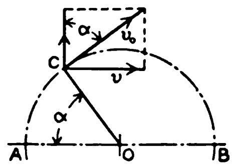


FIG. 108.—Velocity.

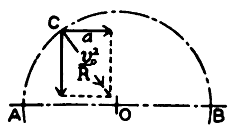


FIG. 109.—Acceleration.

is in feet per second and R in feet, if a_0 is to be in feet per second per second, the same terms as g, the acceleration of gravity. Laying off this a_0 as shown in Fig. 109 and resolving, we get

$$a = a_0 \cos \alpha = \frac{v_0^2}{R} \cos \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (204)$$

(c) ANALYTICAL DERIVATION.—The same results can be obtained by an analytical method which will be especially useful when we come to the actual engine-mechanism. In a very short time dt the piston will travel the distance ds; and, under the action of acceleration, the velocity will change by the amount dv: then from the primary definitions of velocity and acceleration,

$$v = \frac{ds}{dt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

Also, for the relation between the linear motion of a point at the end of radius R and the angular motion of this radius, we have

$$v_0 dt = R d\alpha;$$

both sides of the equation giving the distance traversed by the point C in the time dt, so that

$$\frac{d\alpha}{dt} = \theta = \frac{v_c}{R} \quad \text{. . . . . (207)}$$

Now starting with Eq. (203), $s = R(1 - \cos\alpha)$, we get by successive differentiation,

$$v = \frac{ds}{dt} = R\sin\alpha\frac{d\alpha}{dt} = v_c\sin\alpha, \quad \text{. . . . . . (208)}$$

and

$$a = \frac{d^2s}{dt^2} = v_c\cos\alpha\frac{d\alpha}{dt} = \frac{v_c^2}{R}\cos\alpha. \quad \text{. . . . . (209)}$$

(d) Piston Movement.—The actual engine-mechanism, sometimes called the slider-crank mechanism, is reduced to skeleton-outline in Fig. 110. Crank-travel is estimated from OA, all the way round the circle, the latter being divided into quadrants as indicated, and the piston-movement divided into forward and return strokes. When the crank is at A and the piston near the plain cylinder-head, the engine is said to be on its head-end dead-center; the other end, OB, is called the crank end. The whole piston-slide is represented by the plain block at W.

Fig. 110.—The Slider-crank Mechanism.

The limits of the piston-stroke are determined by making $AM = BN = L$. The piston-travel, as shown in Fig. 111, is not AD, but MW, found by striking off CW from C as a center. A construction which will give this travel s for any crank-position, without the trouble of thus striking off the rod-length each time, is made by drawing the arcs A′A, B′B, tangent to the crank-

circle, from M and N as respective centers. Then for any position of C, it is only necessary to draw A′CB′ parallel to AB, and C will be located on this line just as W is on MN; for since the equal lines MA′, WC, and NB′ are included between parallels, they must be parallel; whence A′C = MW.

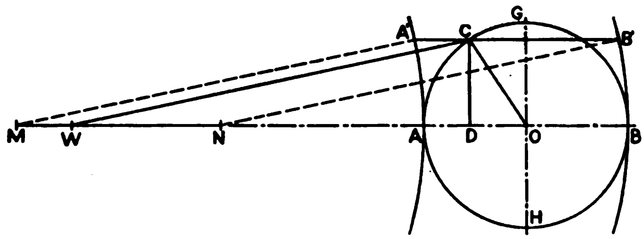


FIG. 111.—Piston-travel.

The analogous construction in Fig. 107 is made by drawing the straight-line tangents A′A, B′B: and it appears that the effect of the connecting-rod is to introduce a curvature into these limit-lines. Since the straight lines in Fig. 107 may be thought of as arcs of infinite radius, the motion there represented is often spoken of as that with infinite connecting-rod. Another view of the same conception is, that if the rod were of infinite length its angular swing would be zero, and the piston would receive harmonic motion without distortion. In Fig. 106, this "infinite rod" is replaced by the simple slide-block W, the piece which forms the connection between the crank-pin and the slide; and to carry out the idea still farther, we can think of the surface of the slot as a small part of the surface of a wrist-pin of infinite radius.

(*e*) VELOCITY AND ACCELERATION.—The analytical method will be employed first, as it is shorter and gives results in a very useful form. From Fig. 111 we get for the travel s the expression

$$\begin{aligned} s &= MW = MO - WO \\ &= (L+R) - (L\cos\beta + R\cos\alpha) \\ &= R(1-\cos\alpha) + L(1-\cos\beta); \quad \ldots \quad (210) \end{aligned}$$

where the second term, $L(1-\cos\beta)$, shows the departure from the harmonic motion represented by (202), or the effect of the connecting-rod. This is evidently the distance, on the line A′B′,

between curved limit-lines AA′, BB′ of Fig. 111 and the respecti— responding straight lines of Fig. 107.

To e te the rod-angle β, we note that, in Fig. 111,

$$CD = R \sin \alpha = L \sin \beta;$$

whence

$$\sin \beta = \frac{R}{L} \sin \alpha,$$

$$\cos \beta = \sqrt{1 - \sin^2 \beta} = \left(1 - \frac{R^2}{L^2} \sin^2 \alpha\right)^{\frac{1}{2}}.$$

If this is developed by the Binomi Formula, the first two terms are

$$\cos \beta = 1 - \frac{1}{2} \frac{R^2}{L^2} \sin^2 \alpha + \quad * \quad * \quad * \quad ; \quad . \quad . \quad . \quad (211)$$

and the succeeding terms, involving high powers of fractions, may be dropped without overpassing the limits of desired accuracy. Then Eq. (210) becomes

$$s = R\left(1 - \cos \alpha + \frac{1}{2} \frac{R}{L} \sin^2 \alpha\right);$$

and from this

$$v = \frac{ds}{dt} = R\left(\sin \alpha + \frac{1}{2} \frac{R}{L} \sin 2\alpha\right)\frac{d\alpha}{dt}$$

$$= v_0\left(\sin \alpha + \frac{1}{2} \frac{R}{L} \sin 2\alpha\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (212)$$

Likewise

$$a = \frac{dv}{dt} = v_0\left(\cos \alpha + \frac{R}{L} \cos 2\alpha\right)\frac{d\alpha}{dt}$$

$$= \frac{v_0^2}{R}\left(\cos \alpha + \frac{R}{L} \cos 2\alpha\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (213)$$

It is possible to work out a strictly exact formula for a, in terms of α, avoiding the approximation of (211); but it is quite complex, and the gain in accuracy is practically inappreciable.

Before discussing the variation of v and of a, and comparing the motions with finite and with infinite connecting-rod, we will develop the graphical methods for determining these same motion-quantities.

(*f*) Velocity by Graphical Relations.—It is a fundamental principle of kinematics, that if a body has motion in a plane, and if the direction of movement of two points of the body can be determined, then the intersection of lines perpendicular to these motion-directions determines the instantaneous center of rotation —a point about which, *at the instant*, the body is rotating as about a fixed pivot. In Fig. 112, CP and WP are drawn perpendicular to the paths (or to the velocity-directions) of C and W, and P is the instantaneous center.

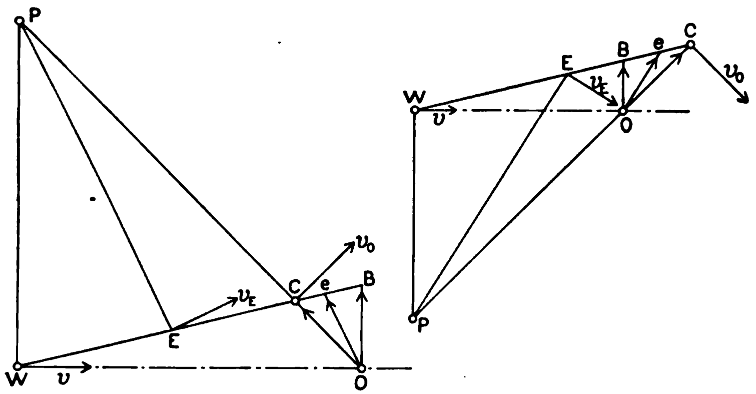


Fig. 112.—The Instantaneous Center of Rotation.

If now the connecting-rod can be thought of as turning, for the instant, about P, the conclusion follows that the velocity of any point on the rod is perpendicular to a radius from P to this point, and is proportional to the length of this radius. The fact that P changes its position from instant to instant, or from position to position of the mechanism, travelling along a curved path or locus, does not at all invalidate the statement just made as to the velocity-relations at a particular instant.

For the point W only the second of these two relations need be used, as the direction of v was part of the data: and we have

$$v : v_0 :: PW : PC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

For any other point as E, we first draw a direction-line for v_E, and then find its value by

$$v_E : v_0 :: PE : PC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

A practical disadvantage of this method is that it involves a lot of troublesome graphical work, in the drawing of long lines to get P, and in satisfying the proportion (214) or (215). A much more convenient construction suggests itself when we note that the line OB, through the shaft-center O, perpendicular to the stroke-line WO, and meeting the rod-line WC at B, makes a triangle OBC similar to PWC, so that

$$r : r_0 :: OB : OC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

And if, further, such a scale be chosen that $OC = r_0$, then at once $OB = r$.

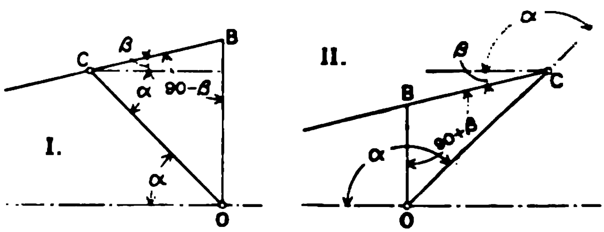


Fig. 113.—Velocity Relations.

The trigonometrical relation between OB and OC is illustrated in Fig. 113, which shows parts of Fig. 112 enlarged. In the triangle OBC,

$$\frac{OB}{OC} = \frac{\sin OCB}{\sin OBC} = \frac{\sin(\alpha + \beta)}{\cos \beta} = \frac{r}{r_0}. \quad . \quad . \quad . \quad . \quad . \quad (217)$$

Velocity of any Point on the Rod.—To apply this reduced construction of OBC, as distinguished from the extended construction of PWC, to the problem of finding the velocity of any point of the connecting-rod, we return to Fig. 112, and draw from O the line Oe parallel to PE; then the triangles OeC, PEC, are similar and

$$Oe : OC = PE : PC = r_E : r_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

From the general similarity of the figures PWEC, OBeC, it follows that e divides CB in the same ratio that E divides CW; and in order to find the velocity of any point E on the rod, we let OC and on the line CB locate a point e, geometrically similar in its position to the point E on the line CW; then Oe is the desired velocity, and just as OC is perpendicular to r_0, so r_E is perpendicular to the line Oe.

In Fig. 114, I. and III. are enlarged from Fig. 112; and from them is derived the construction given in II. and IV. for most conveniently finding the velocity of any point on the connecting-rod at any position of the mechanism. First of all, in I. and III., the scale of the drawing is such that OC represents the value of v_0:

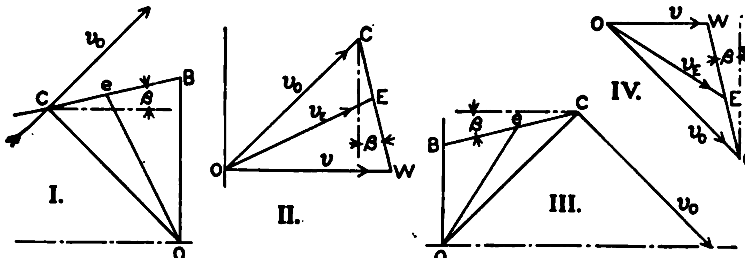


FIG. 114.—The Velocity-image.

and then the figure OCeB is rotated about O, through 90°, until it takes the position OCEW in II. and IV., with OC parallel to v_0, and therefore completely representing this velocity: then all other velocities—the particular value v and the general value v_E—are likewise completely determined, as indicated.

A more concise proof for this construction may be got directly from Fig. 112, reproduced in Fig. 115, as follows:

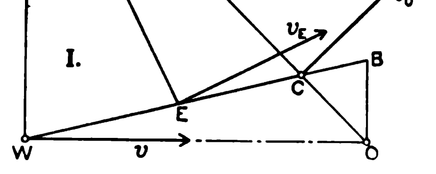


FIG. 115.—Proof of the Image.

In I., lay off from P, along PC, the length PC′ representing v_0 to scale: then the line C′W′, parallel to WC, will satisfy, for the velocities, the condition of proportionality to instantaneous radius; and the length of v and of v_E will be given by PW′ and PE′. Now C′W′ may be thought of as a reduced image of the rod: it is similar to CW, and could be formed by the device of finding the velocities of an indefinite number of points of the rod, laying off these

[illegible] from P as a center or pole, and drawing a locus of their [illegible]. Finally, to show the velocities in direction as well [illegible], the figure PW″C″ is rotated through 90° (here backward against the direction of crank-rotation) into the position [illegible] II.; and the conditions that the velocity shall be determined [illegible] the length and direction of the instantaneous radius are [illegible] most convenient manner.

[illegible] to find the velocity of any point on the connecting-[illegible] of much practical importance in the study of the working [illegible] engine; and this image-construction is given, not so much [illegible] of its own utility as because it prepares the way for [illegible] method of finding the acceleration of any point on the

[illegible] OF THE CONNECTING-ROD.—In Figs. 112 to [illegible] characteristic cases, for the first and second quadrants [illegible] of the crank-motion, are shown. In quadrant [illegible] greater than sin α, in quadrant II. it is less; for [illegible] would be horizontal, and the two quadrants symmetrical [illegible], therefore, that the effect of the connecting-[illegible] greater in the first quadrant, and less in the second. [illegible]. It is to be noted that the determination [illegible] relations need not be carried through [illegible] two quadrants; because for crank-positions [illegible] to the stroke-line, as OC_1 and OC_2 in [illegible] that all geometrical relations and construc-[illegible]

[illegible] contracted constructions in Figs. 113 and [illegible] the rod-angle β be known. To save the [illegible] figure of the mechanism, the reduced [illegible] 116 may be used. Here AGB is the real [illegible] miniature crank-circle within it is drawn

$$OD_1 = \frac{R}{L} R;$$

[illegible] actual radius R is taken to repre-[illegible] in correct proportion, is used [illegible] similarity of EOD to WCO is evident:

and the reader, instead of formulating a rule as to where to put D and E in every case, will do better to keep in mind the form of the mechanism and make EOD correspond.

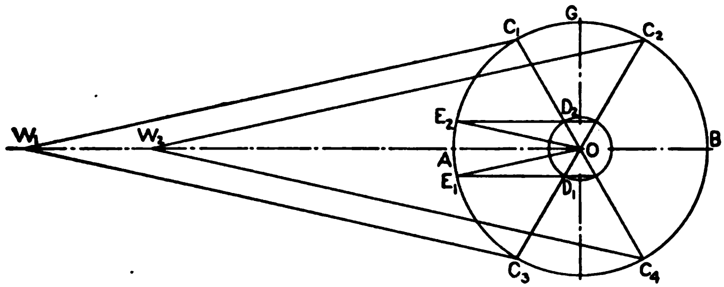


FIG. 116.—Rod-angle Construction.

(i) VELOCITY ANALYSIS.—The relation between v and v_0 can be found without the use of the instantaneous center; and the results of a discussion along this line, involving a more detailed study of the movement of the connecting-rod, form the foundation of a deduction of the acceleration of W from that of C.

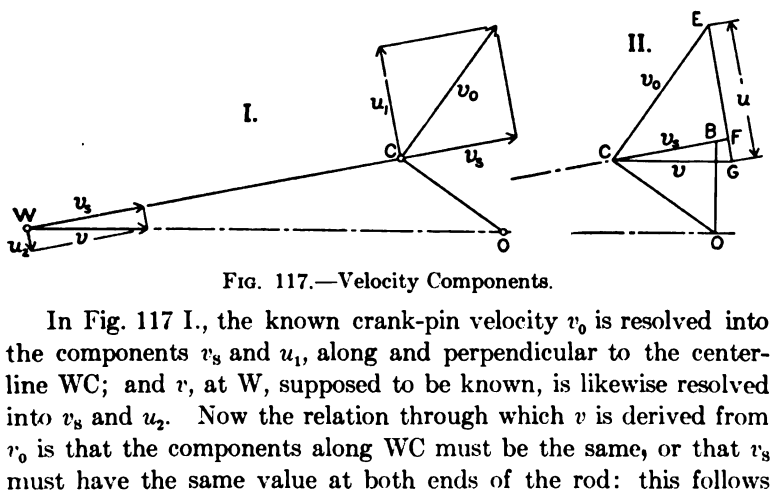


FIG. 117.—Velocity Components.

In Fig. 117 I., the known crank-pin velocity v_0 is resolved into the components v_s and u_1, along and perpendicular to the center-line WC; and v, at W, supposed to be known, is likewise resolved into v_s and u_2. Now the relation through which v is derived from v_0 is that the components along WC must be the same, or that v_s must have the same value at both ends of the rod: this follows from the fact that WC is a line of invariable length. This relation is applied in II.; first v_0 or CE is projected upon the rod-line as CF; then EF is produced until it cuts off the length CG $= v$ from a

horizontal line through C. This last is, in effect, the process of finding the velocity v when we know its direction CG, one component CF, and the direction of the other component, along FG. The triangle CEG is evidently similar to COB, and this similarity leads at once to Eq. (216): further, CEG is identical with OCW in Fig. 115 II.

We must now develop the meaning of the components u_1 and u_2, or of the total velocity u which forms the third side of the triangle CEG, the other two sides being v_0 and v. We get this when we consider the movement of the connecting-rod, determined by the paths of C and W, as a combined translation and rotation. By a translation-motion of a body is meant one in which all the points of the body move in similar paths; which may be straight or curved, but whose simultaneous elements are parallel—in other words, a motion in which successive positions of a line on the body are parallel to each other. In simplest analysis, the translation-component would be a motion right along the center-line WC of the rod, and the rotation-component would be about a point on this line.

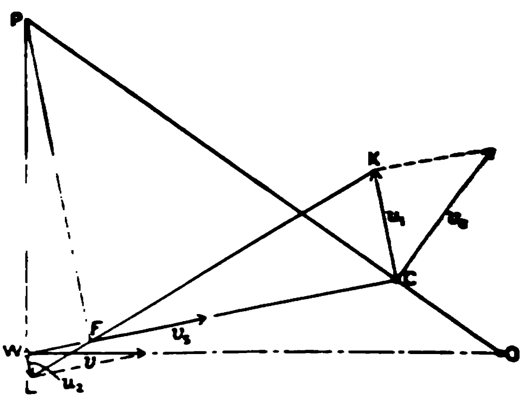


Fig. 118.—Center of Rotation.

To find this point, the one whose actual, total velocity is along WC and identical with v_3, we can either drop a perpendicular from the instantaneous center P upon the line WC, at F in Fig. 118; or else draw the line KL through the ends of the two u's and find

the point F where it intersects WC. An equivalent to the first construction is seen in Fig. 117 II., where CF, from the pole C, perpendicular to the velocity image EG, locates F just as PF does in Fig. 118.

If desired, the angular velocity of the rod could be found, by dividing the radius-length FC into u_1, or FW into u_2, or the whole rod-length WC into the total cross-velocity u, shown at EG in Fig. 117 II.

(j) Derivation of Wrist-pin Velocity.—In the engine-mechanism, the primary moving part is the crank, and the motion of the connecting-rod and of the slide is derived from that of the crank-pin. With this emphasis laid upon C as a driving-point, the ultimate analysis of the motion of the connecting-rod is into translation with C and rotation about C. Then in Fig. 119, W,

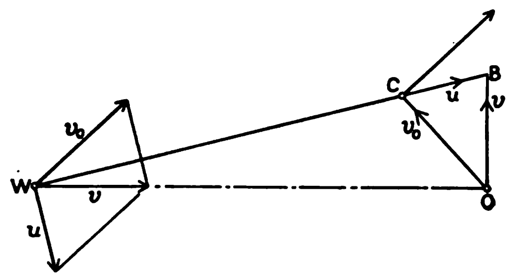


Fig. 119.—Wrist-pin Velocity.

like all the other points on the rod, would have, on account of the translation-component, the velocity v_0; and W would have also, on account of the rotation of WC about C, the linear velocity u; and the resultant of v_0 and u must then be, of course, the actual velocity v, along the stroke-line.

Now we see that, in the triangle OCB, if OC is v_0, then CB is u, and OB is the resultant, having the relative direction marked in Fig. 119.

(k) Acceleration of the Wrist-pin.—With the velocity-analysis shown in Fig. 119, it is easy to proceed to a determination of the acceleration of W from that of C. First of all, if the rod had only a translation-motion with C, then every point on it, including W, would have the same acceleration as C. The accelera-

tion of the crank-pin center. $a_n = v_a^2/R$, is known and is represented by CO in Fig. 120.

The acceleration of W is determinate in direction, as it must be along WO; suppose it resolved into two components along and perpendicular to the rod-line WC; then if we can find the component along the rod, the value of the resultant will at once become known. On account of the translatory component of the rod's motion, the point W would have the acceleration CO [illegible],

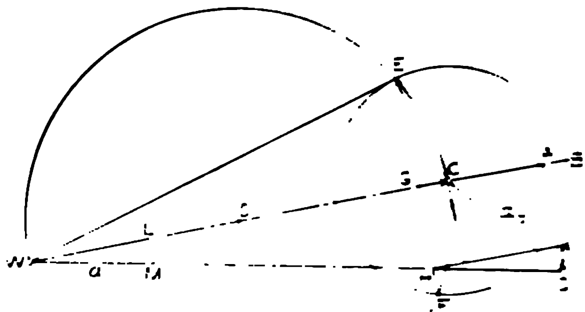


Fig. 120.—Wrist-pin Acceleration.

and resolving [illegible] CA as the component [illegible] CW.

[illegible] W [illegible] acceleration [illegible] W [illegible] component along WC [illegible] C [illegible] centripetal acceleration [illegible] the radius is WC [illegible] CB [illegible] C and WB [illegible]

$$[illegible]$$

[illegible] Fig. 120 [illegible] W [illegible] CB [illegible] CW [illegible] $a' = GC$. For [illegible]

$$WC : CL [illegible]$$

or

$$CG = \frac{\overline{CE}^2}{WC} = \frac{\overline{CB}^2}{WC}.$$

In the figure, GC and CA are in the same direction, so that the total acceleration along the rod is GA: and the corresponding acceleration HO of W is cut from the stroke-line by the chord EF, as appears from the triangle HKO, the same at WLM.

It is a matter of some importance in this connection to show that if CO is used to represent both v_0 and a_0 (in general, to different scales), the acceleration GC will be to the same scale as a_0. If m is the scale of dimensions, so that m inches of drawing $=1$ ft. of R or L; and if n is the scale of the velocity-diagram, so that n inches of drawing $=1$ ft. per sec. of velocity, then

$$a_0 = \frac{v_0^2}{R} = \frac{\overline{OC}^2}{n^2} \cdot \frac{m}{OC} = \frac{m}{n^2} OC.$$

Again,

$$a' = \frac{u^2}{L} = \frac{\overline{CB}^2}{n^2} \cdot \frac{m}{WC} = \frac{m}{n^2} GC.$$

This deduction seems to be easier to grasp than a statement made simply in terms of general principles.

(*l*) The General Case.—The common form of the engine-mechanism is, as to motion, the simplest arrangement of the four pieces or "links" which are its elements, but it is only a particular case. The general form of the mechanism is shown in Fig. 121, where the stroke-line WQ does not pass through the shaft-center O: and a few engines are built with this arrangement. Further, Fig. 121 is generalized in another direction by assuming that the crank is no longer limited to uniform speed of rotation: then the acceleration a_0 will have a tangential component besides the centripetal, and will be off the radius, as shown.

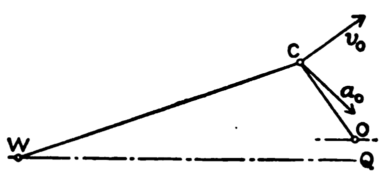


Fig. 121.—The General Case.

The graphical methods of Figs. 117 to 120 can be applied to this general case quite as well as to the standard mechanism—in

which they have a decided advantage over the analytical integration of (x), which would be greatly complicated by the non-symmetrical arrangement. But this phase of the subject is not of sufficient practical importance to warrant a presentation going beyond the mere suggestion; although no difficulty of importance will be encountered in making a full determination of the motion of the point W in Fig. 121.

For the usual mechanism, however, the acceleration of the slide is found with less work by the method of Eq. ([illegible]) than by construction; especially after a table of values of the factor $\left(\cos\alpha + \frac{R}{L}\cos 2\alpha\right)$ has been compiled—see Table IX.

(m) *Acceleration of the Connecting-rod.*—The acceleration of any point on the rod can be found easily after we have located the instantaneous center of acceleration, which is analogous to the instantaneous center of rotation, but differs in this, that whereas the velocity is always at right angles to its radius, the acceleration makes some other angle with its radius from the center of acceleration—the angle being the same for all points in any one position of the mechanism and of the center, but changing as these move. The law of proportionality to radius-length is the same for both cases.

In order to get the acceleration-center, we must know the accelerations of two points, as a_0 and a of C and W, and then find a point which satisfies the geometrical conditions that lines drawn from it to C and W will make the same angle with CO and WO respectively, and in length be proportional to a_0 and a. The construction, developed in Fig. 122, is as follows:

Letting CO represent a_0, measure back Ow equal to a, and join Cw; next produce Cw to W′, making CW′ = CW, draw W′A′ parallel to a, and produce CO to meet it at A′: then in the figure A′CW′,

$$A'C : a_0 :: A'W' : a.$$

Now swing the triangle A′W′C around C, until it comes into the position AWC; then A′C and A′W′, originally having the same directions as a_0 and a, and being parts of a "rigid" figure, must

make the same angles with their respective original positions: so that angle ACO = angle AWO, and A is the center of acceleration.

To find the acceleration of any point as E, we draw the radius AE, make angle AEF = angle ACO, and get the length of EF by the condition

$$EF : AE = OC : AC.$$

(*n*) THE ACCELERATION-IMAGE.—The construction just given is a complete solution of the problem, but involves too much graphical work, in application. The short-cut method, analogous to that in Fig. 114, is also developed in Fig. 122:

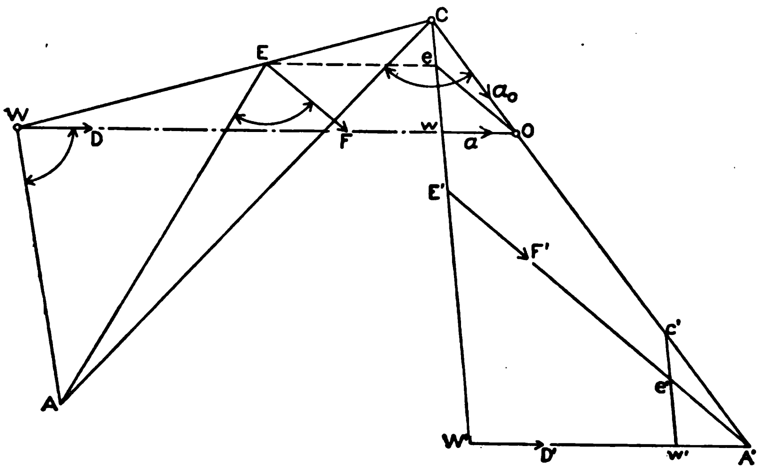


FIG. 122.—The Center of Acceleration.

First, swing the figure AWC back into its original position A′W′C, carrying with it the point E and its radius AE, to A′E′. Imagine the acceleration-lines WD and EF to go with their points to W′ and E′, being pivoted upon the figure at these points, and turning on the pivots so as always to remain parallel to their original positions: then by the time they get to W′ and E′ the acceleration-angle AWO = AEF = ACO will have been eliminated, and each acceleration will point along its radius, CO along CA′, W′D′ along W′A′, E′F′ along E′A′. Now, in order to get the acceleration of any point E on CW, we need only locate the corresponding point E′ on CW′ and take a certain proportion of the radius E′A′.

§ 34. Motion of the Engine.

(b) Acceleration of the Slide.—The similar diagram for acceleration is based on Eq. (213). Evaluating the factor

$$m=\left(\cos\alpha+\frac{R}{L}\cos 2\alpha\right),$$

we have

$\alpha=$	$0°$	$90°$	$180°$	$270°$
$m=$	$\left(1+\frac{R}{L}\right)$	$-\frac{R}{L}$	$-\left(1-\frac{R}{L}\right)$	$-\frac{R}{L}$

Also, for $\alpha=45°$, $135°$, etc. (that is, for any mid-quadrant crank-position), $m=\pm\cos\alpha$.

Now in Fig. 124, the radius of the circle is a_0: and for infinite rod it is evident that the horizontal distance CE′, from the C-point to the line GH, will give a, according to the relation $a=a_0\cos\alpha$. For the actual mechanism, GH must be replaced by a curve analogous to the D-curve in Fig. 123: three points in this curve are got by laying off

$$OE_0=GE_1=HE_2=\frac{R}{L}AO;$$

and two more by noting that the curve must cross GH where $m=\cos\alpha$, at the mid-quadrant points K_1 and K_2. Without any further determinations (unless the figure be very large), a fair curve can be drawn through these five points which will be the locus of the end of a, measured always from C toward the E-curve.

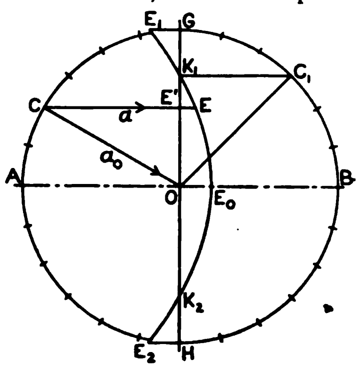


Fig. 124.—Acceleration Diagram.

These two curves—the locus of D in Fig. 123 and that of E * in Fig. 124—show, by their departure from the straight lines AB and GH, the effect of the connecting-rod upon the motion of the slide.

(c) Motion Diagrams.—In Fig. 125 are shown diagrams of displacement, velocity, and acceleration of the piston, first in

* This E-curve is a parabola, but that for D has a more complex equation.

[illegible]ANICS OF THE [illegible]

[illegible] 123 and 12[illegible]

[illegible] developed [illegible]

harmonic motion, and all three are simple sine-curves or sinusoids: while the full lines are for the actual mechanism, with the rod-ratio 5, or $L=5R$.

As to displacement, we note that the piston travels farther during the first and fourth quadrants than during the second and third. Referring to Fig. 111, we see that the mid-stroke position of W is at the distance $L=nR$ to the left of O; and that when C is at G, or $\alpha=90°$, OW will be the base of a right triangle of which L is the hypotenuse and R the other side, so that OW $=\sqrt{L^2-R^2}=R\sqrt{n^2-1}$. Then the distance of the piston from mid-stroke for the crank at 90° is, expressed in terms of the stroke rather than of the radius,

$$s_0=S\frac{(n-\sqrt{n^2-1})}{2}=mS. \quad . \quad . \quad . \quad . \quad . \quad (220)$$

From this we get

$n=$	3	4	5	6	8
$m=$	.0858	.0635	.0505	.0420	.0314

Compare values of s for 90° in Table VIII.

The velocity-curve is similarly distorted: and v has its maximum where the acceleration a is zero, or where the curve $E_1E_0E_2$ crosses the circle in Fig. 124. Using Eq. (213), and solving the equation

$$\cos\alpha+\frac{R}{L}\cos 2\alpha=0,$$

we get

$$\cos\alpha+\frac{R}{L}(2\cos^2\alpha-1)=0,$$

$$\cos^2\alpha+\tfrac{1}{2}\frac{L}{R}\cos\alpha=\tfrac{1}{2},$$

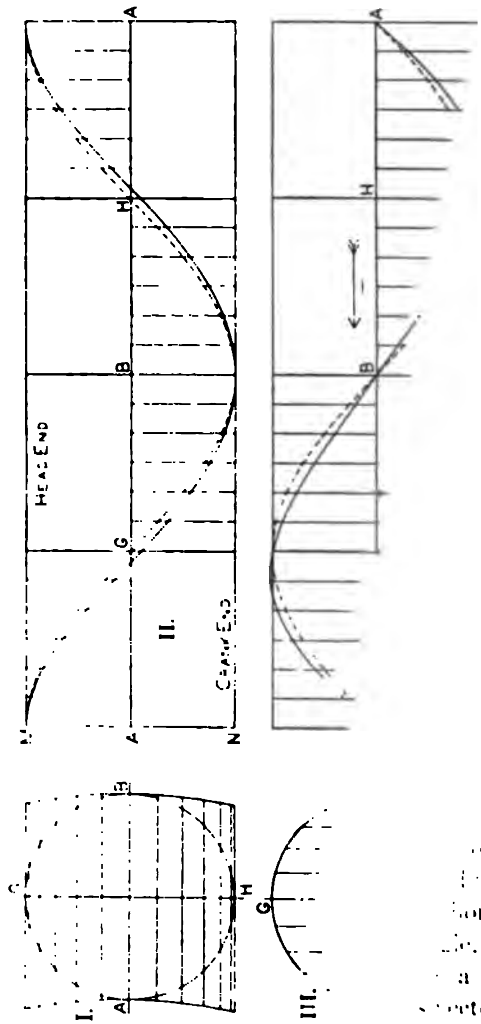
whence

$$\cos\alpha=\tfrac{1}{4}(\sqrt{8+n^2}-n). \quad . \quad . \quad . \quad . \quad . \quad (221)$$

From which the following results can be got:

$n=$	4	5	6	8
$\cos\alpha=$	.2248	.1862	.1583	.1213
$\alpha=$	77° 1′	79° 16′	80° 54′	83° 2′

the circular form of Figs. 111, 123, and :
and then laid out upon the developed ·



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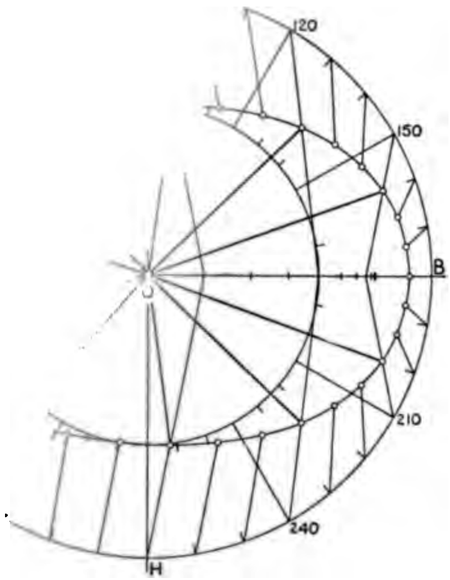


1:27.—Acceleration of the Rod.

·:·ıt of the rod. These figures are drawn with
.5.

:ıatic analysis in these two sections covers the
·.y. and goes rather beyond ordinary requirements:
: are of constant practical application, others are
; mic interest, as leading to a full understanding of
l he very important application in the matter of
:·ccelerating-force and inertia will now be exhibited.

§ 35. The Working-forces in the Engine.

(*a*) INERTIA-FORCE OF THE RECIPROCATING PARTS.—Know the acceleration a and the weight W of the sliding parts—wl are supposed to include a portion of the connecting-rod—we h only to multiply the mass M (equal to $W \div g$) by a in order to the force $F = Ma$ required to accelerate this mass, or its iner force. Then for the mechanisms discussed we have,

For infinite rod,

$$F = \frac{W}{g} \frac{v_0^2}{R} \cos \alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22$$

For actual rod,

$$F = \frac{W}{g} \frac{v_0^2}{R} \left(\cos \alpha + \frac{R}{L} \cos 2\alpha \right). \quad . \quad . \quad . \quad . \quad (22$$

In either case, $\frac{W}{g} \frac{v_0^2}{R}$ is the centrifugal force which the recip cating mass would have if it were concentrated around the cra pin center C: this ideal centrifugal force will be called F_0, and actual inertia-force is then a component of F_0, given by

$$F = F_0 \cos \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2$$

or by

$$F = F_0 \left(\cos \alpha + \frac{R}{L} \cos 2\alpha \right). \quad . \quad . \quad . \quad . \quad (2$$

The value of F can be found, for any crank-angle, by a diagr like Fig. 124, but with F_0 instead of a_0 as the radius. As to dir tion of force, it must be remembered that accelerating force point toward the center O or toward the limit-line GH or E_1E_0 while inertia-force will point outward, or away from the mid of the diagram. This is illustrated in Fig. 128, where the two ca as to form of mechanism are separated, the two diagrams embo ing Eqs. (223) and (224) respectively.

These diagrams are not, however, in a shape suitable for direct combination of inertia-force with steam-pressure, since latter, given by the steam diagram, is laid out on a stroke-line ba

The derivation of an inertia diagram in these terms is shown in Fig. 129, where, in I., the diagram from Fig. 128 II. is surrounded by another diagram, for piston-position, like Fig. 111 or Fig. 125 I. From this we get the two co-ordinates for the curve in II., the abscissa s = MS at FC′; the ordinate F = ST at CE. For the actual mechanism this gives the curve PQR, for infinite rod, the straight line JKL. It must be clearly understood that this is a

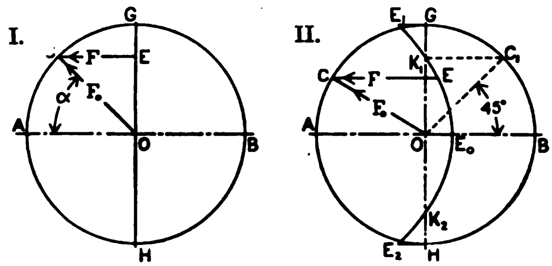


FIG. 128.—Circular Diagrams of Inertia-force.

diagram for both strokes, as is indicated by the numbered positions on both figures. The meaning of plus and minus (up and down) ordinates in II. is shown by the arrows.

That JL must be a straight line is evident when we consider that, with infinite rod, the distance SK from mid-stroke and the inertia-force F are both proportional to cos α. With the curve, the areas PMQ, QNR—the first representing work stored in the moving parts from zero velocity to maximum, the second that given back during retardation—must be equal. Note that the dead-center acceleration is always greater than a_0 at the head end and less at the crank end, the ends being taken according to the conventional arrangement in Fig. 110.

This diagram can also be plotted, very conveniently, with ordinates computed by means of Tables VIII and IX.

(b) CALCULATION OF INERTIA-FORCE.—For combination with steam-pressure diagrams, the curve of Fig. 129 II. must show force per square inch of piston instead of total inertia-force; the equation for its ordinate is then

$$\frac{F}{A}=\frac{F_0}{A}\left(\cos\alpha+\frac{R}{L}\cos 2\alpha\right). \quad . \quad . \quad . \quad . \quad . \quad (225)$$

As a preliminary to the graphical construction of Fig. 129, the value of F_0 or of F_0/A must be known for the particular engine.

Starting with

$$a_0 = \frac{v_0^2}{R},$$

and substituting, from (207), $\theta = \frac{v_0}{R}$, we get

$$a_0 = \theta^2 R; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

where R is in feet and θ in radians per second. If N = R.P.M., then the angular velocity will be

$$\theta = 2\pi \frac{N}{60} = .10472N. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

If we wish to use the stroke S in inches instead of the crank-arm R in feet, we substitute $S/24$ for R in (226) and get

$$a_0 = \frac{\theta^2 S}{24} = C_1 S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

Now, for the ideal centrifugal force F_0 we have

$$F_0 = \frac{W}{g} a_0 = \frac{\theta^2}{24g} SW = C_2 SW. \quad . \quad . \quad . \quad . \quad (229)$$

Values of the constants θ, $\theta^2/24 = C_1$ and $\theta^2/24g = C_2$, are given in Table VII., for different values of N. To get centripetal acceleration, multiply the stroke in inches by the proper value of C_1: to get the centrifugal force of any weight W, multiply it by its rotation-diameter S in inches and by C_2.

Note that

$$\frac{\theta^2 S}{24g} = \frac{a_0}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

is the ratio of the centripetal acceleration to the acceleration of gravity: and since forces are proportional to their acceleration-effects, this must be the ratio of F_0 to W: then we have only to multiply C_2 by S in order to see how the centrifugal force of a given mass will compare with its weight.

If we substitute in (229) W/A instead of W, we get F_0/A directly. In practice W/A varies from 1.5 to 4 lbs. in engines intended to run at any — speed.

EXAMPLE 1.—An engine 14″ diam. by 15″ stroke, 250 R.P.M., has a connecting-rod 42″ long, and its reciprocating parts weigh 325 lbs. Find angular velocity of crank, linear velocity of crank-pin, centripetal

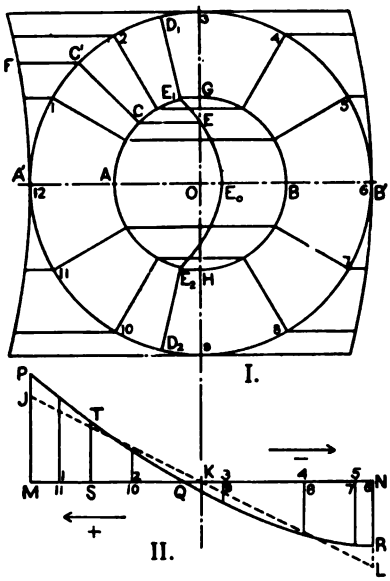


FIG. 129.—The Stroke-line Diagram.

acceleration of crank-pin, F_0 and F_0/A; also the value of F/A at the dead-centers and at 50° and 125° of crank-angle.

Angular velocity of crank:

$$\theta = .10472 \times 250 = 26.180. \quad \text{(See Table VII., col. 2.)}$$

Linear velocity of crank-pin:

$$v_0 = \pi \frac{S}{12}\frac{N}{60} = \frac{3.1416 \times 15 \times 250}{12 \times 60} = 16.36 \text{ ft. per sec.;}$$

or, from $v_0 = \theta R$,

$$v_0 = 26.18 \times \frac{15}{24} = 16.36.$$

Centripetal acceleration of crank-pin: from Table VII., for 250 R.P.M., $C_1 = 28.56$; then

$$a_0 = C_1 S = 28.56 \times 15 = 428.4 \text{ ft. per sec.}^2$$

Centrifugal inertia: again from Table VII, $C_2 = .8880$; then

$$C_2S = \frac{F_0}{W} = 15 \times .888 = 13.32;$$

so that

$$F_0 = 325 \times 13.32 = 4440 \text{ lbs.}$$

From Table VI., $A = 153.9$ sq. in.: and

$$\frac{F_0}{A} = 4440 \div 153.9 = 28.85 \text{ lbs. per sq. in.}$$

Here,

$$\frac{R}{L} = \frac{7.5}{42} = \frac{1}{5.6}: \text{ then at head-end dead-center,}$$

$$\frac{F}{A} = 28.85 + \frac{28.85}{5.6} = 28.85 + 5.15 = 34.00 \text{ lbs.};$$

and at the crank-end dead-center

$$\frac{F}{A} = 28.85 - 5.15 = 23.70 \text{ lbs.}$$

Now we cannot take values of m directly from Table IX. for $L/R = 5.6$: but using cols. 2 and 3 of the table we easily get,

$$\text{For } 50°, \qquad m = .6428 - \frac{.1737}{5.6} = .6428 - .0310 = .6118;$$

$$\text{For } 125°, \qquad m = -.5736 - \frac{.3420}{5.6} = -.5736 - .0611 = -.6347.$$

And the required values of F/A are 17.65 lbs. at 50° and − 18.30 lbs. at 125°.

(c) EFFECTIVE DRIVING-FORCE.—In Fig. 130 a pair of indicator diagrams, from the two ends of the cylinder, is shown at I.: and by means of motion-arrows the fact is made clear that the forward-pressure line, or steam-line of one end, as AB, is simultaneous with the back-pressure line, or exhaust-line of the other end, as GH. So that a subtraction of P_B from P_F for the whole of each stroke, giving P_E—refer to Fig. 101—is made by the combination in II.; where MGHN (crank end) is superimposed on MABN (head-end); and where the effective steam-pressure P_E is given by the ordinate intercepted between the curves AB and GH. Note that whereas the indicator cards in I. are for the two

ends of the cylinder, the two diagrams in II. are for the respective strokes (compare § 20 (b), Ch. IV.): and further that, with high compression, the back-pressure will rise above the forward-pressure toward the end of each stroke, so that the mechanism will have to drag the piston to dead-center instead of being driven by it. In § 32 (b), the pressures P_F and P_B were defined as abso-

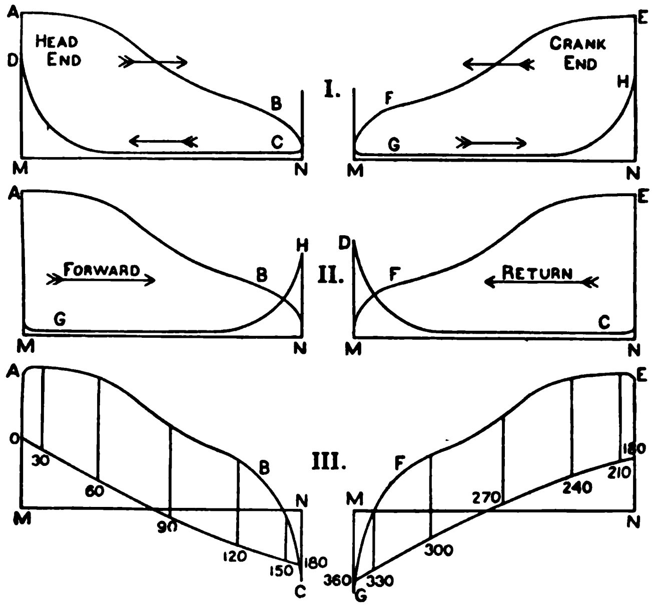
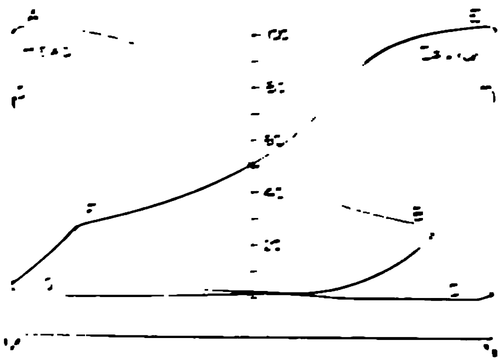


FIG. 130.—Effective Driving-force.

lute pressures, above perfect vacuum; as shown in Fig. 130, on diagrams from a non-condensing engine, they are measured above atmosphere: but since their difference is the result sought, either datum-line can be used.

The diagrams in Fig. 130 II., where the variable ordinate p_E is included between curves, are brought to a more convenient form in III., by measuring the ordinate up (and down) from MN, and getting the p_E-diagrams MABCN, NEFGM, on straight base-lines. The inertia-force diagram can be combined directly with these by laying it off on the same base-line—being inverted for the return stroke because the direction of positive or forward-acting steam-



values; but with an inertia-curve for $F_0/A = 36$ lbs. and $L/R = 6$, the greatest value of the effective driving-pressure is 74 lbs. per square inch on the forward stroke and 77 lbs. on the return stroke.

By measuring areas, and taking the area of the whole figure ABCDEF as unity—this area representing the work done by the

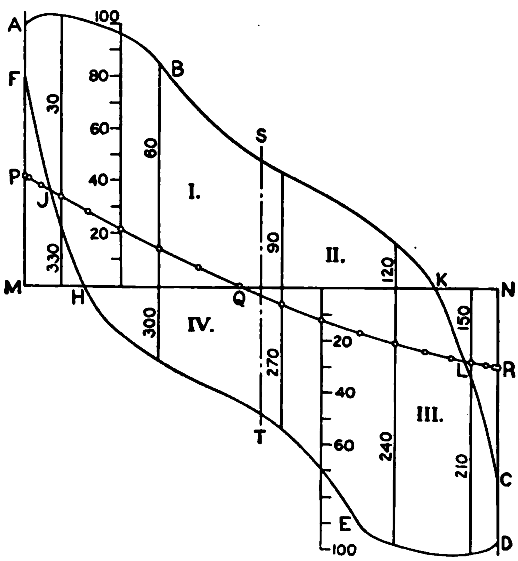


FIG. 132.—The Two-stroke Diagram.

engine in one revolution—the distribution of work among the four half-strokes was found to be as follows:

	I.	II.	III.	IV.
A	.428	.065(−.043)	.438	.069(−.041)
B	.334	.159(−.013)	.344	.163(−.010)

Case A is for the effective steam-pressure alone, the figure being quartered by the base-line MN and the middle line ST: while in B the inertia-effect is taken into account, the division-lines being PQR and ST; and the tendency of the inertia-action to equalize the distribution of work is strongly apparent. The minus quantities in parentheses after II. and IV. are the values of the negative work-

areas at the ends of the strokes, KNC and HMF for A, LRC and JPF for B.

(*d*) TURNING-FORCE RELATIONS.—Two methods of reasoning may be followed in finding, for a known driving-force at the wrist-pin, the turning-effect upon the crank: both are under the assumption that the connecting-rod is a weightless transmitting-link, its inertia having been taken into account according to § 32 (*c*). The first is illustrated in Fig. 133, where the force S transmitted along

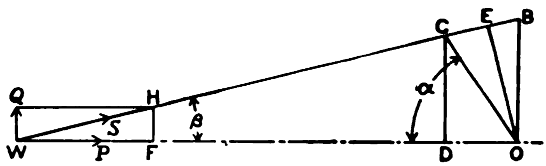


FIG. 133.—Turning-moment on the Crank.

the rod is the resultant of the driving-force P and the guide-reaction Q. The perpendicular OE from O upon the rod-line is the lever-arm of this force, and its moment is

$$M = S \times \mathrm{OE}.$$

Drawing also the vertical line OB, we have that the triangles OEB, WFH, are similar: wherefore

$$P : S :: \mathrm{OE} : \mathrm{OB},$$

or

$$P \times \mathrm{OB} = S \times \mathrm{OE} = M: \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

and the turning effect is just as though the force P acted upon the crank with the lever-arm OB.

In the mechanism of Fig. 106, or with "infinite connecting-rod," the force P would be applied horizontally, or parallel to the stroke-line, at C, and its lever-arm would be CD or $R \sin \alpha$: the increase of arm-length from CD to OB measures the effect of the connecting-rod. In the second quadrant, OB will be less than CD.

Replacing the moment M in (231) by that of a tangential force T at C, with the arm R, we have

$$M = P \times \mathrm{OB} = T \times R,$$

or

$$T=\frac{OB}{R}\times P=\frac{\sin(\alpha+\beta)}{\cos\beta}P; \quad . \quad . \quad . \quad . \quad . \quad (232)$$

the geometrical relation being the same as that for the velocities in Fig. 113, and (232) derived like (217). The equation

$$\frac{v}{v_0}=\frac{T}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (233)$$

is a fundamental relation: for if there are no internal (frictional) losses in the machine, the work-rate at the wrist-pin must be the same as that at the crank-pin: work-rate is, working-force $\times$ velocity of force (along force-line), so we see that the equation

$$vP=v_0T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (234)$$

must be true.

The second derivation of T is through force-analysis strictly, and is shown in Fig. 134. Here the rod-force S is carried over to

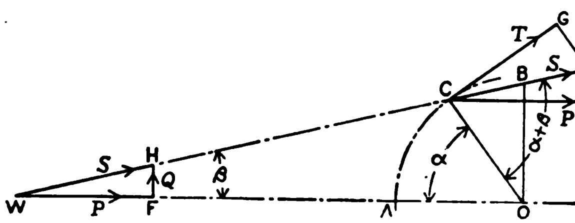


FIG. 134.—Tangential Force on Crank.

C and there resolved, as suggested in Fig. 101. From triangle WFH or CED,

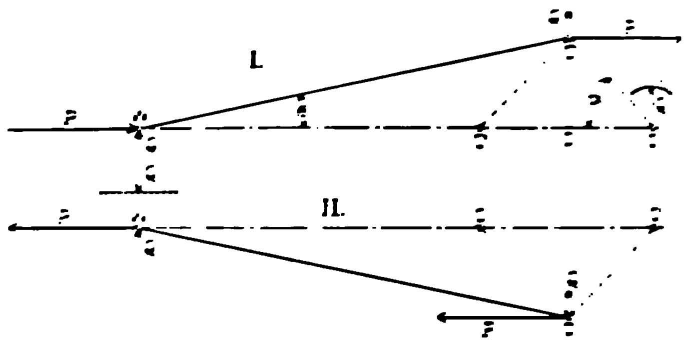
$$S=\frac{P}{\cos\beta};$$

and in triangle CGD, right-angled at G, and with angle CDG = angle OCD $=(\alpha+\beta)$,

$$T=S\sin(\alpha+\beta)=P\frac{\sin(\alpha+\beta)}{\cos\beta}. \quad . \quad . \quad . \quad . \quad (235)$$

If we apply to this expression the effect of making the rod infinite—which makes β zero—we get at once the simple relation indicated by the lever-arm CD or $R\sin\alpha$ in Fig. 133, namely,

$$T'=P\sin\alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$


I.
II.

The measure of this couple is

$$\begin{aligned} M &= Q \times \mathrm{WO}, \\ &= P\frac{\sin\beta}{\cos\beta}(L\cos\beta + R\cos\alpha) \\ &= P\left(L\sin\beta + \frac{R\cos\alpha\sin\beta}{\cos\beta}\right). \quad . \quad . \quad . \quad . \quad (239) \end{aligned}$$

And if we substitute $R \sin \alpha$ for $L \sin \beta$ and combine terms, we come at once to

$$M = PR\frac{\sin(\alpha+\beta)}{\cos\beta};$$

proving that this moment is equivalent to the turning-moment on the crank.

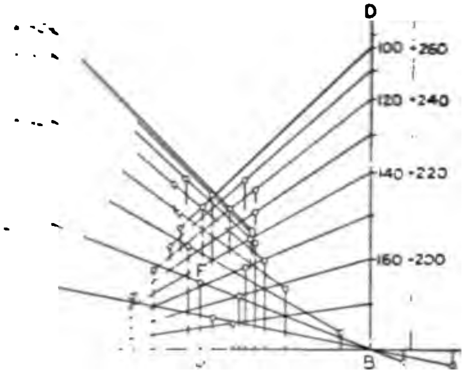
(*f*) Determining Tangential Force.—Having drawn the diagrams of effective driving-force, like Fig. 130 III. or Fig. 132, and located on them a series of ordinates corresponding to a number of equally-spaced crank-angles, the next step is to find the turning-force T for each of these P's. One method, shown in Fig. 137, is a direct application of the relation expressed in (232), which is equivalent to

$$T : P :: \mathrm{OB} : \mathrm{OC} \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

on Fig. 134: and the construction is similar to that for v in Fig. 123. The crank-circle on AB is of any convenient size, but with its radius greater than the largest value of P on the diagram of effective driving-force. Each length of P is measured inward along the corresponding crank-line, as CD; and the length CE, cut from the vertical by DE parallel to the rod-line, is T. The triangle CDE is similar to OCB in Figs. 133 and 134; and the construction of Fig. 116 is used for finding the rod-angle. A separate triangle must be drawn for each crank-position. The T's, when found, are laid off radially outward from the circle, and a curve is traced through their ends.

A second method uses the computed values of the ratio of T to P, or of $m = \dfrac{\sin(\alpha+\beta)}{\cos\beta}$, given in Table X., working through a reduction diagram like those in Fig. 42 III. The base-line AB, Fig. 136,

... drawn; and for any ...
only to measure off from B ...
... diagram, in order to get the ...



Transformation Diagram

... —Values of T ha...
... either from the crank
... -line base, as in Fig. 138. ...
... crank-circle; that is, the ...
... diagram are straightened
... diagram is clearer for
... turning-force, while the
...

turning-force, AQRBST, with this circle of uniform resistance, we get an idea of the duty which the fly-wheel has to perform. Only at four points, Q, R, S, and T, is the driving-force just equal to the resistance: during two periods or phases, marked I. and III., the work done upon the shaft is less than that taken from it, and the deficiency is made up by energy taken from the wheel, which of course slows down; but during phases II. and IV. the wheel has to store up excess-work, and regains kinetic energy.

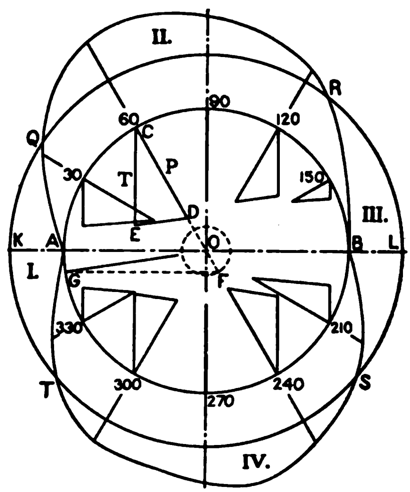
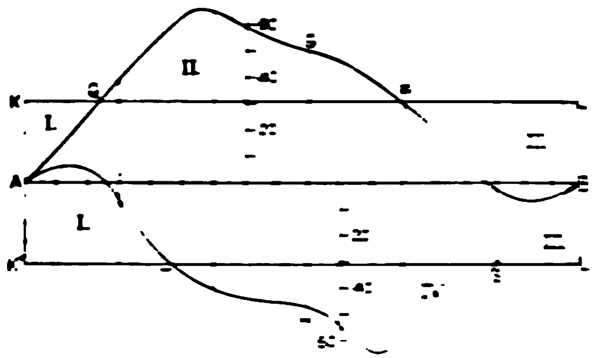


FIG. 137.—Turning-force Diagram.

Knowing the largest amount of work to be taken care of by the wheel, as shown by the diagram, and the greatest permissible variation of speed within the revolution, we can compute the weight of wheel-rim required under any conditions.

(*h*) FLY-WHEEL DATA.—Both forms of the turning-force diagram, Figs. 137 and 138, show force plotted on a distance base: but only in the straight-base diagram, where the ordinates are parallel, is work truly represented by area. Working then on Fig. 138, we can get the data for fly-wheel determination in several forms.

Measuring the area of each phase and dividing area in square inches by base in inches, we get the mean height and reduce it to


II
K
L
A
L

If we know the average M.E.P. p_m, we get W_R by

$$W_R = 2\frac{S}{12}Ap_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (243)$$

Or, having the I.H.P. H and the R.P.M. N,

$$W_R = \frac{33,000H}{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

Finally, knowing the dimensions of the engine and the scales used in laying out Fig. 138, we may calculate a work-scale of foot-pounds per square inch, as follows:

Suppose that Fig. 138 is drawn for a 14″×15″ engine, and that in the figure the circumference-scale is 1″=20°, while that for pressures is 1″=24 lbs. per sq. in. Then the circumference of the 15″ crank-circle is 3.927 ft., and the distance-value of 1″ along the base of the diagram,

$$\frac{20}{360}\times 3.927 = .2182 \text{ ft.}$$

The piston-area being 153.9 sq. ins., the total force-value of 1″ of ordinate is 24×153.9=3694 lbs.: so that 1 sq. in. represents

$$.2182\times 3694 = 806.0 \text{ ft. lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (245)$$

A table of data from Fig. 138, applying it to an engine 14″ by 15″ at 250 R.P.M. instead of the engine from which the diagrams combined in Fig. 131 were actually taken, is shown below. With the degree-scale 1″=20°, the length of the diagram must be 9″: the measured area of the whole figure AGBH was 23.17 sq. ins.: so that the mean turning-force or M.T.F. is

$$T_m = \frac{23.17}{18}\times 24 = 30.9 \text{ lbs.}$$

Applying (201) backward, the M.E.P. corresponding would be

$$P_m = 30.9 \div .6366 = 48.5 \text{ lbs.},$$

as against an average value of 48.03 in Table 20B: the discrepanc is due to inaccuracy in measurement of the small indicator dia grams.

TABLE 35 A. RESULTS FROM FIG. 138.

Phase No.	Area, sq. ins. a	Angle, degrees. γ	Length, feet. l	Mean Force. t	Phase-ratio. k	Work-value. E
I.	−3.34	70.6	.770	−23.4	.1442	−2690
II.	+3.92	95.0	1.036	+19.4	.1715	+3160
III.	−4.23	91.0	.993	−22.3	.1824	−3410
IV.	+3.67	103.4	1.129	+17.0	.1584	+2960

In Table 35 A, the phase-areas a were measured with the pla nimeter, as was that of the whole figure: given as measured, the check by adding up very nearly to zero. Dividing each by its bas in inches, and multiplying by the force-scale 24, gives the mea force t. The phase-angles γ were measured with a scale of 20 t the inch and made to add up to 360°: and the correspondin lengths l on the crank-circle were got by multiplying γ by the valu of 1° in feet, which is $3.927 \div 360 = .01091$ ft. The phase-area a divided by the total area 23.17 sq. ins. gives the ratio k. Finally E is got from a through the work-scale of (245).

The work per revolution, W_R, as found from the total are 23.17 sq. ins., is

$$W_R = 23.17 \times 806 = 18{,}650 \text{ F.P.}$$

By (243) it is

$$W_R = \frac{30}{12} \times 153.9 \times 48.5 = 18{,}650 \text{ F.P.}$$

The supply of data in this table is redundant, more being give than is necessary for the solution of any particular problem. It will be noted that a, γ, t, and k depend only upon the form of the diagram; while l and E involve also the dimensions of the engine.

On Fig. 138 there was made a determination of the division of work among the four quadrants, analogous to that on Fig. 132, by measuring the quarters divided by AB and GH, the latter being

the 90°-line. The result, using the numerals for the four quarters, was

I. 0.357; II. 0.136; III. 0.322; IV. 0.185.

This same proportion would result from the use of the 90°-line instead of ST in Fig. 132: and we show here, in a different way, how the effect of the swing of the connecting-rod is to transfer work from the 2d and 3d to the 1st and 4th quadrants respectively.

§ 36. Fly-wheel Action.

(*a*) WEIGHT OF WHEEL.—The variation-work E, just determined, is to be taken up or given off by the wheel within a certain prescribed limit of speed-change. This limit is generally defined by stating that the range from the greatest velocity V_1 to the least velocity V_2 of the wheel-rim is not to exceed a certain fraction f of the average velocity V. The change in kinetic energy of a mass whose weight is W, in a variation of speed from V_1 to V_2 (or the reverse), is

$$E = \frac{W}{g}\frac{(V_1^2 - V_2^2)}{2} = \frac{W}{g}\frac{(V_1 + V_2)}{2}(V_1 - V_2)$$

$$= \frac{W}{g} \times V \times fV = f\frac{WV^2}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

This E is identical with that in (241) and (242): so that the method of solving the fly-wheel problem consists in getting from the diagram an expression for, or value of, the work to be taken care of by the wheel, and equating this to the expression for change in kinetic energy.

The fraction f of permitted fluctuation varies with the character of the work done by the engine: a good degree of uniformity, suitable for ordinary high-grade work such as driving electric generators and textile mills, is secured by making f = about .01. In slow-running engines on rough work, the fluctuation often greatly exceeds this value, rising to 5 or 10 per cent. or more; while in the work requiring the greatest delicacy of regulation—

the driving of alternating-current generators in parallel—the fl wheel must be very powerful.

It must be clearly understood that the regulation of the spe within the revolution, by the fly-wheel, is a different matter fr the regulation of the average speed by the governor, through continuous accommodation of the power of the machine to its loa

EXAMPLE 1.—Find the weight of fly-wheel at an effective radi of 36″ which will regulate the 14×15—250 engine of Fig. 138 witl 1 per cent.

Circumference of wheel $=6\times 3.1416=18.85$ ft.

Velocity of rim $=18.85\times\frac{250}{60}=78.54$ ft. per sec.

$$\frac{V^2}{g}=\frac{6169}{32.16}=191.8;\qquad f=.01.$$

From Table 35 A, the greatest value of E is 3410 F.P.; then for we get by (246)

$$W=\frac{3410}{191.8\times .01}=1725 \text{ lbs.}$$

If this all goes into the rim, and we make the latter 12″ wide, th with cast-iron at 450 lbs. per cu. ft., the rim would be about 2.5″ thick.

EXAMPLE 2.—An engine 24″×48″, at 80 R.P.M., has a wheel 18 in diameter and weighing 12,500 lbs. How close will be the regulati if the M.E.P. is 40.2 lbs. and the phase-ratio k is 0.16?

The work per revolution is

$$W_R=2\times 4\times 452.4\times 40.2=145,490 \text{ F.P.}$$

The work to be absorbed by the wheel is then

$$E=145,490\times 0.16=23,280 \text{ F.P.}$$

The velocity of the rim is

$$V=56.55\times\frac{80}{60}=75.40 \text{ ft. per sec.}$$

Then

$$\frac{V^2}{g}=\frac{5685}{32.15}=176.8$$

and

$$f=\frac{23,280}{12,500\times 176.8}=.01056.$$

(b) Effective Radius of Wheel.—A mass-particle m, at the end of a radius r which makes n turns per second, has the kinetic energy

$$e=\frac{mv^2}{2}=2\pi^2mr^2n^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

The total kinetic energy of the fly-wheel, the summation of the e's of all the particles, will then be equal to the product of a constant by $\int mr^2$: which integral is the polar moment of inertia of the mass of the wheel about its rotation-axis. As always in such a case, an equivalent effect would be got by imagining the whole mass of the wheel to be concentrated in a ring at the end of the radius of gyration: so that this radius of gyration is the effective radius of the wheel, for which the velocity V in (246) is to be calculated.

Usually, the rim of the wheel contains by far the greater part of its mass; and the parts near the shaft, at short radius, have very little value as energy vehicles: so that no great error is caused, especially with wheels of the belt-pulley shape, by taking only the rim of the wheel into account, and using that as if concentrated at its outer circumference.

When, however, great accuracy is required, the polar moments of rim, arms, hub, and crank-disks can be approximately computed and an equivalent mass at the outer radius found. And when there are other rotating bodies attached to the shaft, as for instance the armature of a generator, these also must be reduced to the wheel-rim. The relation through which this reduction is made is that the energy-value of any mass varies as the square of its radius from the axis, or that the mass for any energy-value is inversely as the square of the radius.

(c) Multiple-crank Arrangements. — Besides using a fly-wheel to restrain the fluctuations in speed due to irregularity in turning-force, there is another method of securing uniformity of running: this consists in the use of two or more cranks at angles with each other (that is, not opposite, or at 180°). Then the excess-phase of one crank can be made to coincide with the deficiency-phase of the other; and not only will there be no dead-center, so that the engine will start from any position, but there will be

a much smaller variation in the total turning-force or turni
moment on the shaft. The freedom from dead-centers is especi
important in engines that have to start frequently against t
full pressure, as locomotives, hoisting-engines, and the l
and these are always made duplex, with cranks at right an
Of course, a compound engine, with each cylinder driving its cr
will have the same action; except that in engines of the loco
type class, provision must be made for admitting steam dire
to the low-pressure cylinder at starting, without waiting for i
get through the high-pressure cylinder.

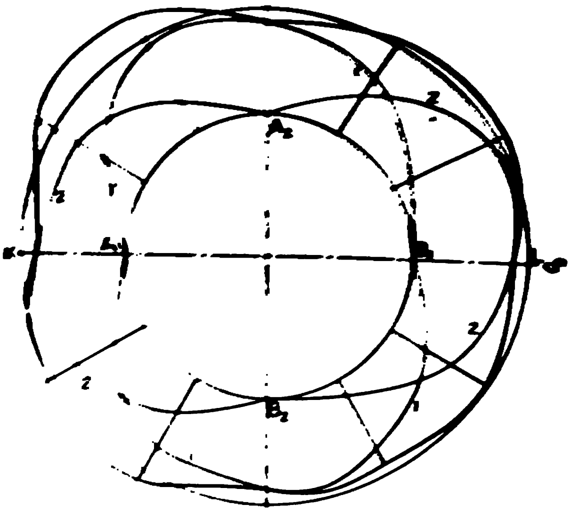


Fig. 139. Circular Diagrams Combined.

An example showing the simplest case of this combined acti is given in Figs. 139 and 140, which represent the working o duplex simple engine with cranks at right angles. The two tu ing-force curves are supposed to be alike, and are taken from Fig. 1 The second crank will be at dead-center when the leading cra is at 90°, hence the location of A_2 and B_2 on the drawings. T resultant curve of total turning-force is got by adding the ordina of the simple curves.

The circle or line of total resistance is drawn on the results

curve, and it at once appears that the variations in driving-force on the crank-pin are much smaller, especially in comparison with the mean force, than in a single engine. The form of curve here shown, with a minimum at each quarter-point and a maximum near the middle of the quadrant, is characteristic of this type of engine.

In the larger multiple-expansion engines, with three or four cranks, especially in marine engines, there is a considerable variety in the arrangement of the cranks, as to angles and as to order of

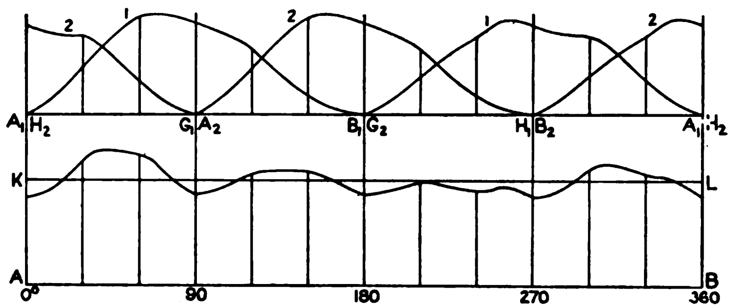


FIG. 140.—Turning-force in Quarter-crank Engine.

sequence. Combined turning-force diagrams for these engines are made in the same manner as Fig. 139 or 140. An additional complication is encountered, in that the pistons are not of the same size; so that if the separate curves are to be combined, they must either be worked out for total force on crank, or else one piston must be taken as the chief and the pressures on the others be referred to that, or expressed as equivalent pressures per unit of area of that piston. These matters will be more fully taken up in connection with the study of the compound engine.

(*d*) FLUCTUATION IN SPEED.—The discussion in (*a*) determines only the limits of speed variation. A more detailed investigation into the motion of the wheel is given in Fig. 141, where curve I. is the turning-force diagram of Fig. 138, plotted on the line of average force instead of an absolute base, and with the whole crank-circle developed along a continuous line.

To derive from I. the curve of speed-variation at II., we find

the effect of successive small increments of free work, as repre sented by areas between ordinates of I., and as taken up or givei off by the wheel. The general expression for kinetic energy o the wheel is

$$K=\frac{W}{2g}V^2=\frac{W}{2g}\left(\frac{\pi DN}{12\times 60}\right)^2=.000,000,296WD^2N^2, \quad . \quad . \quad (248$$

where D is the diameter in inches. For any particular engine w can work out the numerical value of $m=1\div.000,000,296WD^2$

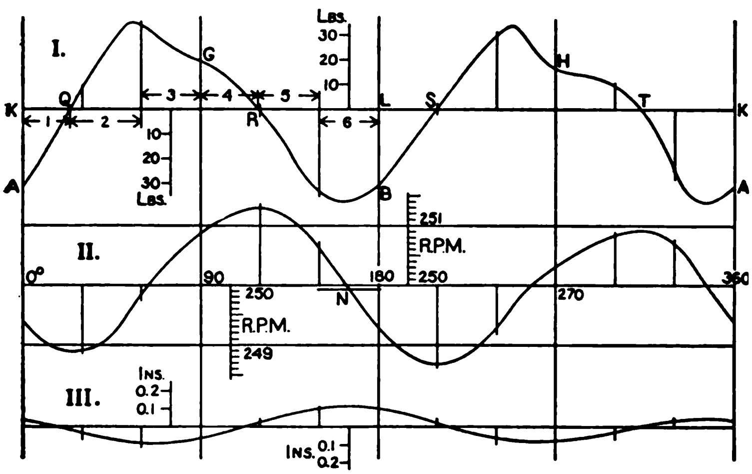


FIG. 141.—Diagrams of Fly-wheel Motion.

and then, putting (248) into the form $N^2=mK$, we get, for simul taneous changes in N^2 and K,

$$\Delta N^2=m\Delta K. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (249$$

For the engine of Fig. 138 and Example 1, the value of m i

$$m=\frac{1}{.000000296\times 1725\times 5184}=\frac{1}{2.641}=.3786.$$

The base-line of Fig. 141 is two-thirds as long as that of Fig. 138 then with the same pressure-ordinates, the area-scale will b $806\times 1.5=1209$ ft. lbs. per sq. in.—see (245). And since one F.P

of free work will change N^2 by .3786, 1 sq. in. of area will have the influence

$$n = 1209 \times .3786 = 458;$$

so that for working directly from the diagram in I., to get curve II., we have the relation, particular as to its numerical constant,

$$\Delta N^2 = 458 \Delta a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (250)$$

Table 36 A, covering half the length of the diagram, but less closely—that is, with wider intervals between ordinates—than in the determination actually used in laying out curve II., will illustrate the method of computing ordinates for this curve. The intervals are indicated by the numbered spaces on I., and the table is self-explanatory through ΔN^2: at the beginning of space 1 the speed is 249.38 R.P.M., according to curve II., and the number

TABLE 36 A. COMPUTATION OF SPEED.

Interval.	Δa sq. ins.	ΔN^2	N^2	N
			62189	249.38
1	− .55	−252	61937	248.87
2	+1.07	+490	62427	249.85
3	+1.10	+504	62931	250.86
4	+ .48	+220	63151	251.30
5	− .73	−334	62817	250.63
6	−1.48	−678	62139	249.28

and its square are entered at the top of Cols. 5 and 4: then the values in each line show the effect of the interval, or the condition at the end of the interval. Values of N can be got from N^2 by interpolation in the table of powers and roots given in any engineers' handbook.

As to the relations between the curves, it will be noted that where I. passes zero II. has a maximum or minimum; and where I. passes a maximum point there is a point of inflection in II.—the regular relations between a derivative curve and its primary. Further, II. passes through zero, or the speed is at its mean value, near the middle of each phase of I. This last fact was used in getting a starting-point for the cumulative addition of ΔN^2:

phase III. of the turning-force diagram, or RBS (compare Table 35 A), has the largest area, and is of nearly symmetrical form: and it was assumed that the mean speed of 250 R.P.M. would exist at the middle of space 6, or at 255°. This gave in II. the base-line passing through the point N: and after the curve was drawn, its net area, measured from this base, was +.86 sq. in.; so that the base-line, 12″ long, had to be moved up .07″ in order to be a true average-line of the curve.

(*e*) IRREGULAR MOVEMENT OF THE WHEEL.—Curve III. in Fig. 141 shows the effect of the variation in rotative speed upon the movement of the wheel, or the manner in which a point on the rim oscillates about the mean position which it would have if the speed were absolutely uniform—which can, perhaps, be best realized by imagining this reference-point to be carried on another wheel rotating beside the first at uniform speed. To get the time-effect of the small variant velocity represented by the ordinates of curve II., we proceed as follows:

The ordinate-scale of II., at full size, is 1″ = 1 R.P.M.; the distance travelled by a point on the rim of the 72-inch wheel, on account of a rotative speed of 1 R.P.M., is 226.2 ins. in one minute or 3.77 ins. in one second. The time represented by the whole base-line, 12 ins. long, is $60 \div 250 = 0.24$ sec., so that 1 in. stands for .02 sec. Then 1 sq. in. of area under II. represents an effect of velocity × time which will move the wheel

$$\Delta S = 3.77 \times .02 = .0754 \text{ in.};$$

giving the more general relation

$$\Delta S = .0754 \Delta a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (251)$$

By means of this formula the ordinates of III. are computed from measured areas under II. The scale of this curve is such that the displacements are represented at about true size in the reduced figure.

(*f*) ACCELERATION OF THE WHEEL.—This second idea, of determining the time-effect of a function of the motion, could have been used with equal facility for getting the velocity curve from

one representing acceleration: the latter would be similar to curve I. in form, differing only in its ordinate-scale; and the acceleration represented might be either angular or lineal—the second, that of the end of the wheel-radius, or of a point on the rim. To get an expression for angular acceleration under the conditions of our problem, we have first that a free force of t pounds per square inch of piston is equal to a total force At on the crank; its moment is $Q = AtR/12$, and it acts upon a rotating mass whose polar moment of inertia it Mk^2, where M is W/g and the radius of gyration k is the wheel-radius in feet, or $D/24$. The general relation is

$$Q = Mk^2\omega, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

in which ω is the angular acceleration in radians per second per second; and substituting the values above, we get

$$\omega = C\frac{At}{W}\frac{R}{D^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (253)$$

where the coefficient C is $48g$ or 1544 for D in inches, and 10.72 for D in feet, R being always in inches.

Linear acceleration at the rim is got directly, by substituting for the actual turning-force At on the crank an equivalent, "reduced" force at the rim, having the same moment, and found by the relation

$$F = At\frac{2R}{D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

R and D being, of course, in the same unit. Then the acceleration a of the mass M is

$$a = 2g\frac{At}{W}\frac{R}{D}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (255)$$

and dividing by the radius $\frac{1}{2}D$, expressed in feet, we should get a formula identical with (253).

Applying (255), as most convenient, we see that, in Fig. 141, 1" of ordinate represents a force of the value $At = 24 \times 153.9 = 3694$ lbs.; substituting this with the other particular values, (255) becomes

$$a' = 64.32\frac{3694}{1725}\frac{7.5}{72} = 14.35. \quad . \quad . \quad . \quad . \quad (256)$$

And 30° or 1″ of base-line represents .02 sec.; so that the velocity-change value of 1 sq. in. is $\Delta V=0.287$. Then for interval no. 3 in Table 36 A the change in velocity would be $1.10\times.287=.316$ ft. per sec. This bears to the mean velocity 78.54 (see Example 1) the ratio $.316\div78.54=.00402$; and in Table 36 A the corresponding change in rotary speed is 1.01, and this bears to 250 the ratio $1.01\div250=.00404$, which is practically the same.

The method actually used, of working from force on a distance-base, through kinetic energy, to velocity, seems more in line with other computations on the fly-wheel than does that involving the acceleration. In some problems analogous to this, however, where the free force is on a time-base only, the second method is the one available.

(*g*) VELOCITY BY MOMENTUM.—Slightly different in idea from the use of the acceleration, but the same in result, is a method based on the momentum equation

$$FT=MV,$$

where T is time in seconds; and which, put into a form more directly applicable, becomes

$$\Delta(FT)=M\,\Delta V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (257)$$

In our example, the force represented by 1″ of ordinate in Fig. 141, reduced to the wheel-rim, or to the path of the moving mass, is, by (254),

$$F'=3694\times\frac{15}{72}=770;$$

and for 1 sq. in., the time-value of 1″ of abscissa being .02 sec., we have

$$\Delta'(FT)=15.4.$$

Dividing this by the mass, $M=1725\div32.16=53.64$, we get 0.287, the same constant as in the last article.

That this is only a variation on the acceleration method appears when we take the general relation

$$a\,\Delta T=\Delta V$$

and substitute for the acceleration a its value F/M.

(*h*) Non-uniform Resistance.—In some engines, where the load is applied directly to the cross-head, the simple condition of a uniform tangential or torque resistance does not exist. In pumping-engines, this axial load is nearly uniform throughout the stroke, while in air-compressors it is variable. In either case, the duty to be performed by the fly-wheel can be most easily determined by diagrams of the type of Fig. 132.

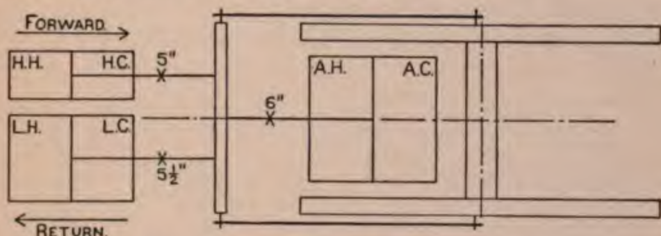


Fig. 142.—Outline of Blowing-engine.

To illustrate the methods of procedure in a case of this sort, an example will now be worked out. The engine, outlined in Fig. 142, is a compound blowing-engine, with the two steam-cylinders lying side by side, and with the ends of the cross-head connected to the two fly-wheels. The diameters are 30″ and 54″ for the steam-cylinders and 80″ for the air-cylinder, with a stroke of 80″.

The indicator diagrams to be worked up are given in Fig. 143; and while the steam-action which they show might be better, they serve very well as a basis for the mechanical discussion of the performance of the engine. The pressure-scales are laid off on the cards, and the R.P.M. was 23.5.

Table 36 B. Constants for Diagrams.

	1	2	3	4	5	6
Cyl. End.	Piston Area, Sq. ins.	Cylinder Volume, Cu. ft.	Ratio to L. H.	True Scale.	Correction Factor.	Reduction Factor.
H.H.	706.8	32.72	.3087	40.0	1.00	44.45
H.C.	687.2	31.82	.3001	40.4	1.01	43.65
L.H.	2290.2	106.03	1.0000	20.4	1.02	146.9
L.C.	2266.5	104.93	.9897	20.4	1.02	145.4
A.H.	4998.2	231.40	2.182	16.67	1.042	327.4
A.C.	5026.5	232.71	2.195	16.0	1.00	316.1

The constants useful for present purposes are given in Table 36 B. For combination, all the pressures are to be reduced to pounds per square foot of the low-pressure piston of full area (not diminished by the piston-rod, or on the head-end side): and in Col. 3 the ratio of each piston-face to this reference-area is given. Col. 4 contains the results of a calibration-test of the indicator springs used, and the factors in Col. 5 are to be used for correcting ordinates when measured with the nominal scale. Finally, Col. 6 is a combination of the factors in Cols. 3 and 5 with 144: thus for high-pres-

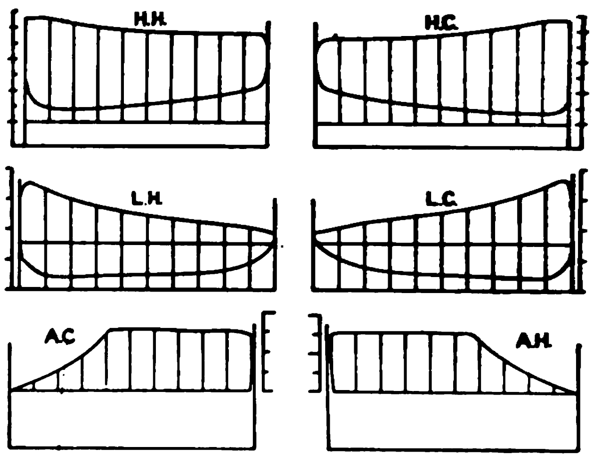


FIG. 143.—Indicator Diagrams.

sure head-end, or H.H., the factor $44.45 = .3087 \times 144 \times 1.00$ is to be used for transforming pressures measured from the H.H. card with a scale of 40 to the inch, or in pounds per square inch by the nominal scale, to pounds per square foot of the L.H. piston.

By measuring, reducing by multiplication, tabulating, and combining the ordinates drawn on the indicator cards, data for the curves in Fig. 144 were obtained. At full size, the scales of this figure are, for pressures, 1″ = 1000 lbs. per sq. ft. of the full L.P. piston; for volumes, 1″ = 10 cu. ft. displaced by this same piston; so that 1 sq. in. of area represents 10,000 F.P., or 10 of the 1000-F.P. work units of § 5 (d).

The whole of Fig. 144 is on the same system as Fig. 132, the base MN being the stroke-line. The primary curves are:

1. Effective steam-pressure, H.P. cylinder.
2. Effective steam-pressure, L.P. cylinder.
3. Total steam-force, the sum of 1 and 2.
4. The air-resistance.

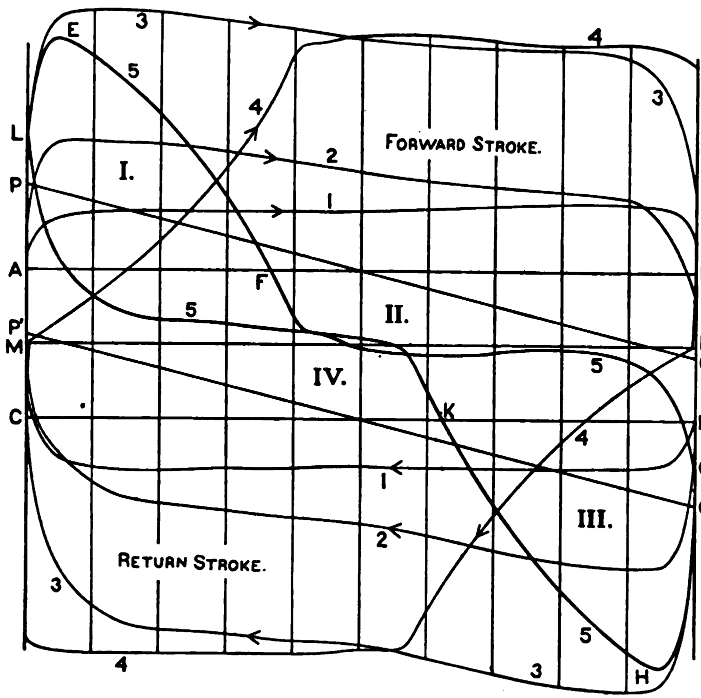


FIG. 144.—Curves of Effective Pressure.

And the derived curve is

5. The difference between 3 and 4, brought to the straight-line base MN.

The results of a calculation of the work done in one revolution, from areas under curves, are given in the following condensed statement; where the two partial steam-works and the total for each stroke were got by independent measurements with the planimeter, so that the sums do not quite check up.

H.P., forward,	225.3 W.U.	return,	204.4 W.U.
L.P., forward,	285.7 "	return,	300.2 "
Steam, forward,	511.1 W.U.	return,	505.0 W.U.
Air, forward,	383.0 "	return,	384.4 "

Total steam work................ 1016.1 W.U.
Total air work.................. 767.4 W.U.
Lost work....................... 248.7 W.U.

$$\text{Mechanical efficiency} = \frac{767.4}{1016.1} = .756$$

Dividing the total friction-work, 248,700 F.P., by the piston-displacement for both strokes, or 212.1 cu. ft., we get a mean resistance of 1174 lbs. per sq. ft. or 8.15 lbs. per sq. in. This average friction-force is measured up and down at MA and MC, and its action is represented by the lines AB, CD, the rectangles MB and NC standing for the work of friction. Actually, the friction-line would be a curve, lower in the middle and rising at the ends; because the friction-resistances of the crank-pins and of the shaft-bearings keep on moving at full speed when the pistons are near dead-center and the driving-force is moving very slowly. However. the straight friction line is a useful approximation.

Now the lines AB, CD, are means of the curves 5, or of the curves of unbalanced force. Then, without considering the inertia of the reciprocating parts, the four phases of free work to be cared for by the wheels, and their values, are, by measurement,

I.	AEF,	+ 90.9 W.U.	II.	FGB,	− 88.2 W.U.
III.	DHK,	+100.7 W.U.	IV.	KCL,	−103.1 W.U.
		+191.6 W.U.			−191.3 W.U.

As a rough estimate, the total weight of the reciprocating parts is about 15 lbs. per sq. in. of the L.P. piston. At 23.5 R.P.M., the angular velocity θ is 2.46, by (227); and the constant $C_2 = \theta^2 \div 24g$ of (229) is .00785. Multiplying the latter by $S=80$, we get the ratio of F_0 to W to be 0.628; so that F_0/A will be $15 \times .628 =$ 9.4 lbs. per sq. in., or 1350 lbs. per sq. ft. Inertia diagrams for "infinite rod" are drawn on AB and CD, and show a considerable

diminution in the irregularity of the unbalanced force, the values of the four phases given above changing to

I.	+ 57.8 W.U.	II.	− 54.0 W.U.
III.	+ 66.8 W.U.	IV.	− 70.8 W.U.
	+124.6 W.U.		−124.8 W.U.

Again roughly, the two wheels have an effective weight of 60,000 lbs. at a radius of 10 ft.; then

$$\frac{V^2}{g}=\frac{24.6^2}{32.2}=\frac{606.0}{32.2}=18.7;$$

and for $E=70{,}800$ F.P., we get

$$f=\frac{70{,}800}{60{,}000\times 18.7}=.063,$$

as the ratio of fluctuation of speed.

(*i*) Stress in Rim of Wheel.—The condition of the rim of the wheel under the action of centrifugal force is illustrated in Fig. 145: it is subjected to a uniformly distributed radial load; and the force tending to cause rupture at any section, as at AB, is found by taking the sum of the components at right angles to AB of all these radial forces—as indicated by F_1, F_1. The mathematical deduction of the value of this resultant force on the half-ring is as follows:

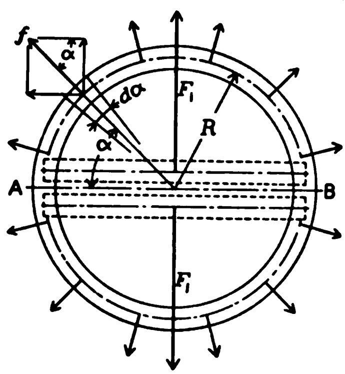


Fig. 145.—Centrifugal Force on Wheel-rim.

Let a be the area of cross-section of the rim in square feet, l any length in feet measured along its circular center-line, and w the weight per cubic foot of the material. Then the centrifugal force of a piece of unit-length will be

$$f=\frac{aw}{g}\frac{V^2}{R}: \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (258)$$

and for an element of the length $dl = Rd\alpha$, we have

$$dF = f dl = f R d\alpha.$$

Now the component perpendicular to AB is $df \sin \alpha$, so that have

$$dF_1 = fR \sin \alpha \, d\alpha:$$

and integrating for the half-circle we get

$$F_1 = fR \int_0^\pi \sin \alpha \, d\alpha = 2fR. \quad . \quad . \quad . \quad . \quad . \quad (25$$

The total centrifugal force on the half rim is

$$\tfrac{1}{2}F = \pi f R;$$

and we see that the bursting tendency is as if the force f were di tributed along a bar of length equal to the diameter of the whe and that the force F_1 bears to the whole centrifugal force F of tl rim the ratio $1 : \pi$.

This load is taken up at two sections of the rim, so that tl tension at either section is $F/2\pi$.

EXAMPLE 3.—A wheel 16′ in diameter, with rim 24″×3″, is ma in sections and bolted together with five 1½″ bolts at each joint: wh will be the tensile stress in the cast-iron rim and in the bolts, c account of centrifugal force at 100 R.P.M., disregarding any holdir effect which the arms may exert?

Taking $C_2 = .1421$ from Table VII and using the diameter $D = 189$ instead of S in (230), we get V^2/gR to be $.1421 \times 189 = 26.86$; whic means that the centrifugal force of each pound of rim is 26.86 lbs.

The weight of the rim is

$$W = \frac{3 \times 24 \times 3.142 \times 189 \times 450}{1728} = 11{,}150 \text{ lbs.}$$

And the tension on the rim is then

$$T = \frac{11{,}150 \times 26.86}{6.283} = 47{,}600 \text{ lbs.}$$

On the 72 sq. ins. of rim section this brings a stress of only 681 lbs per sq. in. On the five bolts, of an effective diameter (under the threads of 1.25″ and with 1.23 sq. ins. of cross-section, the stress will be

$$S = \frac{47{,}600}{1.23 \times 5} = 7750 \text{ lbs. per sq. in.}$$

(j) LIMIT OF SPEED.—Knowing the working-stress in the wheel and that the centrifugal load-force varies as the square of the speed, it is easy to calculate, in any case, the speed at which danger of bursting will begin. At present we shall not go into a discussion of the effect of various methods of construction in this respect. But by extending the deduction of the last article, we can arrive at certain general relations of considerable interest.

Multiplying (258) by the circular length πD of the ring, we get

$$F=\pi Df=2\pi R\frac{aw}{g}\frac{V^2}{R}. \qquad (260)$$

Taking out the factor 2π, we have the tension T and can equate it to the expression for the strength of the ring, getting

$$T=\frac{aw}{g}V^2=144aS: \qquad (261)$$

from which

$$V^2=\frac{144g}{w}S. \qquad (262)$$

It will be noted that in (260) the volume-factor $2\pi R$ was necessarily in feet, while S is in pounds per square inch; and if we take a in square feet and w in pounds per cubic foot, then the factor 144 must be introduced, as above.

For cast-iron, with a maximum allowable working-stress of, say, 4000 lbs. per sq. in., the greatest safe speed would be

$$V=\sqrt{\frac{144\times 32.16\times 4000}{450}}=203 \text{ ft. per sec.}$$

While for a high-grade steel, where S might be as much as 20,000 lbs., the value of V would be about $200\times\sqrt{5}=450$. This is greatly exceeded in turbines of the De Laval type: but there the radial tension of the solid wheel becomes a principal element of strength.

The reason for the disappearance of R from (260), expressed in other than purely mathematical terms, is that while, for a given linear velocity V, the centripetal acceleration varies inversely with the radius R, on the other hand the weight of a ring of given cross-section increases as R.

§ 37. Force-action in the Connecting-rod.

(*a*) THE COMPONENTS OF ACCELERATION.—In the developme of this subject, already set forth in general terms in § 32 (*d*), tl first step is the determination of the exact inertia-force of the ro The conditions as to acceleration are closely analogous to those i regard to the velocity which are shown in § 33 (*i*). That is, ju as the movement of the rod can be resolved in a translation-motic with some point, and a rotation about that point, so also is tl total acceleration made up of a direct, or linear, and an angul component.

Thus in Fig. 146 the components are, direct acceleration wit C and angular about C: the first alone gives to W, or to any poir E, the same acceleration a_0 that C has; and the total acceleratio of these points is got by combining with a_0 another component, whic is evidently the whole or a part of Cw—as appears from the equiv alence of the triangles HBW, COw, and of DFE, COe. Keepin in mind the manner of locating the center of acceleration (as i Fig. 122), we see that the angle WCw is the same as ACO or AWO that is, it is the angle which each acceleration makes with its radiu from the center A. And further, comparison with Fig. 120 show that what we found there, in GC, is the component along the rod line of wC or WH in Fig. 146.

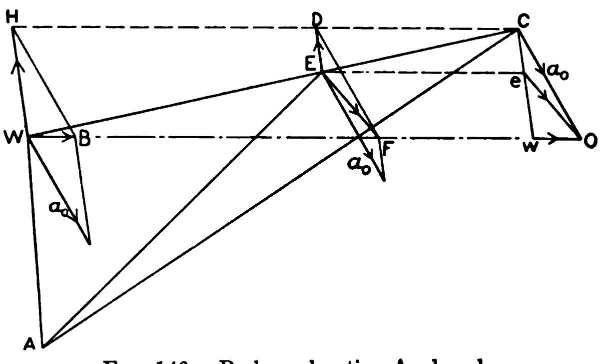


FIG. 146.—Rod-acceleration Analyzed.

The general facts as to the acceleration of the rod, and the conclusions which may be drawn from Fig. 146, are as follows:

First, and fundamentally, the acceleration-center A is the one point of the rod (or on a plane attached to the rod) that has no acceleration at all, or that is moving, at the instant, with unchanging velocity.

Second, the acceleration of any point is proportional to the radius from A and makes a certain angle with that radius. Each of these point-accelerations is made up of a component at right angles to its radius, due to the angular acceleration proper about A; together with a centripetal component toward A, due to the rotation of the radius about A with an angular velocity the same as that with which the rod turns about its instantaneous center. The above analysis of the determination of Fig. 120, where just this idea is followed in getting one component of HW, Fig. 146, establishes this proposition with reference to the secondary center C; and it is easy to see that the same relation would hold for the primary center A. This explains why the total acceleration is not at right angles with its radius from A.

Third, if the motion of the rod be separated into a translation with some point as C, and a rotation about C, then the acceleration of any other point is the resultant of C's acceleration and of another component due to an angular acceleration about C equal to that about A—the latter appearing at WH and ED.

The important practical result of this discussion is that the angular acceleration of the rod can be found by dividing Cw—taken to represent acceleration to the same scale as CO represents a_0—by the perpendicular distance from W to Cw, expressed in feet. Dividing linear acceleration in feet per second per second by radius in feet, we should get angular acceleration in radians per second per second.

(b) Total Acceleration.—The force required to produce a linear acceleration a in a body of mass M is

$$F=Ma; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (263)$$

while for an angular acceleration $\omega=\frac{d\theta}{dt}=\frac{d^2\alpha}{dt^2}$, the torque or moment required is

$$T=Mk^2\omega: \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (264)$$

where k is the polar radius of gyration of the body with respect to the center of angular acceleration, so that Mk^2 is the polar moment of inertia of its mass.

In passing from acceleration (a geometrical quantity) to force, a regard for simplicity of relations limits our choice of the center of angular inertia to the center of mass of the body. It is only when the force F is applied at the center that it can produce the simple linear acceleration of (263). Resolving the total acceleration of the rod, then, into a linear component, that of the center of gravity, and an angular component about this center, we have k as the principal radius of gyration.

In Fig. 147, the two force-actions of (263) and (264) are at first shown separately, by $F_L = M \times Og$ at the center of gravity G,

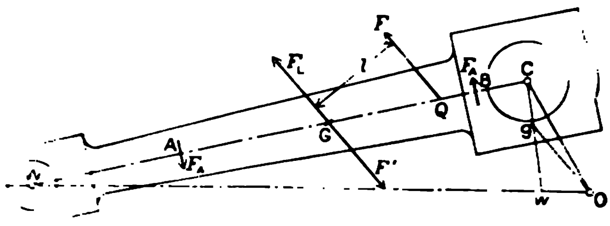


FIG. 147. Inertia-force of Rod.

[illegible] $F_L - F_A$ of the value T. Now there is an in[illegible] [illegible] effects of a force-couple; and the simplest [illegible] to $F - F'$, where each of these forces [illegible] placed on the line of, and opposite, F_L. [illegible] expressed by the single force F; or, [illegible] produced by the application [illegible] F.

[illegible] principles of Fig. 146, II. [illegible] the accelerating force F [illegible] does not pass through the center of mass of [illegible] are two effects, the first a linear acceleration of [illegible] [illegible]

$$a = F \div M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2[illegible])$$

co.

the second an angular acceleration about the center, of the value

$$\omega = Fl \div Mk^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (266)$$

In order, then, to determine completely the inertia-force of the connecting-rod, we get its direction and intensity from the known acceleration of the center of gravity, and must then locate it by an application of (264).

Following the conclusion stated at the end of (a), we should get ω, and knowing k could easily find l; but this method, while perfectly feasible and giving a complete solution of the problem, is less convenient in application than one along somewhat different lines, which will now be developed.

(c) Concentration of Mass of Rod.—If we are concerned with only the linear acceleration of a body, we may think of the whole mass as concentrated at the center of mass (or of gravity), where it will have a simple linear inertia, as F' in Fig. 148. But if angular acceleration is to be considered, then the simplest arrangement that can be substituted for the actual, distributed mass of the body consists of two concentrated masses, on opposite sides of the center and on a rigid straight line through the center. The determining conditions for these masses are, letting M_1 and M_2 be their respective quantities and h_1 and h_2 their distances from G,

First, the total mass must remain unchanged, or

$$M_1 + M_2 = M. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (267)$$

Second, the original center of mass must be preserved, or

$$M_1 h_1 = M_2 h_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (268)$$

Third, there must be no change in the polar moment of inertia about G, or

$$M_1 h_1^2 + M_2 h_2^2 = Mk^2. \quad . \quad . \quad . \quad . \quad . \qquad (269)$$

By interchanging the terms of (268) in (269) we get

$$M_2 h_2 h_1 + M_1 h_1 h_2 = Mk^2;$$

$$(M_1 + M_2) h_1 h_2 = Mk^2;$$

$$h_1 h_2 = k_2: \quad . \quad . \quad . \quad . \quad . \quad . \qquad (270)$$

which is the determining condition of what is known in Mechanics as the "compound pendulum."

In applying these conditions to the connecting-rod, one of the simpler cases is that shown in Fig. 148, where the partial mass M_1

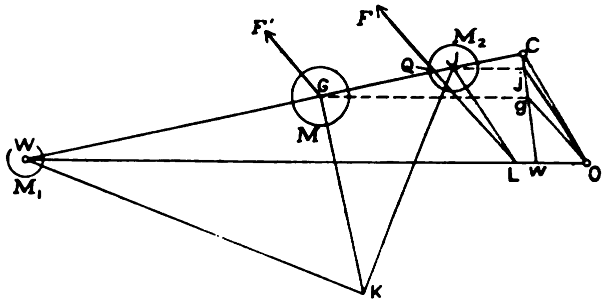


FIG. 148. Concentrated Masses.

is at the wrist-pin W, and the other, M_2, is at J. This second point is located graphically by drawing GK perpendicular to the rod and equal to k, then making WKJ a right angle; for in the triangles WGK, KGJ, where WG $= h_1$ and GJ $= h_2$,

$$h_1 : k :: k : h_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (271)$$

which satisfies (270).

Now the inertia-force of M_1 will act along OW, that of M_2 along a line whose direction is found by drawing Jj horizontally over to the image and joining Oj. Then the line of this force is JH parallel to jO; and the resultant of the two partial inertia-forces, F_1 and F_2, must go through the point L. A complete solution by this method would involve the determination, first of M_1 and M_2 by (268), next of a_1 and a_2 from the image, finally of F_1 and F_2 by (263), and the graphical combination of these forces. But since the direction and intensity of the resultant is already known, from the acceleration gO, we need not go beyond the finding of the point L: the line LQ, parallel to Og, locates the rod-force.

(d) ROD-FORCE CONSTRUCTION.—A further development of this method into the form most convenient of application is illustrated in Fig. 149. The work of locating the action-line of F—deter-

mined by the intersection L in Fig. 148—is reduced to a minimum by choosing one point of concentration at B (on I.), where its acceleration will be right along the rod, and will pass through the other point, at Q. Now it matters not what is the direction of the second component F_2, since it is bound to pass through this point Q. Out of the infinite choice of points of mass concentration—not limited, of course to the center-line WC—this leads most directly to the desired result.

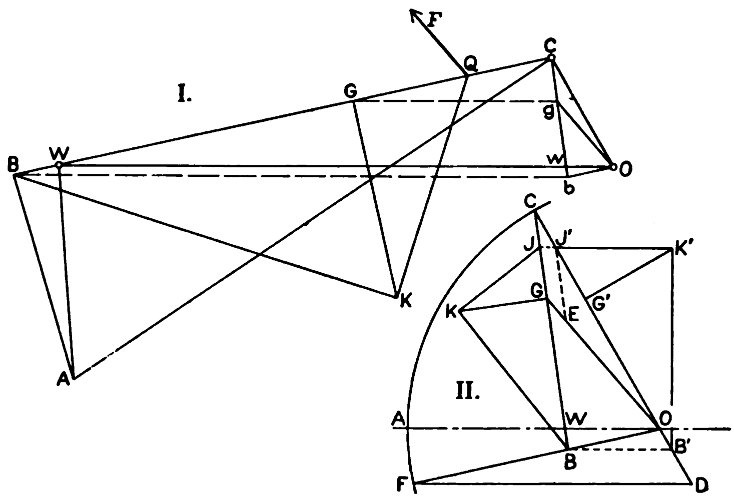


Fig. 149.—Rod-force Construction.

The next step is to reduce or contract the construction to the image, as shown at II. in Fig. 149. The point B, corresponding to b in I., is found by drawing OB parallel to the rod, by the method of Fig. 116: then KG is drawn perpendicular to the image CW at its "center of gravity," its length in the proper ratio to CW: and BKJ is made a right angle. The only disadvantage of this method is that CW changes in length as we go round the crank-circle: and it is better to make the similar construction B′K′J′ on the radius, considering it to be, for this purpose, a sort of constant-length image of the rod; BB′ is, of course, a horizontal line, as is also Bb in I.

Finally, by drawing J′E parallel to CW, we divide the inertia-

force of the rod into its parallel components at the pins; for, evidently,

$$OE : EG = OJ' : J'C = WQ : QC \quad \text{on I.};$$

or

$$OE \times CQ = EG \times WQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (272)$$

Here we treat OG as a force, represented to a proper scale by this line: while in all the preceding part of the discussion it is used only as an acceleration.

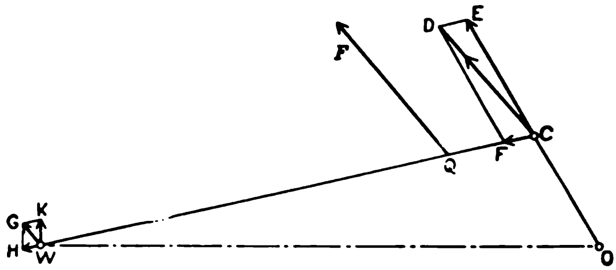


FIG. 150. Rod-inertia Resolved.

(e) EFFECTS OF ROD-INERTIA.—In Fig. 150 the rod-inertia F, a single force at Q, is replaced by its parallel components, CD at the crank-pin and WG at the wrist-pin, being resolved according to (272). At C the force CD is further resolved into two components, CE along the crank-line, where it will have no turning-effect, and CF along the rod-line, where it will combine with the force transmitted, S in Fig. 133 or 134. At W, WG is resolved into WH along the rod and WK in the direction of the guide-pressure. In other words, the components of inertia are resolved, at the respective pin-centers, in the direction of force transmitted and of restraint of motion.

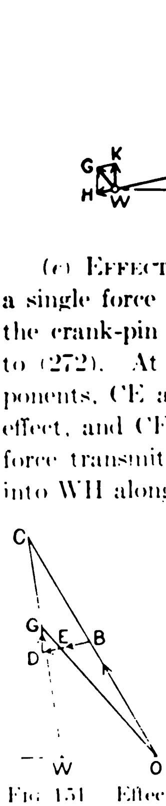


FIG. 151 Effect of Rod-inertia.

In Fig. 151 these force-resolutions are brought together on the image construction, taken from Fig. 149 II. The triangle CDE of Fig. 150 is reproduced at OEB, and WGH at EGD. It appears now that by the simple device of drawing BD through E, parallel to the rod-line and meeting a vertical from G at D, we have the rod-force OG resolved into its three component-effects; namely, OB along the

crank, outward, and with no influence upon work-performance; BD along the rod and modifying the transmitted force S; and DG perpendicular to the guide and affecting only the guide-bar pressure.

(*f*) Division of Rod-mass in Approximate Method.—The construction just described is applied in Fig. 152 to a rod of typical proportions, in order to see how nearly the approximation

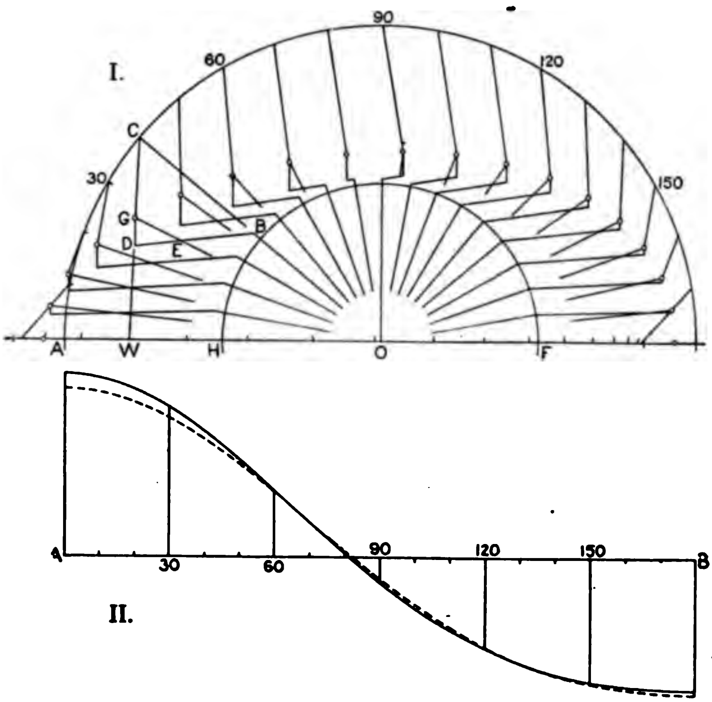


Fig. 152.—Components of Rod-inertia.

of § 32 (*c*) agrees with a true analysis, the comparison alluded to in § 32 (*d*) being made. The rod is six cranks long; the center of gravity is at four-tenths of the length from C to W, or CG =0.4CW in Fig. 147; and the radius of gyration k has this same length, $0.4L$. The construction of Fig. 149 II. is first used for getting OEG at each crank-angle, and then the DB-lines of Fig. 151 are

drawn. The construction need not be made for more than two quadrants, as remarked in § 33 (*h*).

For comparison with the radial component OB, the circle on HF is drawn with the radius OH equal to half of OA: and it appears that to assume half the mass of the rod to be concentrated at the crank-pin will give a very close approximation to true results. In II., the horizontal component of DB is plotted on the developed semicircle, giving the dotted-line curve; while the full-line curve is laid out from a diagram like Fig. 124, likewise for a radial force equal to one-half of the full F_0 for the rod, or of OA. This shows how very nearly correct it is to concentrate the other half of the rod-mass at the wrist-pin.

Departures from the rod-proportions used above, as encountered in various designs of engine, are not large: and it is safe to make a general rule that the mass of the rod shall be divided into equal parts, for concentration at the two pin-centers in the approximate determination of inertia-effect.

§ 38. Pressures on the Bearings.

(*a*) EXACT PIN-PRESSURES.—The method of determining the true pin-pressures, as indicated in Fig. 102, is now to be worked out. The first step is to show, by means of Fig. 153, certain relations existing in the case of a two-joint link which transmits force and is at the same time under the action of a transverse force of

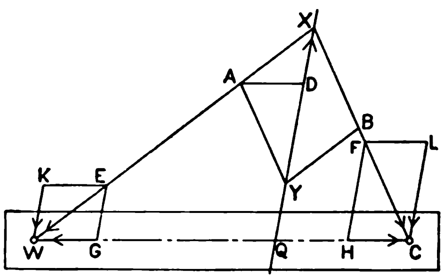


FIG. 153.—Pin-pressure Analysis.

its own. The conditions of the case are set forth in this figure on the rigid bar WC acts a force F, along the line QX; and this is

to be held in equilibrium by forces applied at (or through) W and C. These two forces may act along any pair of lines which meet in a point on QX. Suppose X to be one such point, then make $XY = F$ and resolve it into the components AX and BX: forces equal and opposite to these components, at W and C, will give equilibrium.

By a further analysis, the pin-pressures, P_W at W and P_C at C, are resolved into the components KW and LC parallel to F, and GW and HC along the center-line WC: then the first two are in equilibrium with F, the second two balance each other, showing the longitudinal stress in the rod due to force transmitted.

Now the only effect of changing the position of X, or of using another pair out of the infinite number of possible force-lines, is to change the internal stress components GW and HC: so long as the force-line QX remains in one definite position with reference to WC, the system of parallel forces made up of F and the equilibrium components $P_{WE} = KW$, $P_{CE} = LC$, will remain unchanged. This ought to be self-evident; but it can also be proved geometrically by drawing AD parallel to WC: then in the similar triangles ADX, WQX,

$$DX : QX :: AD : QW;$$

and in ADY, CQX,

$$DY : QX :: AD : QC;$$

from which we can easily get

$$DX : DY :: QC : QW.$$

That is, no matter what the position of X, D divides F into the components P_{WE} and P_{CE}, inversely as the distances QW and QC.

(*b*) Construction for Pin-pressures.—The condition of equilibrium—namely, that each pin-pressure, besides exerting an effect along the center-line, shall balance its share of F—is put into graphical form in Fig. 154; where the line E_1E_2 is a locus of the ends of all the possible pressures at the wrist-pin W, if laid back from W as a pole; while F_1F_2 is a similar polar diagram for the crank-pin.

We approach the problem of finding the true pin-pressures with two quantities unknown; and the method of solution is given in Fig. 155. Coming from the piston-slide, we know completely one component of P_W, the effective driving-force P; but the other, the guide-reaction Q, is known only in direction. Measuring off WA $=P$, and drawing the vertical AJ, we have that AJ is a polar

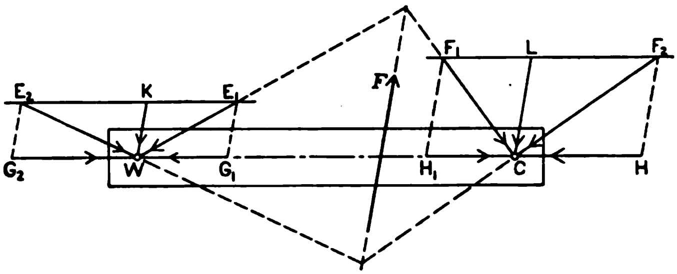


FIG. 154.—The Equilibrium Locus.

locus of the ends of all wrist-pin pressures which meet the conditions external to the rod. Knowing the rod-force F and its parallel components at the pins, from Fig. 149, we make WE equal F_w (the same as WK in Fig. 154), and draw EF parallel to WC. This locus embodies the internal conditions of equilibrium: and its intersection with AJ at F determines, in WF, the one wrist-pin pressure that satisfies all conditions.

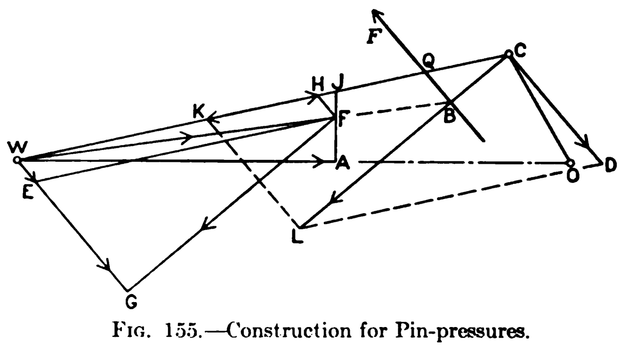


FIG. 155.—Construction for Pin-pressures.

The pressure at the crank-pin can be found by making CK = WH, or transferring the internal stress component to C, and combining it with CD $=F_C$ so as to get CL $=P_C$: or, more simply, by closing

the triangle WFG, where WG is equal (but opposite) to F. In this construction, we find the pin-pressures, in direction, as acting upon the rod, using the balancing components, WE, EG, as a part of each.

It is interesting to note that the triangle HJF is identical with DGE in Fig. 151; showing how one component of F_W or FH diminishes the guide-reaction Q from the full value AJ which it would have if the rod were a weightless link, while the other component JH shortens the transmitted force. A similar analysis might be made at the crank-pin, but to no useful result.

(c) DETERMINATION OF THE PIN-PRESSURES.—In finding the crank-pin pressures shown on Fig. 157, the construction in Fig. 156,

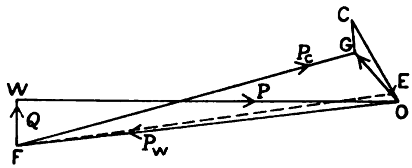


FIG. 156.—Pressure on the Crank-pin.

a slight modification of Fig. 155, was used. Here OC is F_0 for the rod, and OG is F, after the manner of Fig. 149 II.; but the division-point E is moved to the other end of F. From O, taking it to represent the wrist-pin center, we measure back P, taken from Fig. 132 (or rather, from Fig. 158): then EF is drawn parallel to the center-line of the rod, and FG is the crank-pin pressure P_C, in the direction of action of rod on pin.

For these figures the same proportions of rod were used as for Fig. 152. In Fig. 132, the value of F_0/A was 36 lbs.; but this is due to a reciprocating mass which includes half the connecting-rod. Of the total weight of the reciprocating parts, a good average assumption gives the piston-slide six-tenths and the connecting-rod four-tenths. So that three-fourths of the 36 lbs. above will be considered as due to the slide alone, and we have:

For the slide, $F_{10}/A = 27$ lbs.

For the rod, $F_{20}/A = 18$ lbs.

In Fig. 158, otherwise the same as Fig. 132, the inertia-curve SQT is drawn for 27 lbs.; and ordinates measured from it were used

in making the construction of Fig. 156. The results of this work are shown in Fig. 157, where each P_C or CD is parallel and equal to an FG of Fig. 156. Forces on these two figures are twice as long as those on Fig. 158.

Fig. 157 shows some interesting facts, as, for instance, how the inertia of the rod combines with its slant so as to throw the pin-

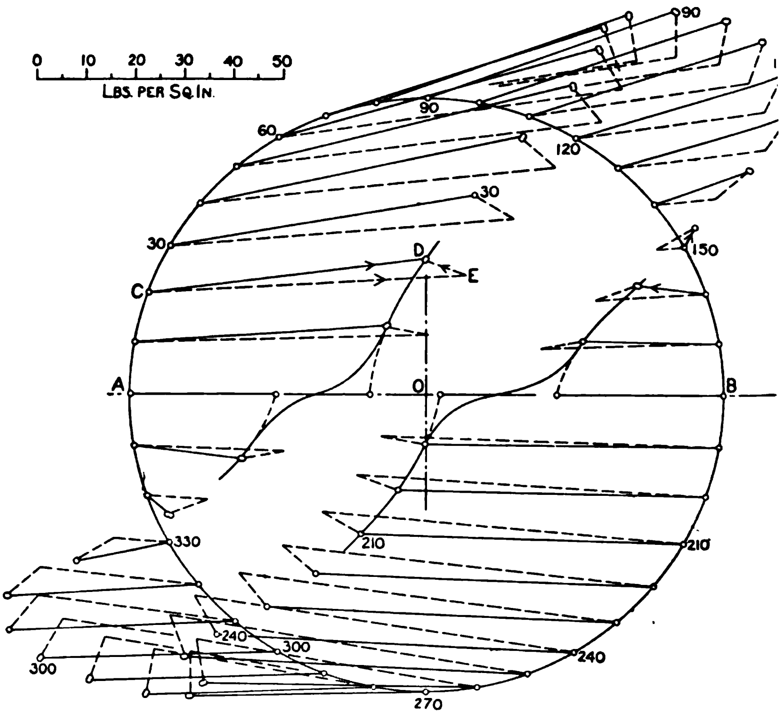


FIG. 157.—Diagram of Crank-pin Pressures.

pressures farther from parallelism with the stroke-line throughout the forward stroke, but to bring them nearer to this parallelism during the return stroke. Its chief value, however, lies in the comparison which is made between results got by the exact and by the approximate method.

The latter is applied to this case by laying off along the rod-line in each position the length CE $=S$, the force along the rod

after the manner of Figs. 133 and 134, as got from the ordinates of Fig. 132. These forces include the effect of half of the mass of the rod; and the other half, at the crank-pin, is taken into account by combining its centrifugal force ED with CE. The resultant is so nearly the same as CD that the difference hardly goes beyond the limit of accuracy of the drawing.

Now CD is very nearly the same in length as CE less the horizontal component of ED; this subtraction would be made by adding the other half of the rod-mass to the sliding-mass which gave the curve PQR in Fig. 132: and we come to the very useful

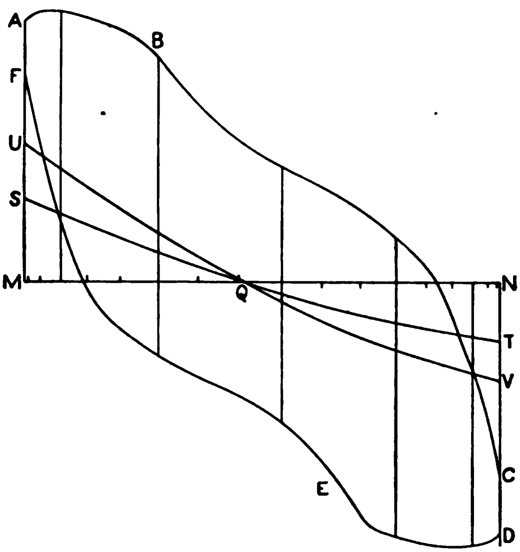


FIG. 158.—Approximate Pin-pressures.

practical result, that the variable pressure on the wrist-pin is given very closely by ordinates measured from the curve SQT in Fig. 158; while for the crank-pin pressure the curve UQV is to be taken as the base-line. Curve SQT shows the inertia-force of the slide-mass alone, UQV includes the whole of the connecting-rod. To get turning-force we continue, however, to use the intermediate curve PQR on Fig. 132.

The error in the determination of pin-pressures by Fig. 158

is greater where the pressures are small, and the rod-angle large, in the second and fourth quadrants. But the large pressures are closely determined, and absolutely all the data that could be desired for the purposes of the not very exact science of designing the machine are given by this figure.

(*d*) PRESSURES ON THE CRANK-PIN.—The results given in Fig. 157 are shown according to another scheme in Fig. 159, where

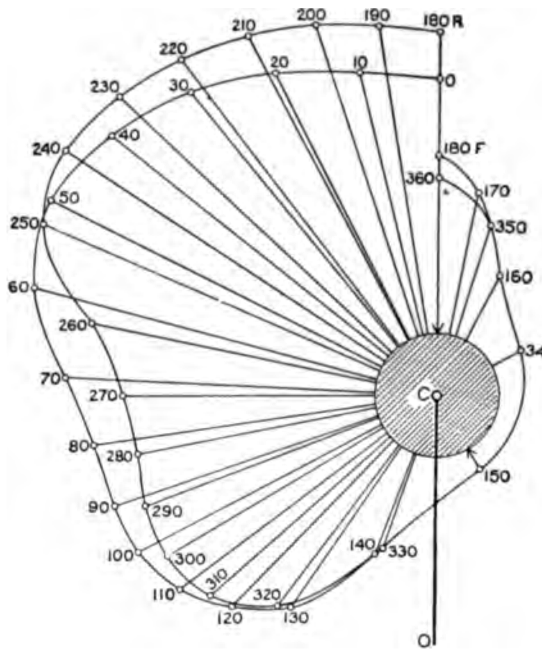


FIG. 159.—Diagram of Pressures on the Crank-pin.

each crank-pin pressure is drawn on its line of action with reference to the crank, and we see the point on the surface of the crank-pin at which the pressure is central for each position of the mechanism. The diagram can be most conveniently drawn by putting the figure of the crank on a sheet of tracing-paper, pivoting it at O on Fig. 157, then bringing C to successive positions, tracing the direction of CD, and measuring back its length. This figure

shows how, when the engine runs in one direction, the pressure and the wear are nearly all on one side of the crank-pin.

The changes in pressure at the dead-center, from 360° to 0°, and from 180 F (forward) to 180 R (return) are not really instantaneous, as shown here and by the dotted lines on Fig. 157; but are made more as indicated by the full-line curves on the latter figure.

(*e*) Guide-bar Pressures.—In Fig. 160, laid out on a stroke-line base, are given curves showing how the guide-reaction varies throughout both strokes. The full-line curve is plotted with FW on Fig. 156 as ordinate; that in dotted line shows Q as complementary to S, as in Fig. 133, got from the same values of P that were used for Fig. 156. The effect of the vertical inertia-component at the wrist-pin (WK on Fig. 150) in tending to lift the

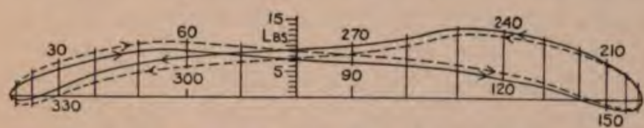


Fig. 160.—Diagram of Guide-reaction.

rod during the forward stroke and to force it down during the return stroke, is clearly shown; the difference between the two curves being simply this vertical inertia, as explained under Fig. 155. This action respectively diminishes and increases the forces which the guide must exert upon the cross-head in the two strokes.

(*f*) Pressures on the Shaft-bearings.—The method of § 32 (*e*) and Fig. 103 III. gives an entirely satisfactory solution to the problem of finding the pressures on the main bearings of an engine: but as there presented it is applicable only to the simple case where all the force-actions are symmetrical with respect to the plane of motion of the crank—that is, to a center-crank engine with equal wheels and with the same load-forces on both wheels. The more general case of the ordinary side-crank engine is partly illustrated in Fig. 161 I.

Considering only th[illegible]ve driving-force P as acting upon the crank-pin, we ha[illegible] force is held in equilibrium by

the two bearing-pressures B_1 and B_2: the important dimensions being the overhang c of the crank-pin beyond the middle of the main bearing O_1, and distance b between the latter and the outboard bearing O_2. Taking the origin of moments at O_2, we get B_1 by the relation

$$B_1 = \frac{b+c}{b}P; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

and since B_1 is the middle one of a set of three parallel forces, we get B_2 as $(B_1 - P)$: or, taking moments about O_1,

$$B_2 = \frac{c}{b}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

The graphical method of determining parallel forces when one out of three is known is also shown on Fig. 161 I. To get B_1 from P, with the center of moments on the line of B_2, we draw any convenient base-line FG across the three force-lines (not necessarily at right angles to them): then on the line of B_1 measure off the length of P at DE, and draw FEH, cutting off the length of B_1, at GH, from the line of P. From the proportion

$$\mathrm{DE} : \mathrm{GH} = \mathrm{DF} : \mathrm{GF},$$

we have

$$P \times (b + c) = B_1 \times b;$$

the interchange of the two forces between their respective lines of action fulfilling the requirement of inverse proportionality to distances from the origin. The third force is given, of course, by subtraction, as KH. To get it primarily, we should use LK as base-line, transfer P to FL, and cut off B_2 at KH.

(*g*) EXTRA PRESSURE ON THE BEARINGS.—The effect of this turning-moment in the plane of the shaft-axis, due to overhang of the cranks, is particularly strong in locomotives which have the connecting-rods outside of the coupling-rods, so that the distance c is long in comparison with b, in Fig. 161 II. When the two P's act in the same direction, the two B's are together equal to their sum, and there is very little extra-pr[illegible] when, as here,

they are opposite, their turning-effects unite to make the bearing-pressures much larger than those on the cranks.

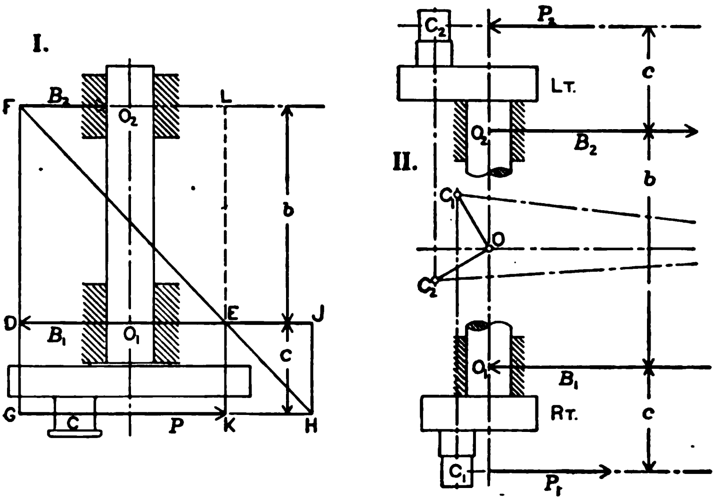


FIG. 161.—Unequal Pressures on the Bearings.

To find B_1 on Fig. 161 II., we take moments about O_2 and get

$$B_1 \times b = P_1 \times (b+c) + (-P_2) \times c;$$

whence

$$B_1 = P_1 + \frac{c}{b}(P_1 - P_2). \quad \text{. . . . . . .} \quad (275)$$

Similarly

$$B_2 = P_2 + \frac{c}{b}(P_2 - P_1);$$

and we see that the extra-pressure, or the excess of each B over the P on its side, is the same for both sides. This excess, $(c/b)(P_1 - P_2)$, is small when P_1 and P_2 are in the same direction, ... when their directions are opposite, so that their algebraic ... their linear sum.

... trates the variation of these forces throughout one ... shaft. The base-line is the developed crank-

circle; and curves I. and II. are plotted for the respective sides from diagrams like Fig. 130 III. Now the difference $(P_1 - P_2)$ is given by the ordinate-length intercepted between these two curves: and to get the extra-pressures we use the reduction-diagram at V., which embodies the ratio $c:b$, and is got by making

$$CB : AB = c : b.$$

Taking any intercept as DE, we measure it off from A along AB; and the vertical at its end, up to AC, is then laid off at DF and EG, so as to get points on the B-curves, III. for B_1, IV. for B_2.

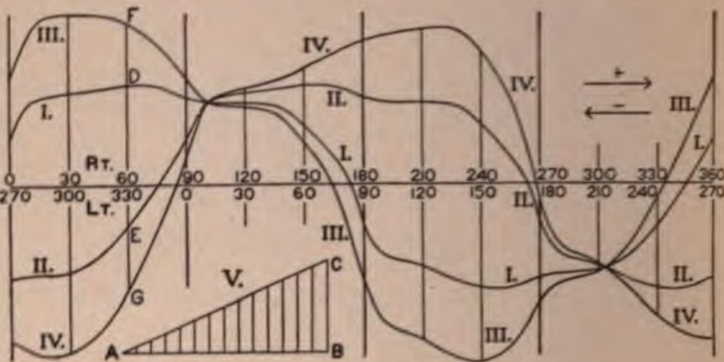


FIG. 162.—Bearing-pressures in a Locomotive.

In this locomotive, the total width $(2c+b)$ is about twice b; so that c/b is one-half, and the maximum bearing-pressures are about twice the corresponding crank-pin pressures. This is an extreme case; and in stationary engines it is likely that the excess will not be more than from 10 to 30 per cent.

(h) DIAGRAMS OF BEARING-PRESSURE.—To illustrate the application of Fig. 103 III., as modified by the conclusions in (f), we apply the force-actions represented in Figs. 132, 157, etc., to the engine-shaft outlined in Fig. 163, belonging to a side-crank engine with the armature of a generator mounted beside the wheel. The load being in the form of a torque only, if the armature is properly centered in the field, the forces to be combined are the crank-pin pressure P, the free counterforce F_B and the weight W. With

the dimensions on the figure, these forces are divided between the bearings in the following proportions:

Force	at O_1	at O_2
P.......	$\frac{73}{59}=1.24$	$\frac{14}{59}=.24\ (-)$
F_B.......	$\frac{69}{59}=1.17$	$\frac{10}{59}=.17\ (-)$
W.......	$\frac{35}{59}=\ .59$	$\frac{24}{59}=.41$

These ratios are got by the method of (273).

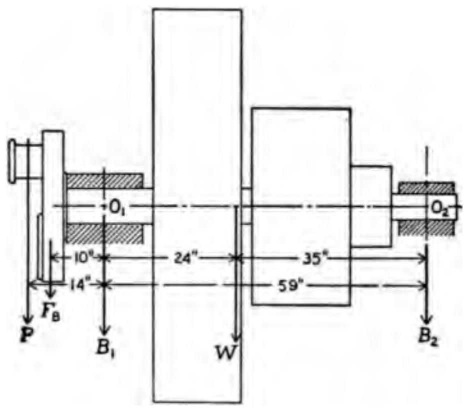


FIG. 163.—Outline of Engine-shaft.

In Fig. 164 is shown a series of force polygons like Fig. 103 III., taken at equal intervals around the crank-circle, and determining the pressure B_1 at the main bearing of Fig. 163. These diagrams, like all the others, show force per square inch of piston. The weight, W/A, is about 15 lbs., of which 9 lbs. are carried at O_1, and 6 lbs. at O_2; F_B/A is 17 lbs., equivalent to $17\times1.17=20$ lbs. at O_1; while P is taken from Fig. 157 and multiplied by 1.24. The resultants give B_1, in the direction of action of shaft on bearing. It will be noted that the pressure on the crank-pin is decidedly the predominating force, and that the prevailing direction of bearing-pressure is nearly along the stroke-line of the engine.

Similar diagrams can easily be drawn for the out-board bearing, keeping in mind the reversed directions of the components of P and F_B at O_2.

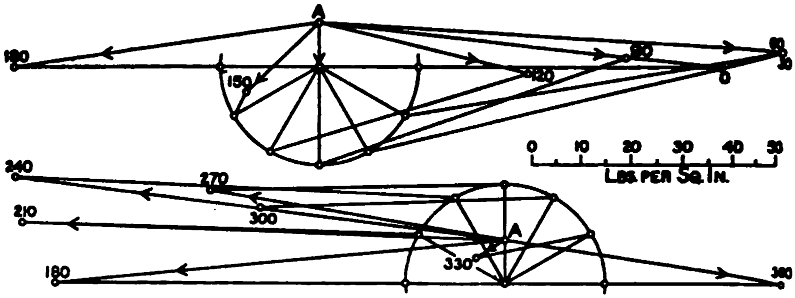


FIG. 164.—Pressures on the Main Bearing.

§ 39. Friction and Efficiency.

(*a*) TWO KINDS OF FRICTION.—In undertaking an analysis of the frictional resistances in the engine, as to their manner of action and as to their amount, we encounter questions of considerable complexity, which cannot be very closely answered. The first is, If we know the normal pressure between two rubbing surfaces, or the force which presses one upon the other, what is the tangential resistance to their relative motion? Determined by the conditions of working, the answer to this question lies somewhere between the two extreme cases which will now be set forth.

On the one hand, the older theory of friction, based chiefly on Morin's results which were published about 1835, makes the resistance to movement a certain fraction of the total normal pressure between the surfaces. This fraction, or the "coefficient of friction," is affected by the material and by the character of the surfaces and by the degree of lubrication; but the resistance is independent of the area of contact, unless the latter is so small as to cause an excessive specific pressure. It is now known that this applies to dry, smooth surfaces under moderate pressure, and to the case of partial lubrication under heavier pressure: but that any particular coefficient is applicable only over a narrow range of conditions, or that the ratio of friction to pressure is a much more variable quantity

than was supposed. Under the conditions just stated, the initial resistance, or the friction of starting, is greater than that which exists after a good velocity of movement has been established; at first, the decrease is rapid with small increments of speed, then the coefficient tends to become constant.

As the result of a later body of experiments, made within a few years of 1880, it was found that with full lubrication—that is, with a film of oil between the metallic surfaces, and separating them so as to prevent actual contact of the solid bodies—the resistance to motion followed laws far different from those before accepted. In this case, the friction depends upon the area of contact, and increases with the velocity; but is only slightly affected by intensity of pressure. For the maintenance of this condition of working, the lubricant must be freely and continually supplied, and must have a "body" or viscosity bearing some relation to the intensity of the bearing-pressure. A thick, sticky oil will carry a far higher specific pressure, but with a correspondingly higher resistance, than will a more fluid lubricant: and it appears that a close adaptation of lubricant to pressure would tend toward a following of the older law.

(*b*) Friction in Machines.—In most machines, the actual relation of friction to working-force lies somewhere between these extremes. To take a common case, the pressure between bearing and axle-journal under railroad cars varies from about 100 lbs. per sq. in. when empty to 400 lbs. when loaded; and it is found that the tractive resistance, on level track, is about 8 lbs. per 2000 lbs. of empty weight, about 4 lbs. per ton of total weight when loaded. The radius of the wheel is about seven times that of the journal: so that the resistances just given are equivalent to 56 lbs. and 28 lbs. per 2000 lbs. of normal pressure, or to a coefficient dropping from .028 for the light load to .014 for the heavy load. But while the ratio becomes smaller, this decrease is not fully complementary to the increase in load, so that the total resistance becomes greater, changing from 2.8 lbs. to 5.6 lbs. per sq. in. of bearing-surface.

It is a pretty well established fact that in a well-made steam-engine the friction-load varies but little, if at all, with the useful load, or with the power developed by the engine. A number of

reasons can be given to account for this apparently anomalous action. With truly-formed surfaces of journals and bearings, of ample size and fully supplied with oil, the conditions are favorable to the existence of simple fluid friction, which is independent of the pressures acting. Further, the forces in the mechanism vary less widely than do the effective driving-forces which work against the load. The degree of tightness of the bearings, necessary to running without lost motion, combined with the capillary action of the oil, may give quite a high oil-pressure and initial resistance, without regard to external forces. If the bearings run at a higher temperature under heavier loads, the tendency to increased friction may be partly balanced by the greater fluidity of the oil. And some of the resistances, as those to the sliding of the piston and piston-rod, and to the movement of the valve, may be very little affected by the change of power.

(c) ESTIMATING THE FRICTION OF AN ENGINE.—A fair idea of the probable frictional resistance in an engine, as far as determinable, may be got by using .02 to .06 as the coefficient of friction for the rod-pins and the bearings, and .08 for the cross-head, with the engine at about its rated power. Thus for the crank-pin of the engine which we have been using as our example, the pressures are given by Fig. 158, but not quite on a basis which fits them for direct application to this case. Approximately, disregarding the secondary effect of the swing of the connecting-rod, these forces are plotted on a base proportional to distance travelled by one rubbing-surface upon the other by laying them out on the developed crank-circle, in Fig. 165—which is very similar to curves I. and II. on Fig. 162.

Measuring the areas under these curves, and counting in the reversed pressures near the end of each half-revolution as if they were positive forces—because friction is not a matter of direction of pressure, but only of its intensity—we get the average force shown by the lines CD, EF: AC, equal to 44.9 lbs. per sq. in. of piston, is less than the M.E.P. of 48.5 lbs.: AE is almost the same.

The crank-pin, 6″ or 6.5″ in diameter in a center-crank engine of this size, will have a circumference of about 20″. With an average pressure of 46.8 lbs. and with .04 as the coefficient of

friction, the tangential resistance at the surface of the pin will be 1.87 lbs. While this travels 20″, the M.E.P. on the piston required to overcome it will travel two strokes, or 30″: so that the latter force will be two-thirds of 1.87, or 1.25 lbs. This would be 2.3 per cent. of the total M.E.P.; and if the total friction were known to require 10 per cent. of the M.E.P. to overcome it, then the crank-pin would be responsible for 23 per cent. of the friction of the engine.

This method, while representing a pleasantly complete theory of the subject, is greatly diminished in practical value by uncertainty as to what coefficient of friction shall be used. Also,

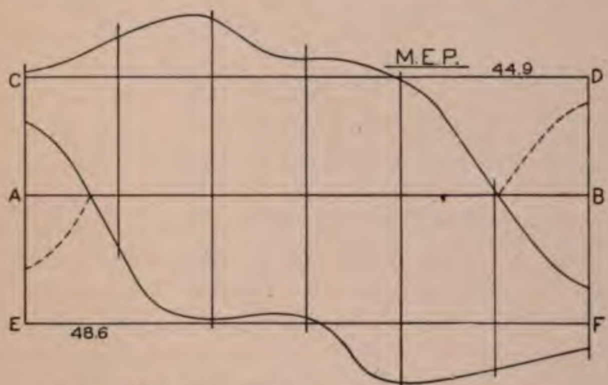


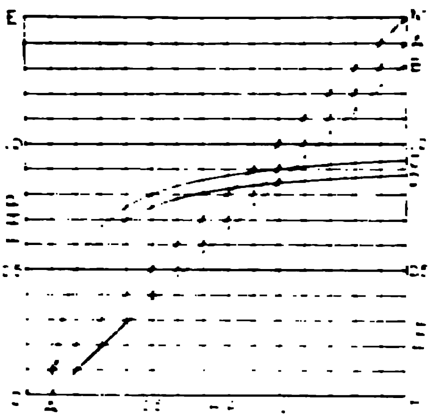
FIG. 165.—Crank-pin Pressures on Distance-base.

some of the resistances, notably the internal sliding frictions, are not determinable to any satisfactory degree. The actual procedure, in designing and building good engines, is to proportion the bearing-surfaces, after strength has been given due consideration, so as to produce specific pressures which experience has shown to be allowable; then make them as true in form as practicable, and provide for free lubrication: and estimate the mechanical efficiency from the performance of other engines of the same sort and under the same conditions.

(d) MECHANICAL EFFICIENCY OF ENGINES.—With small engines, and up to several hundred horse-power, the effective work delivered at the shaft can be measured by substituting a friction-brake, or absorption dy[illegible] the usual load. In many cases.

where the load is directly applied, it can be measured—as by the indicator in pumps and compressors, or by the readings of the electrical instruments when driving a generator. But usually some friction-effect besides that of the engine proper is included in this determination; and with the generator, the electrical loss is added to the friction. In many large engines, where the load is inseparable, it is not possible to measure the friction, even by the expedient of running the engine light (without external load) and taking what is called a "friction-card" with the indicator.

The best method of representing the results of a mechanical efficiency-test is shown in Fig. 166. The base-line is the indicated power, the ordinate the effective power, and the latter is shown here by the line AA or AB. [illegible] power OM is drawn at an angle of 45° [illegible] AA [illegible]



[illegible]

AB [illegible]

[illegible] AA [illegible]

[illegible] constant.)

[illegible] of the rated power [illegible] numerical expression [illegible] speed—is not [illegible] diagram should [illegible] rather than [illegible] force be[illegible]

either M.E.P. on the piston, or mean turning-force on the crank, or tangential resistance at the rim of a wheel, as desired; and values from the indicator card and from the effective load being compared.

As to the efficiency realized in engines, it will be found to vary from 75 to 95 per cent., under a load that is a fair proportion of the rated power; with 90 per cent. as a good value to assume for high-grade engines. Data on this point will be presented and discussed in Part II., in connection with the thermodynamic performance of the engine; and a great deal of information on the subject of friction in general, as well as engine-friction in particular, will be found in any good handbook for mechanical engineers.

§ 40. Counterbalancing.

(a) Diagrams of Shaking-force.—Under the approximation of assuming part of the mass of the connecting-rod to be concentrated at the wrist-pin and the rest at the crank-pin, the moving masses which we have to consider in this connection, and their inertia-forces, are shown in Fig. 167. Here

M_1 = total reciprocating mass, including part of the rod.

M_2 = mass at crank-pin, which may include, besides the rest of the rod, the crank-pin and the crank-arm or hub.

M_3 = mass of counterbalance, here taken at the radius OB, which is not necessarily the same as OC or R.

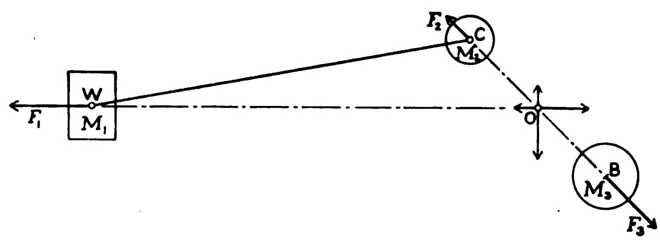


Fig. 167.—Accelerated Masses.

Now F_1 is a component of F_0, the ideal centrifugal force of M_1 according to (222), or

$$F_1 = F_0\left(\cos\alpha + \frac{R}{L}\cos 2\alpha\right).$$

As to F_2 and F_3, their resultant $F_B = F_3 - F_2$ (see Fig. 105) is the free counterforce whose horizontal component partly balances F_1. This component is

$$H = F_B' \cos \alpha.$$

By subtraction, we get the horizontal shaking-force to be

$$S_H = (F_0 - F_B) \cos \alpha + \frac{R}{L} F_0 \cos 2\alpha. \quad . \quad . \quad . \quad (276)$$

A circular diagram which will give S_H for any crank-angle is derived from Fig. 128 by changing the radius of the circle to $(F_0 - F_B)$ without changing the distance of points on the curve E_1E_2 from the line GH. Thus, in Fig. 168 I., the diagram on AB is

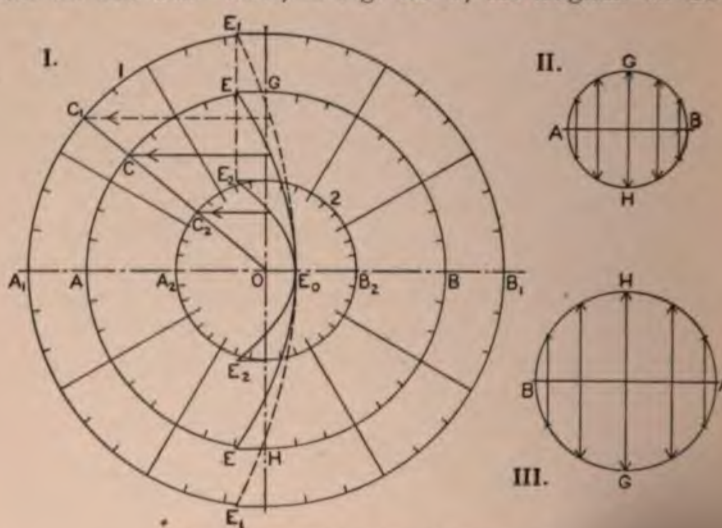


FIG. 168.—Shaking-force Components.

drawn with F_0 as radius, and gives F_1; which would be identical with S_H if M_3 were made equivalent to M_1, or F_3 equal [illegible]— that is, if the rotating masses on the crank were brought to [illegible] balance. If F_3 were absent, F_B would be a negative qua[illegible] we should use the radius $OA_1 = F$[illegible] F_2: with this cir[illegible] the E-curve is stretched out vertic[illegible] $E_1E_0E_1$. Diag[illegible] is drawn for $F_B = \frac{1}{2}F_0$ and the E-cu[illegible]w squeezed [illegible]

The amount of counterweight necessary to bring the crank-disk, with its attached rotating mass (part of the connecting-rod), to a state of centrifugal balance about O is well called the dead counterbalance: and the excess of F_3 over F_2 is then the free counter-force.

Similar diagrams for the vertical shaking-force,

$$S_V = -F_B \sin \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (277)$$

are given at II. and III. In the first, for no counterweight at all, the minus sign is neutralized by that of F_B, and the angle-scale has its zero-point A at the left, as usual: but where F_B has a positive value, we must measure α from the other dead-center in order to get S_V, in direction as well as intensity, by direct measurement from the figure.

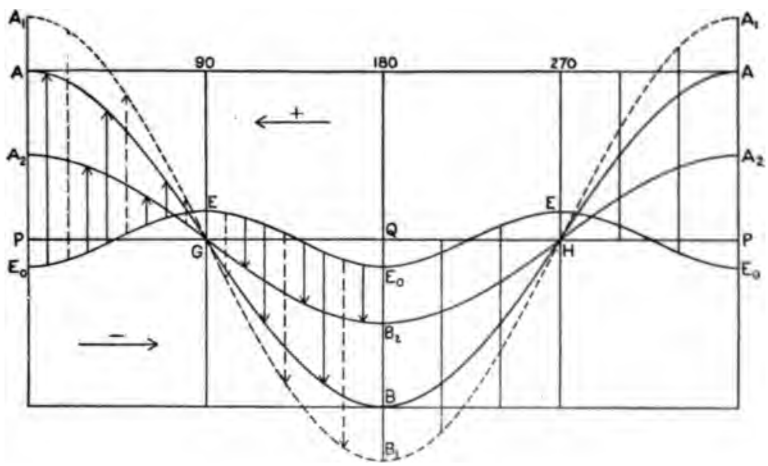


FIG. 169.—Shaking-force Curves.

Diagrams on the developed crank-circle are given in Fig. 169. Here the first term of (276), $(F_0 - F_B) \cos \alpha$, is laid off from the base PQP in one direction, in a simple sine-curve for each value of $(F_0 - F_B)$; and the second term, $\frac{R}{L} F_0 \cos 2\alpha$, is measured from PQP in the opposite direction, in a sine-curve of half the principal period. Then S_H is given by the ordinate measured from this

E-curve to the particular AB-curve, as indicated by the arrow-heads. It is made apparent that with a high degree of balancing—with F_B a large fraction of F_0—the effect of the connecting-rod, as shown by curve E_0E, becomes of greater relative importance.

(b) THE POLAR DIAGRAM.—Using the exact determination of the rod-force, a good graphical representation of the combined inertia-effect of the moving masses is got by the method of Fig. 170.

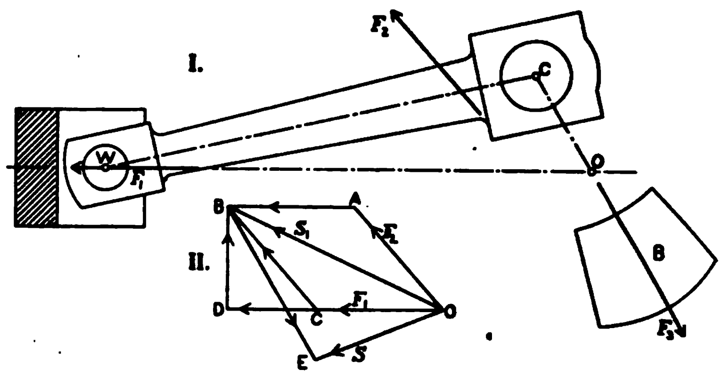


FIG. 170.—Exact Construction for Shaking-force.

The forces are shown on I., and the manner of combination in II. Having F_2 determined on Fig. 152 and F_1 by Fig. 128 or by Eq. (222), we combine them along either OCB or OAB in II., as convenient; or it may be easier to transfer F_2 by means of its components, getting B by laying off OD and DB. In any case, the resultant OB is the shaking-force without any free counter-balance; and after BE has been laid off equal and parallel to F_B, the final resultant is OE.

In Fig. 171, this method is applied to our 14×15 engine, with $F_{10}/A = 27$ lbs., $F_{20}/A = 18$ lbs., for the slide and the connecting-rod respectively, as in § 38 (c). The complete construction, just like OABE in Fig. 170 II., is shown at 30°. The radial rod-force F_2 was determined in the construction for Fig. 157; while F_1 is got from the usual circular diagram. Curve I. corresponds with diagram No. 1 on Fig. 168 I.; and its vector-ordinates are very nearly the same as the resultants of horizontal components from

this circle No. 1 and vertical components from Fig. 168 II. For curve II. the same counterforce is used as for diagram No. 2 of Fig. 168 I.

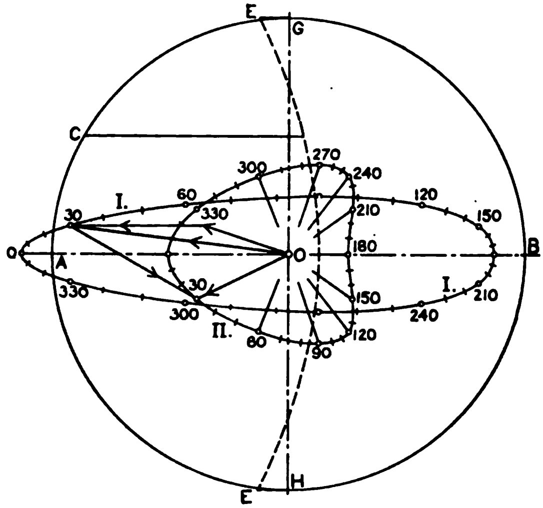


FIG. 171.—Polar Diagrams.

(c) DIVISION OF THE ROD-MASS.—Figs. 168 and 171 are drawn for the same fundamental force-values, with the purpose of comparing the approximate and the exact method of determining the inertia-force of the rod. Inspection of Figs. 149 and 150, and especially the location of the E-points on Fig. 152, shows that for combination with the other inertia-forces, more than half of the rod should be brought to the crank-pin. In the figures under discussion, about five-eighths of the rod is at C, three-eighths at W: that is, with symbols having the same meaning as on Fig. 167, F_{10} is 34 lbs. and F_{20} is 11 lbs. Then for curve II. of Fig. 171, F_3 is 28 lbs.; while the radii of the circles in Fig. 168 II. and III. are 11 lbs. and 17 lbs. respectively.

To compare the two methods, the abscissas of points on curve I., Fig. 171, or their distances from GH, are measured off from the corresponding crank-pin positions on the circle: and the resulting E-curve, drawn through th[illegible] ends, is practically indistinguishable from [illegible] I.

It appears then that a good general rule for the division of the rod-mass is:

To find turning-force and pin-pressures, put half the mass of the rod at each pin—as already stated in § 37 (*b*).

To find shaking-effect, put five-eighths of the rod at the crank-pin, and add the rest to the slide-mass.

(*d*) THE SIDE-CRANK, DUPLEX ENGINE.—The preceding development of the subject of shaking-force is full and complete for the single center-crank engine, where the counterbalance on the two crank-disks is symmetrically placed, so that it can be represented by a single mass right in the plane of the crank. We will now extend the discussion to cover the case of a non-symmetrical balance and of two engines working on one shaft.

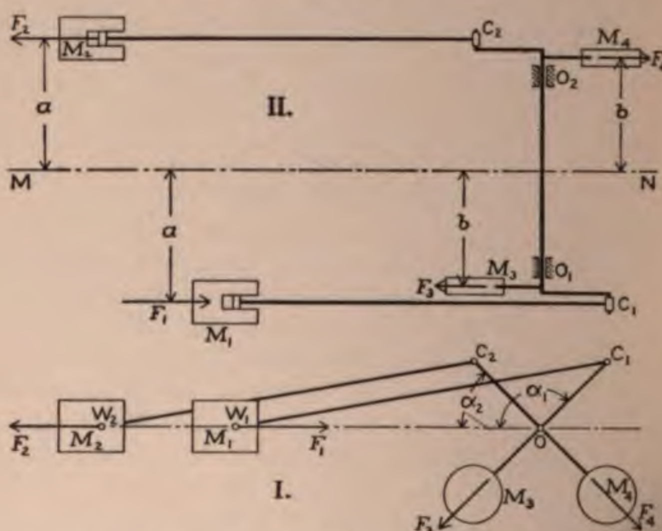


FIG. 172.—Inertia-force Outline of the Duplex Quarter-crank Engine.

The mechanism of the double engine with cranks at right angles is outlined in Fig. 172: and we wish to determine first the combined effect of the two slide-inertias F_1 and F_2, and then to see how the action of the balance-weights is modified by the fact that they are nearer the center-line MN than are the slides. It will be

noted that only the free or effective counterweight is here shown.

The principle of the double effect of a non-central force—given in Fig. 103 II., and extended under Fig. 147—is the foundation of this discussion. Viewing the plan of the engine, Fig. 172 II., we see that none of the forces under consideration acts upon the center of mass, which is on the line MN: then each of these forces, as F_1, will have two tendencies; first, to give the center of the whole engine-mass a direct acceleration, as though the forces were along the line MN, or in the vertical central plane; second, to give an angular acceleration about this center, exerting the moment $F_1 a$.

To get the combined shifting tendency of the slide-inertias, we simply consider the plane of the elevation, Fig. 172 I., to be the center-plane, imagining the two forces F_1 and F_2 to be transferred or projected to the center-line MN. Then we have only to take their algebraic sum to get the total shifting, or direct-accelerating, force.

Taking the right-hand as the principal or leading crank, whose angle determines the position of the mechanism, and considering the inertia-force positive toward the left, we have

$$F_1 = F_0\left(\cos \alpha_1 + \frac{R}{L} \cos 2\alpha_1\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

$$F_2 = F_0\left[\cos (\alpha_1 - 90) + \frac{R}{L} \cos (2\alpha_1 - 180)\right]$$

$$= F_0\left(\sin \alpha_1 - \frac{R}{L} \cos 2\alpha_1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

Adding these and dropping the subscript from α_1, we get, for the total force,

$$S_H = F_0(\cos \alpha + \sin \alpha), \quad . \quad . \quad . \quad . \quad . \quad (280)$$

the two rod-effects neutralizing each other.

(e) Graphical Determination of Shifting-force. — Now just as the principal part of F_1 and of F_2—respectively $F_0 \cos \alpha_1$ and F_0 [illegible] α_1—is the horizontal component of an ideal radial force, so als[illegible] sum of these two, $F_0(\cos \alpha + \sin \alpha)$, the horizontal comp[illegible] single radial force, which force is the resultant of

the two F_0's. This is proven in Fig. 173, where OC_1 and OC_2 are the respective F_0's, and OC is their resultant: then DOC_1 is α, and

$$OD_1 = F_0 \cos\alpha, \qquad OD_2 = F_0 \sin\alpha;$$

also

$$OD_1 = C_2F = D_2D:$$

wherefore

$$OD = F_0(\cos\alpha + \sin\alpha) = S_H;$$

and OD is the horizontal component of OC. This OC is $\sqrt{2} \times F_0$ or $1.414F_0$, and the value of S is therefore

$$S_H = 1.414F_0 \cos(\alpha - 45°). \quad . \quad . \quad . \quad . \quad . \quad (281)$$

The two counterforces, shifted to the center-plane and represented by OB_1 and OB_2 in Fig. 173 II., can likewise be replaced by a single resultant OB, whose horizontal component OE will oppose OD, while the vertical component EB will be a free shaking-force.

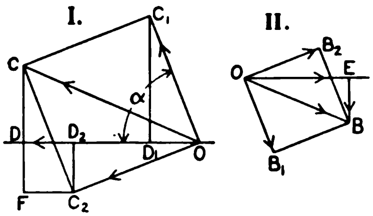


FIG. 173.—Combined Radial Effects.

The results of this investigation are expressed in complete graphical form by Fig. 174. The circle in I. is drawn with 1.414 $(F_0 - F_B)$ as radius, F_B standing for either F_3 or F_4 on Fig. 172. The zero of the angle-scale is located at the position which OC,

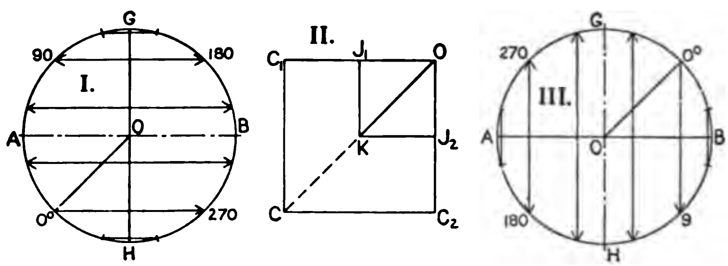


FIG. 174.—Diagrams of Shifting-force.

Fig. 173, will occupy when α for crank No. 1 is zero, or when OC_1 is on its zero dead-center: and to get S_H we locate the actual value of α on this angle-scale and measure over from the circle to GH.

In Fig. 174 II., the diminution of OC_1 and OC_2 by C_1J_1 and C_2J_2, each equal to F_B, is shown with the purpose of giving a clear idea of the size of the primary forces. Then the resultant OK is the radius in I.; while CK, the same as OB on Fig. 173 II., is used as radius in III. This second circle shows the vertical shaking-force S_V in the same way that I. gives S_H; its zero of angle being diametrically opposite to that in I.

(*f*) TURNING-EFFECT OF THE COMBINED INERTIAS.—Referring to Fig. 172 II., we note that F_1 and F_2 both act at the end of the lever-arm a; and that when they point in the same absolute direction their moments oppose each other. The best way to combine them is to imagine F_2 to be swung, at the end of its radius a, through 180° about some point on MN, until it comes into the line of F_1: then the algebraic sum of the two forces in this position—which is the same as their algebraic difference with F_2 in its actual direction—is a resultant free force acting at the end of the radius a to give the engine an angular acceleration about a vertical axis through the center of mass. Calling this turning or twisting force T_H, we have, without counterbalance, from (278) and (279),

$$T_H = F_1 - F_2 = F_0(\cos\alpha - \sin\alpha) + 2\frac{R}{L} F_0 \cos 2\alpha, \quad . \quad (282)$$

the two rod-effects acting together in this combination.

In Fig. 175, the reversal of F_2 in bringing it into the line of F_1 is represented by the reversal of the F_0 for the left side, from OC_2 to OC_3: then the major part of the force T_H is the horizontal component of OC: for

$$OD = OD_1 - DD_1 = OD_1 - D_3O = F_0(\cos\alpha - \sin\alpha).$$

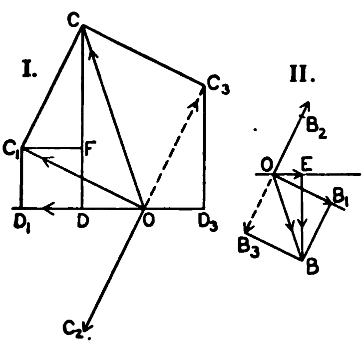


FIG. 175.—Radial Twisting Effects.

The two counterforces are combined in the same way at II., after reversing that for the left side, just as in Fig. 173 II.: except that a reduced value of F_3 and F_4, or of F_B, must be used.

That is, F_B at the radius b will be represented by a smaller force F_B' at the radius a, according to the equation

$$F_B b = F_B' a,$$

or

$$F_B' = \frac{b}{a} F_B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (283)$$

This reduced force is laid off as OB_1 and OB_2 in II., b/a having the value 0.8.

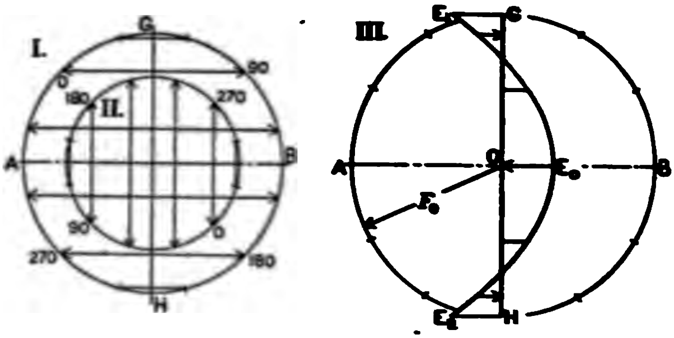


FIG. 176.—Diagrams of Twisting-force.

In Fig. 176, diagrams like I. and III. of Fig. 174 are drawn together, about the same center. The radius of circle I. is the difference between OC and OB of Fig. 175, or it is their algebraic sum, having the value

$$T_0 = 1.414\left(F_0 - \frac{b}{a} F_B\right); \quad . \quad . \quad . \quad . \quad . \quad (284)$$

and the horizontal ordinate to GH shows what T_H would be with infinite connecting-rod. Calling this T, its value is, as above,

$$T = (F_0 - F_B')(\cos \alpha - \sin \alpha). \quad . \quad . \quad . \quad . \quad . \quad . \quad (285)$$

The radius of circle II. is 1.414 F_B', and its ordinate from AB is a free force which, acting at the end of the lever arm, tends to rock the engine on its foundation, or to give it an oscillation about the center-line MN of Fig. 172.

Diagram III. on Fig. 176 shows the double rod-effect,

$$t = 2\frac{R}{L} F_0 \cos 2\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (286)$$

It is an inertia-diagram of the usual sort: the radius, here F_0, can really have any length that may be convenient, provided that the distances GE_1, OE_0, and HE_2 are each equal to $2nF_0$, where n stands for R/L. Intercepts between the E-curve and GH give values of t.

(*g*) DEVELOPED DIAGRAMS FOR THE DUPLEX ENGINE.—The forces shown in Figs. 174 and 176 are represented in Fig. 177 by curves on the developed crank-circle, similar to those in Fig. 169.

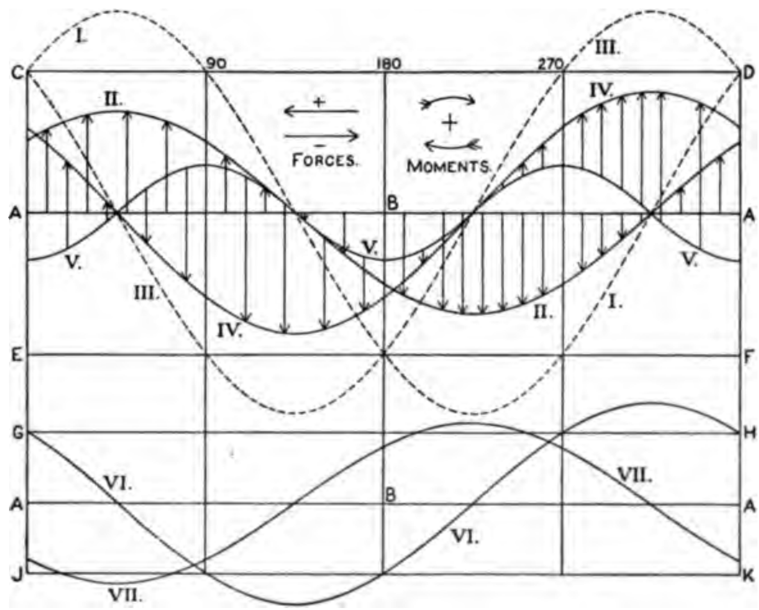


FIG. 177.—Curves from Figs. 174 and 176.

By combining all these curves on one base, we get a better idea of the magnitude of the force-actions, and especially of their relative periodicity: and where separate components of a force are given by the circular diagrams, as in I. and III. of Fig. 176, it is only by

drawing resultant curves that we can clearly see the manner of variation of the whole force.

Curve I. shows the full shifting-force of the two slides, and would be got by using the whole radius OC from Fig. 173 for a circle like Fig. 174 I.; and II. shows the actual S_H, after this radius has been shortened by the resultant counterforce to the length used in Fig. 174 I.

Similarly, III. shows the undiminished twisting force due to the two slides; while IV. is laid out from Fig. 176 I. Curve V. represents the rod-effect t, its ordinates being taken from Fig. 176 III., and measured off in a direction opposite to that used for all the other curves, in order that the total twisting-force, $T_H = T - t$, may be given directly by the intercepts between IV. and V. Values of S_H and T_H are indicated by alternate force-ordinates; and the most noteworthy fact is the marked influence of the rod-effect in modifying the symmetrical, sinusoidal force-action shown by IV.

The vertical actions are plotted from the lower base-line, curve VI. from Fig. 174 III. to show S_V, VII. from Fig. 176 II. for T_V.

(*h*) DETERMINATION OF THE ROD-EFFECT.—The method of the radial resultant, developed from Fig. 172 and applied in the set of diagrams just discussed, is very useful and convenient when we wish to determine the action of the shaking-forces in a complex engine. But to make it complete, we must have an equally simple and general way of getting the resultant of the several rod-effects. Of the inertia-force of the sliding mass, the component which is due to the rod-action has the value

$$f = \frac{R}{L} F_0 \cos 2\alpha = nF_0 \cos 2\alpha : \quad . \quad . \quad . \quad . \quad (287)$$

and in Fig. 178 is evolved a simple graphical method of finding this f for any crank-angle.

In Fig. 178 I., F_0 has the usual meaning, while f_0 is a similar ideal radial centrifugal force of the value $f_0 = nF_0$, so that (287) becomes

$$f = f_0 \cos 2\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (288)$$

The form of this equation suggests at once that f, being a sine-function, can be shown by a circular diagram of t[illegible]e that we

have been using right along for $F = F_0 \cos \alpha$; but we see that the determining point on the circle, or the radius f_0, will have to rotate twice as fast as the crank. The detailed illustration in I. should be self-explanatory, showing how f_0 makes a complete rotation while F_0 turns through 180°. At 0°, f_0, which really lies along F_0, is also shown a little to one side of F_0, for clearness. The circle

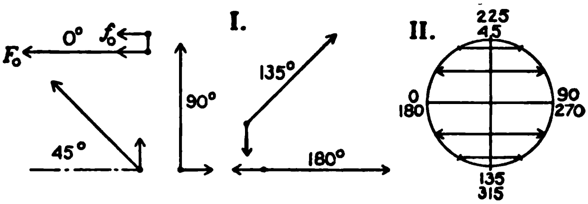


FIG. 178.—Circular Diagram of Rod-effect.

in II. is drawn with f_0, enlarged, as radius: and the use of the double angle-scale, going twice around the circle, is self-evident. When the crank is at any angle α, the end of f_0 is at the correspondingly numbered point on the scale: and the distance to the vertical diameter is f.

Applying this method to our first case of the complex engine, we represent in Fig. 179 I. the assumption that the cranks are together at the zero dead-center; under which condition all the ideal radial forces will point in the same direction. Now when crank No. 2 is turned backward through 90° in order to realize the actual arrangement, f_{20} will turn twice as far as F_{20}; and we see, in II., that the two rod-effects will oppose each other as regards shifting tendency. But when we reverse F_{20} by the device of turning it bodily into the plane of F_{10}, we likewise reverse f_{20}; and then, as shown at III., the rod-effects combine.

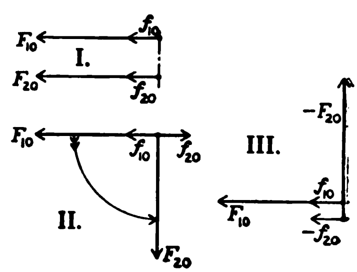


FIG. 179.—Rod-effect in the Quarter-crank Engine.

In comparison with the trigonometrical method of Eqs. (280) and (282), this has the advantage of avoiding complicated formulas when the angles between the

cranks are not quadrants; and it also obviates the need of interpreting the direction-meaning of the algebraic sign of the angle-functions—which is likely to be mentally confusing.

It must be clearly understood that these radial forces are only ideal, as are the F_0's from which they are derived: and that only their components along the axis or stroke-line of the engine are actual forces.

(*i*) DUPLEX ENGINE WITH CRANKS OPPOSITE. — Considering now this second case of the engine with two cranks, we have the conditions as to shifting-force represented in Fig. 180 I. In rotating F_{20} through 180°, from the zero-position as shown in Fig. 179 I., we turn f_{20} through 360°: then the main effects are self-balanced, and only the double rod-effect is active in this respect. Reversing F_{20}, in II., we see that the two slide-masses work together to produce angular shake; so that, unless the cylinders are so close together as to give this force T a very short lever-arm, counter-balancing cannot well be omitted. The rod-effects, however, neutralize each other in this action.

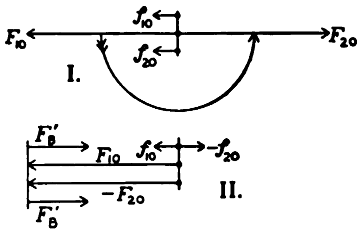


FIG. 180.—Radial Analysis of the Engine with Two Cranks at 180°.

(*j*) THE THREE-CRANK ENGINE.—The typical case of a three-cylinder triple-expansion engine, with cranks at intervals of 120°, is outlined in Fig. 181. Engines of this type are usually vertical, especially in marine service, where they are most common. To give the problem its full complexity, the weights of the several slides are taken to be different: for while the external reciprocating parts are generally made alike, the weight of the piston will increase with its diameter.

In Fig, 182 I., the three F_0's are combined after the manner of Fig. 173 I., to get their total shifting-effect: the resultant of OC_2 and OC_3 is found to be OC_4; and when this is combined with OC_1 the final resultant is OC_5. If the original radial inertias had been equal, their resultant would be zero; and the engine would then be self-balanced against the main shifting-tendency.

The angular distances of OC_2 and OC_3 from the zero-position are indicated by the arrows on I.; doubling these angles in II., we

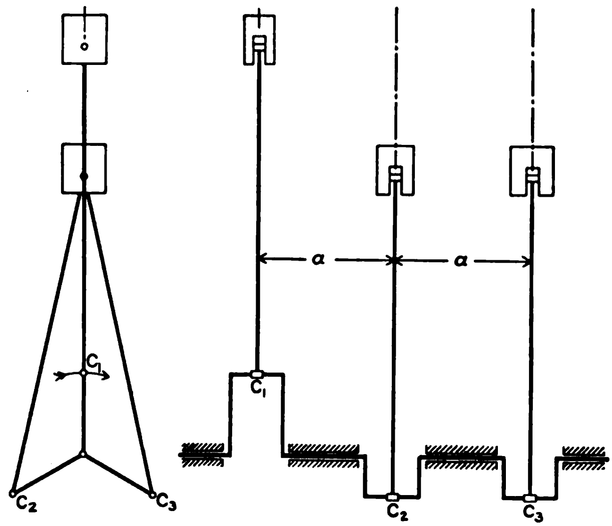


FIG. 181.—Outline of a Triple-expansion Marine Engine.

get the arrangement of the radial f_0's to be as there indicated by numbers. Combining these three forces in a polygon, at III.,

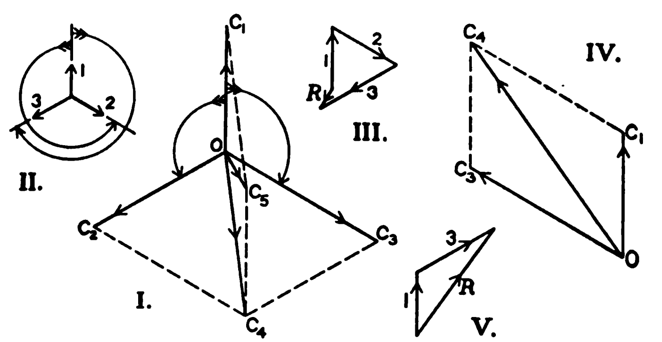


FIG. 182.—Radial Analysis of Fig. 181.

the short resultant R is determined. With equal f_0's, this R would be zero: and we see that an entirely symmetrical engine with this arrangement of cranks is completely self-balanced against shifting.

The middle slide, No. 2, exerts no turning-effect, since its line of motion goes through the center of mass of the engine. Bringing F_{30} into the plane of F_{10}, we combine OC_1 and OC_3 reversed in Fig. 182 IV., and get OC_4 as the radial resultant. Similarly, reversing f_{30} and combining it with f_{10} at V., we determine the radial force R for the twisting-effect due to the combined rod-actions.

In Fig. 181, the ratio of rod to crank is 4, the rod being usually shorter in marine than in stationary engines. This increases the amount of the rod-effects; but the latter are exaggerated in Fig. 182 III. and V., where the forces are laid out double the true size as shown in II.

Having the four resultants, OC_5 in I., OC_4 in IV., R_{III} and R_V, we can draw circular diagrams like Figs. 173 and 175 and Fig. 178 II., and from these plot curves as in Fig. 177 to get total effects in each action. Introducing equal counterbalances will change OC_5 in I., and will materially shorten OC_4 in II. An investigation into the effect of any particular arrangement or proportions of the balance-weights can be made very easily by this radial method.

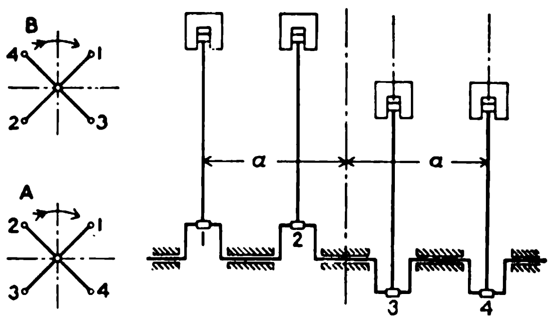


FIG. 183.—Skeleton of a Four-crank Engine.

(*k*) WHEN THE ENGINE HAS FOUR CRANKS, it is most symmetrical to have them spaced at quadrant-intervals around the circle. In the matter of sequence along the axis, two arrangements are shown in Fig. 183: at A and in the main view, any pair of successive cranks is quartered; while in B the outside pairs are opposite, with Nos. 2 and 3 at 90°.

Radial determinations for the first case are made in Fig. 184.

In I. the four F_0's are laid out, and it appears that the principal shifting tendencies are self-balanced—which is true, further, no matter what the order of sequence. The rod-effects, shown at II. and located after the manner of Figs. 179 and 180, also have a zero resultant.

To get the turning or tipping effect—in this case tending to give the engine a fore-and-aft oscillation in a vertical plane—we

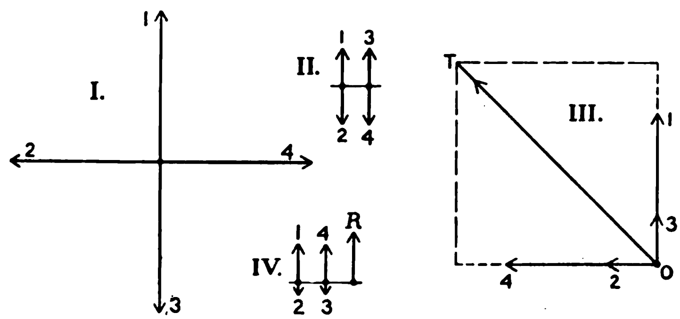


FIG. 184.—Diagrams for Case A.

reduce or reverse all the forces into the stroke-line of slide No. 1: then 2 and 3, having a lever-arm equal to only one-third of a, will be represented by forces of one-third their actual length. The combination is shown at III., and the resultant has the value

$$OT = 1.414 \times \tfrac{4}{3} F_0 = 1.885 F_0. \quad . \quad . \quad . \quad . \quad . \qquad (289)$$

Reversing f_{30} and f_{40} from II., and using one-third of f_{20} and f_{30}, we get the combined rod-effect to be as in IV., with the resultant equal to four-thirds of f_0.

The second arrangement, Fig. 185, gives a better action. As before, the engine is completely self-balanced against shifting. The principal radial turning-force, OT in III., is only half as great as in Case A; but the rod-effects are now all in the same direction, and their resultant is eight-thirds of f_0.

Case C, not shown, would have the cranks in the order 1, 2, 4, 3; or the outer pairs quartered, the middle one at 180°. This will give a principal turning resultant the same as OT in Fig. 184:

but the resultant rod-effect is zero for turning as well as for shifting. This arrangement is better than A, but not quite so good as B.

The vertical or axial components of OT and of R on Fig. 185 are plotted on Fig. 186, in curves I. and II., with the rod-effect reversed

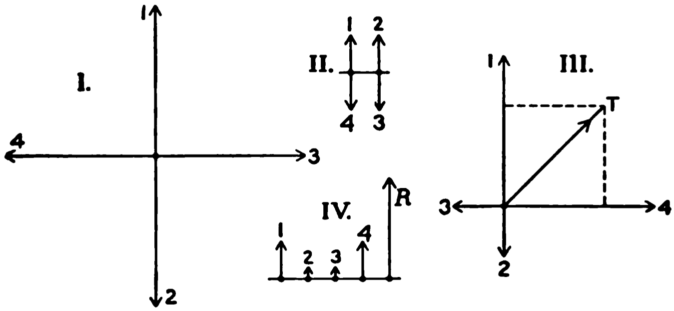


FIG. 185.—Diagrams for Case B.

in direction, as on Fig. 177, so that the total force is given by the ordinate intercepted between II. and I. This resultant is then laid off from the base-line, or its diagram rectified, and we have the shaking-effect represented by curve III.

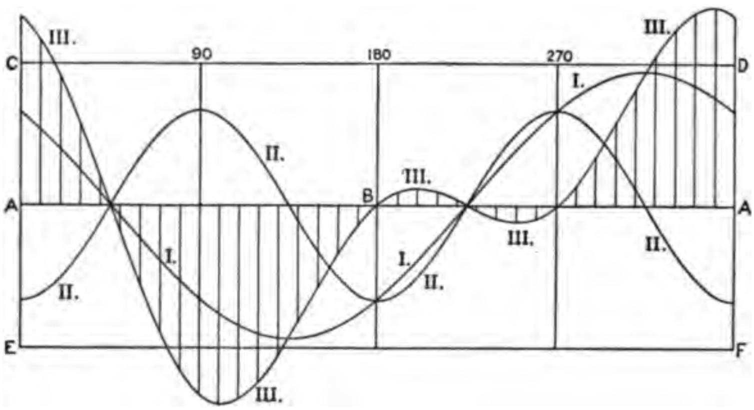


FIG. 186.—Curves for Case B.

It will be noted that curves I. and II. in this figure are related as to period just like IV. and V. on Fig. 177: but with the rod-effect of greater relative magnitude, we have two small extra phases of

force variation, here in the third quadrant. On this figure, the lines CD and EF are drawn at the distance F_0 from ABA. Comparing the other two crank-arrangements, we see that Case C would give a curve like I., but with its ordinates twice as great; while Case A would have this same double-size curve for the main effect, combined with a rod-effect curve of half the amplitude of II.

(*l*) Complex Engine with two Stroke-lines.—Engines are sometimes built with two or three cylinders disposed radially around the shaft, and all acting upon one crank. As to turning-force upon the crank, this is equivalent to a common engine with two or three cranks. The balancing of the most important example of this type is as shown in Fig. 187: and it appears from the diagram at II. that a counterforce F_B equal to F_0 will completely balance the principal part of the two slide-inertias, leaving only the rod-effects free. This gives about the most perfect balancing attainable in a reciprocating engine: and it seems strange that an engine on this principle, with two pairs of cylinders working on quartered cranks, has not been extensively used in torpedo-boats, where vibration from the engine has caused the greatest trouble.

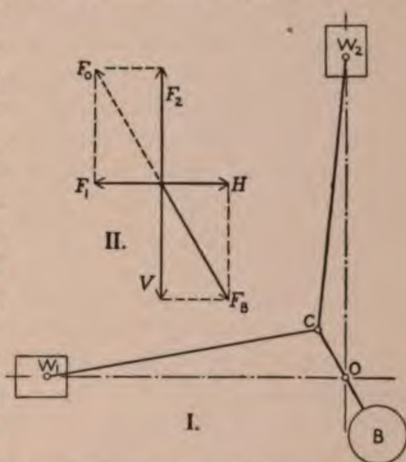


Fig. 187.—Engine with Cylinders at Right Angles.

(*m*) Effects of the Shaking-force.—It is far easier to determine the unbalanced inertia-force of the moving-parts of an engine than to predict what effect this force will have in producing undesired motion of the machine as a whole: in fact, an answer to this latter question can be given only in general terms.

If the engine were supported in such a way that it could move freely in any direction, then the recoil of the moving-parts would give to it a motion similar to that of these internally-moving bodies,

but smaller in the inverse ratio of the masses. Thus if an engine weighed 10,000 lbs. and the unbalanced portion of the reciprocating-parts 400 lbs., the whole machine would have a reciprocating movement one-fiftieth of the stroke in length.

This tendency to motion is resisted by the foundation; or, in other words, the amplitude of the movement is diminished by greatly increasing the "fixed" mass. With earth-borne foundations the shaking-effect may be a mere tremor, which only the delicate instruments used in earthquake observations would detect—a great deal depending upon the character and structure of the earth or rock upon which the foundation rests.

With a more elastic support, as in a ship or, occasionally, on the floor of a building, a large part of the structure will take up the motion of the engine-bed, and the highest attainable degree of internal balancing becomes desirable. The worst disturbance is produced when the period of variation of the shaking-force coincides with that of the elastic vibration of the structure. The latter is something that can be found only by trial.

In the preceding discussion of the combined effects in a complex engine, it is assumed that the engine is so compact and rigid that the several forces can be truly represented, as to their external effect, by a single resultant. Where the several parts are connected only by the foundations, as in many stationary engines of the "spread-out" type of construction, the separate force-actions in the respective single engines are of greater interest than the combined effect. But engines in which the inertia-forces are large —high-speed engines of any class—are usually of the close-constructed, self-contained type.

To show the perfection of balance in a high-speed engine, it is sometimes run on exhibition without being bolted down, or even resting on smooth blocks. Stability under such conditions depends simply on the answer to the question whether the greatest shaking-force is less than the frictional resistance to movement on the support.

[illegible] The C[illegible].—The construction [illegible] of attachment of the balance-weights will be [illegible] chapter. But at this point it seems well [illegible]

the fact, as best shown by Eq. (226), that the centrifugal force-value of any mass varies directly as its radius: so that for a given force, the mass is inversely as the radius. The problem of securing a certain relation of the forces acting from the shaft-center is equivalent to that of getting a certain gravity-balance of weights hung on the horizontal crank-line. And a mechanical method of adjusting the balance in locomotive driving-wheels, sometimes used, consists in hanging on the crank-pin a weight equal to the fraction of the reciprocating-parts that is to be balanced, and then putting into the counterweight enough lead to give equilibrium about the center-line of the axle. Usually, however, the dimensions of the counterbalance are determined in the drafting-room, when the engine is designed; choice being made, as convenient, between a large weight at a short radius or a small weight at a longer radius.

APPENDIX.

TABLES FOR REFERENCE.

TABLE I.

CONSTANTS FOR THE CURVE $pv=C$.

REFERENCES, § 7 (*e*), Fig. 18; § 17, Fig. 29.

Col. 1. r = ratio of expansion $= v_2/v_1 = p_1/p_2$ in Fig. 18.

Col. 2. $1/r$ = fraction of cut-off $= v_1/v_2$ in Fig. 29.

Col. 3. $\log_e r$ = hyperbolic logarithm of r = 2.3026 times common log.

Col. 4. This gives the ratio of the mean (total) pressure to the initial pressure in a diagram like Fig. 29: see Eq. (88).

1	2	3	4	1	2	3	4
r	$\frac{1}{r}$	$\log_e r$	$\frac{1+\log_e r}{r}$	r	$\frac{1}{r}$	$\log_e r$	$\frac{1+\log_e r}{r}$
1.05	.9524	.0488	.9989	2.55	.3922	.9361	.7593
1.10	.9091	.0953	.9957	2.60	.3846	.9555	.7521
1.15	.8696	.1398	.9911	2.65	.3774	.9746	.7451
1.20	.8333	.1823	.9853	2.70	.3704	.9933	.7383
1.25	.8000	.2231	.9785	2.75	.3636	1.0116	.7315
1.30	.7692	.2624	.9711	2.80	.3571	1.0296	.7249
1.35	.7407	.3001	.9630	2.85	.3509	1.0473	.7184
1.40	.7143	.3365	.9546	2.90	.3448	1.0647	.7120
1.45	.6897	.3716	.9459	2.95	.3390	1.0818	.7057
1.50	.6667	.4055	.9370	3.00	.3333	1.0986	.6995
1.55	.6452	.4383	.9279	3.05	.3279	1.1151	.6934
1.60	.6250	.4700	.9188	3.10	.3226	1.1314	.6875
1.65	.6061	.5008	.9096	3.15	.3175	1.1474	.6817
1.70	.5882	.5306	.9004	3.20	.3125	1.1632	.6760
1.75	.5714	.5596	.8912	3.25	.3077	1.1787	.6704
1.80	.5556	.5878	.8821	3.30	.3030	1.1939	.6648
1.85	.5405	.6152	.8731	3.35	.2985	1.2090	.6594
1.90	.5263	.6419	.8642	3.40	.2941	1.2238	.6541
1.95	.5129	.6678	.8553	3.45	.2899	1.2384	.6488
2.00	.5000	.6931	.8466	3.50	.2857	1.2528	.6437
2.05	.4878	.7178	.8380	3.55	.2817	1.2669	.6386
2.10	.4762	.7419	.8295	3.60	.2778	1.2809	.6336
2.15	.4651	.7655	.8212	3.65	.2740	1.2947	.6287
2.20	.4545	.7885	.8130	3.70	.2703	1.3083	.6239
2.25	.4444	.8109	.8048	3.75	.2667	1.3218	.6191
2.30	.4348	.8329	.7969	3.80	.2633	1.3350	.6145
2.35	.4255	.8544	.7891	3.85	.2597	1.3481	.6099
2.40	.4167	.8755	.7815	3.90	.2564	1.3610	.6054
2.45	.4082	.8961	.7739	3.95	.2532	1.3737	.6010
2.50	.4000	.9163	.7665	4.00	.2500	1.3863	.5966

TABLE I. CONSTANTS FOR THE CURVE $pv=C$—*Continued.*

1 r	2 $\frac{1}{r}$	3 $\log_e r$	4 $\frac{1+\log_e r}{r}$	1 r	2 $\frac{1}{r}$	3 $\log_e r$	4 $\frac{1+\log_e r}{r}$
4.05	.2469	1.3987	.5923	6.60	.1515	1.8871	.4374
4.10	.2439	1.4110	.5881	6.70	.1493	1.9021	.4331
4.15	.2410	1.4231	.5839	6.80	.1471	1.9169	.4290
4.20	.2381	1.4351	.5798	6.90	.1449	1.9315	.4249
4.25	.2353	1.4469	.5758	7.00	.1429	1.9459	.4208
4.30	.2326	1.4586	.5718	7.10	.1408	1.9601	.4169
4.35	.2299	1.4702	.5679	7.20	.1389	1.9741	.4131
4.40	.2273	1.4816	.5640	7.30	.1370	1.9879	.4093
4.45	.2247	1.4929	.5602	7.40	.1351	2.0015	.4056
4.50	.2222	1.5041	.5565	7.50	.1333	2.0149	.4020
4.55	.2198	1.5151	.5528	7.60	.1316	2.0281	.3984
4.60	.2174	1.5261	.5492	7.70	.1299	2.0412	.3950
4.65	.2151	1.5369	.5456	7.80	.1282	2.0541	.3916
4.70	.2128	1.5476	.5420	7.90	.1266	2.0669	.3882
4.75	.2105	1.5581	.5385	8.00	.1250	2.0794	.3849
4.80	.2083	1.5686	.5351	8.20	.1220	2.1041	.3785
4.85	.2062	1.5790	.5318	8.40	.1190	2.1282	.3724
4.90	.2041	1.5892	.5284	8.60	.1163	2.1518	.3665
4.95	.2020	1.5994	.5251	8.80	.1136	2.1748	.3608
5.00	.2000	1.6094	.5219	9.00	.1111	2.1972	.3552
5.05	.1980	1.6194	.5187	9.20	.1087	2.2192	.3499
5.10	.1961	1.6292	.5155	9.40	.1064	2.2407	.3448
5.15	.1942	1.6390	.5124	9.60	.1042	2.2618	.3398
5.20	.1923	1.6487	.5093	9.80	.1020	2.2824	.3349
5.25	.1905	1.6582	.5063	10.00	.1000	2.3026	.3303
5.30	.1887	1.6677	.5033	11.0	.0909	2.3979	.3089
5.35	.1869	1.6771	.5004	12.0	.0833	2.4849	.2904
5.40	.1852	1.6864	.4975	13.0	.0769	2.5649	.2742
5.45	.1835	1.6956	.4946	14.0	.0714	2.6391	.2599
5.50	.1818	1.7047	.4918	15.0	.0667	2.7081	.2472
5.55	.1802	1.7138	.4890	16.0	.0625	2.7726	.2358
5.60	.1786	1.7228	.4862	17.0	.0588	2.8332	.2255
5.65	.1770	1.7317	.4835	18.0	.0556	2.8904	.2161
5.70	.1754	1.7405	.4808	19.0	.0526	2.9444	.2076
5.75	.1739	1.7492	.4781	20.0	.0500	2.9957	.1998
5.80	.1724	1.7579	.4755	22.0	.0455	3.0910	.1860
5.85	.1709	1.7664	.4729	24.0	.0417	3.1781	.1741
5.90	.1695	1.7750	.4703	26.0	.0385	3.2581	.1638
5.95	.1681	1.7834	.4678	28.0	.0357	3.3322	.1547
6.00	.1667	1.7918	.4653	30.0	.0333	3.4012	.1467
6.10	.1639	1.8083	.4604	35.0	.0286	3.5553	.1302
6.20	.1613	1.8245	.4556	40.0	.0250	3.6889	.1172
6.30	.1587	1.8405	.4509	45.0	.0222	3.8067	.1068
[illegible]	.1562	1.8563	.4463	50.0	.0200	3.9120	.0982
[illegible]	.1538	1.8718	.4418	60.0	.0167	4.0944	.0849

TABLE II.

CONSTANTS FOR THE CURVE $pv^n = C$.

SEE § 7 (*j*) for the application of this Table; also § 10 (*i*) and § 13 (*h*), (*j*), for the particular curves represented.

For the columns marked "Exp.", Col. 1 gives a set of assumed values of the increasing volume v to the initial volume v_1; then the Table gives values of the factor a in

$$p = ap_1 = p_1\left(\frac{v_1}{v}\right)^n.$$

For the columns marked "Comp.", Col. 1 shows ratios of the increasing pressure p to the initial pressure p_1; and the Table givès values of a in

$$v = av_1 = v_1\left(\frac{p_1}{p}\right)^{\frac{1}{n}}.$$

The use of the Table is illustrated in Fig. 19.

TABLE II. CONSTANTS FOR THE CURVE $pv^n = C$.

1	2	3	4	5	6	7	8	9	10
Ratio.	Constant Steam Weight.		Adiabatic of Saturated Steam, for $x_1 =$ 0.7	0.9	1.0	Adiabatic of Superheated Steam.		Adiabatic of Air.	
	$n = 1.065$		1.105	1.125	1.135	$n = 1.333$		$n = 1.406$	
	Exp.	Comp.	Exp.	Exp.	Exp.	Exp.	Comp.	Exp.	Comp.
1.125	.8821	.8953	.8780	.8759	.8749	.8547	.9155	.8476	.9196
1.25	.7885	.8110	.7815	.7780	.7763	.7427	.8459	.7307	.8533
1.5	.6493	.6843	.6389	.6337	.6312	.5824	.7378	.5655	.7495
1.75	.5510	.5913	.5388	.5328	.5299	.4742	.6572	.4553	.6716
2	.4780	.5216	.4649	.4585	.4553	.3969	.5946	.3774	.6108
2.25	.4216	.4670	.4082	.4016	.3984	.3393	.5443	.3198	.5617
2.5	.3769	.4230	.3633	.3567	.3535	.2947	.5030	.2757	.5212
2.75	.3405	.3868	.3270	.3204	.3172	.2596	.4683	.2412	.4870
3	.3104	.3565	.2970	.2906	.2874	.2311	.4387	.2134	.4578
3.5	.2634	.3084	.2505	.2443	.2413	.1882	.3908	.1718	.4102
4	.2285	.2721	.2161	.2102	.2073	.1575	.3536	.1424	.3731
4.5	.2015	.2436	.1898	.1841	.1814	.1346	.3237	.1207	.3431
5	.1801	.2206	.1689	.1636	.1609	.1170	.2991	.1041	.3183
6	.1483	.1859	.1381	.1332	.1309	.0917	.2609	.0805	.2796
7	.1259	.1609	.1165	.1120	.1099	.0747	.2324	.0648	.2506
8	.1092	.1419	.1005	.0964	.0944	.0625	.2102	.0537	.2274
9	.0963	.1271	.0882	.0844	.0826	.0534	.1925	.0455	.2096
10	.0861	.1151	.0785	.0750	.0733	.0464	.1778	.0393	.1944
12	.0709	.0970	.0642	.0611	.0596	.0364	.1551	.0304	.1708
14	.0602	.0839	.0541	.0514	.0500	.0296	.1382	.0245	.1531
16	.0522	.0740	.0467	.0442	.0430	.0248	.1250	.0203	.1392
18	.0460	.0663	.0410	.0387	.0376	.0212	.1144	.0172	.1280
20	.0412	.0600	.0365	.0345	.0334	.0184	.1057	.0148	.1188
25	.0324	.0487	.0285	.0268	.0259	.0137	.0894	.0108	.1013
30	.0267	.0410	.0233	.0218	.0211	.0107	.0780	.0084	.0890

TABLE III.

CONSTANTS FOR HEATED WATER.

FOR any temperature t in ° F. are given:

p = corresponding absolute pressure of steam-formation;
D = weight of one cubic foot of water;
w = volume of one pound of water.

Here p is taken from the Steam-table;

D is found by an approximate formula given by Rankine,

$$D = \frac{2D_0}{\frac{T}{500} + \frac{500}{T}};$$

where D_0 is the weight at the temperature of maximum density, 40° F., and is 62.425; and T is the absolute temperature, $t + 460$. This formula is not exact for high temperatures; but since the quantities which it determines are of relatively insignificant magnitude, it is quite good enough for all practical purposes.

w is the reciprocal of D.

Table III. Constants for Heated Water.

Temperature, Fahrenheit. t	Vapor Pressure, Lbs. per Sq. In. p	Weight per Cubic Foot, Lbs. D	Volume of 1 Lb., Cu. Ft. w	Temperature, Fahrenheit. t	Vapor Pressure, Lbs. per Sq. In. p	Weight per Cubic Foot, Lbs. D	Volume of 1 Lb., Cu. Ft. w
40	.12	62.43	.01602	310	77.9	57.03	.01754
50	.18	.41	02	20	90.0	56.72	63
				30	103.5	.41	73
60	.26	62.37	.01603	40	118.4	.10	82
70	.36	.31	05	350	135.1	55.79	92
80	.51	.23	07				
90	.69	.13	09	360	153.5	55.49	.01802
100	.94	.02	12	70	173.9	.18	12
				80	196.3	54.87	22
110	1.27	61.87	.01616	90	221	.56	33
20	1.68	.72	20	400	248	.25	43
30	2.21	.56	25				
40	2.88	.38	30	410	277	53.94	.01854
150	3.71	.19	35	20	309	.62	65
				30	343	.31	76
160	4.73	60.99	.01640	40	381	.00	87
70	5.98	.78	45	450	442	52.69	98
80	7.46	.55	51				
90	9.43	.32	57	460	465	52.38	.01909
200	11.52	.08	64	70	512	51.07	20
				80	563	.76	32
210	14.1	59.83	.01671	90	618	.45	43
20	17.2	.58	78	500	676	.15	55
30	20.8	.32	86				
40	25.0	.05	94	510	737	50.84	.01967
250	29.9	58.77	.01702	20	803	.54	79
				30	873	.24	91
260	35.5	58.49	.01710	40	947	49.94	.02003
70	42.0	.21	18	550	1025	.64	14
80	49.3	57.92	27				
90	57.7	.53	36				
300	67.2	.33	45				

TABLE IV.

PROPERTIES OF SATURATED STEAM.

FOR the derivation and relations of the quantities in Columns 1 to 8, see §§ 9 and 10; for Cols. 9 and 10, see § 13.

NOTES.—This Table is taken from the same sources as that given in Kent's Mechanical Engineer's Pocket-book. From 1 lb. pressure up to 220 lbs., the table of Professor Dwelshauvers-Dery, printed in Vol. XI.; Trans. A. S. M. E., is used; above 220 lbs., the values given are taken from, or based upon, the upper part of Buel's table.

Dery's table is based on Regnault's formulas and Zeuner's methods; Buel's is founded chiefly on Rankine's work.

Regnault's formula for p in terms of t is of the general form

$$\log p = a + bm^x + cn^x,$$

where a, b, c, m, and n are determined constants, with one set of values for the range from 0° C. to 100° C., another set for 100° to 200° C.; and x is t or a simple function of t.

Rankine's formula is

$$\log p = A - \frac{B}{T} - \frac{C}{T^2}.$$

That the two tables are in this respect, however, derived from the same source, appears from the following comparison:

	p	100	150	200	220	lbs. abs.
Dery,	t	327.57	358.16	381.57	389.66	° F.
Buel,	t	327.63	358.22	381.64	389.74	° F.

The values given by Buel are used above 220 without change.

The three heat-quantities q, H, and r are found by Regnault's formulas, Eqs. (57) and (58) in § 10, and by Eq. (47): [illegible] involves the substitution of newly computed values of q and [illegible] hose of [illegible] Buel, above 220 lbs.

The volume-increase during expansion, u, is found by the rational formula

$$u=\frac{r}{AT}\frac{dt}{dp},$$

which is the one referred to in § 10 (a). Dery's values were worked out with the old value of A, 1/772; changing to 778, we increase u (or s) in the ratio of 778 to 772, or by the fraction 6/772=.00777 of itself. Similarly, d is decreased in the ratio of 772 to 778, or by the fraction 6/778=.00771. With these corrections, Dery's values are used up to 220 lbs.

Since the density d varies as $1/s$, and the product ps is not far from constant, d varies nearly as p: inspection of the Table will show that the difference between successive values of d decreases very slowly as the pressure rises. Above 220 lbs., d and s are here gotten by continuing the law of variation of d, and then deriving s from d.

These volume-functions are less definitely determined than the heat-quantities, the various authorities showing quite appreciable differences.

The external work of vaporization, APu, can be taken directly from Dery's table, the change made in u being compensated by the use of 778 instead of 772 in A: but above 220 lbs. newly computed values are given.

Then l is found by subtraction from r.

The two entropies, a and b, calculated by the writer, are explained in § 13.

As to the degree of numerical accuracy, it may be remarked that the quantities in Cols. 1 to 8 are all carried to one more decimal place than is of any practical significance when the Table is to be used in connection with any actual tests or experiments. But there are computations to be made in some lines of theoretical investigation where results depend upon small differences between large quantities, and where an even greater accuracy of expression —or, it might be better to say, a more accurate spacing of the tabular values—would be desirable. Some annoying minor discrepancies are likely to be encountered in using the Table as here given, but they could not be eliminated without a re-working of the whole mass of numerical matter.

TABLE IV. PROPERTIES OF

Absolute Pressure, Lbs. per Sq. In. p	1 Temperature, Fahrenheit. t	2 Heat in the Water above 32°, B.T.U. q	3 Total Heat in the Steam above 32°, B.T.U. H	4 Total Latent Heat, B.T.U. r	5 External Work of Evaporation, B.T.U. APu
1	102.00	70.09	1113.05	1042.96	62.34
2	126.27	94.44	20.45	26.01	4.62
3	141.62	109.88	25.13	15.25	6.01
4	153.07	121.40	28.63	07.23	7.06
5	162.33	130.72	31.45	00.73	7.89
6	170.12	138.58	1133.83	995.25	68.58
7	6.91	145.43	5.90	0.47	9.18
8	182.91	151.48	7.73	986.25	9.71
9	8.32	6.94	9.38	2.44	70.18
10	193.24	161.92	1140.88	978.96	0.61
11	197.77	166.50	1142.26	975.76	70.99
12	201.96	170.74	3.54	2.80	1.34
13	5.86	4.68	4.73	0.05	1.68
14	9.55	8.43	5.86	967.43	2.00
14.7	212.00	180.90	1146.60	965.70	72.20
15	213.03	181.94	1146.91	964.97	72.29
16	216.30	185.25	1147.91	962.66	72.57
17	9.41	8.41	8.86	0.45	2.82
18	222.38	191.42	9.76	958.34	3.07
19	5.20	4.29	1150.63	6.34	3.30
20	7.92	7.03	1.45	4.42	3.53
21	230.52	199.68	1152.25	952.57	73.74
2	3.02	202.22	3.01	0.79	.94
3	5.43	4.67	3.74	949.07	74.13
4	7.75	7.03	4.45	7.42	.32
25	240.00	9.31	5.14	5.83	.51
26	242.18	211.52	1155.80	944.28	74.69
7	4.28	3.67	6.45	2.78	.85
8	6.33	5.75	7.07	1.32	75.01
9	8.31	7.76	7.67	939.91	.17
30	250.25	9.73	8.26	8.53	.33
31	252.12	221.65	1158.84	937.19	75.47
2	3.95	3.51	9.39	5.88	.61
3	5.74	5.33	9.94	4.61	.75
4	7.48	7.10	1160.47	3.37	.89
35	9.18	8.84	0.99	2.15	76.02

SATURATED STEAM.

6 Inner Latent Heat, B.T.U. l	7 Volume of 1 Lb. of Steam, Cu. Ft. s	8 Weight of 1 Cu. Ft. of Steam, Lbs. d	9 Entropy of the Water, B.T.U. ÷ ° F. a	10 Entropy of Evaporation, B.T.U. ÷ ° F. b	Absolute Pressure, Lbs. per Sq. In. p
980.62	336.9	.00297	.13322	1.8558	1
61.39	174.6	573	.17561	1.7500	2
49.24	118.9	841	.20157	1.6876	3
40.17	90.6	.01104	.22054	1.6414	4
32.84	73.34	363	.23565	1.6080	5
926.67	61.77	.01619	.24818	1.5795	6
1.29	53.42	872	5895	551	7
916.54	47.08	.02124	6846	340	8
2.26	42.15	373	7695	153	9
908.35	38.14	621	8457	1.4986	10
904.78	34.88	.02867	.29155	1.4836	11
1.46	32.14	.03112	9797	696	12
898.37	29.81	355	.30392	567	13
5.43	27.80	597	0954	449	14
893.50	26.56	.03765	.31322	1.4370	14.7
892.68	26.07	.03838	.31479	1.4339	15
890.09	24.52	.04078	.31969	1.4234	16
887.63	3.16	318	2433	137	17
5.27	1.95	557	2876	044	18
3.04	0.87	794	3296	1.3956	19
0.90	19.88	.05031	3696	874	20
878.83	18.987	.05267	.34077	1.3796	21
6.85	8.174	502	443	720	2
4.94	7.429	737	796	648	3
3.10	6.743	972	.35137	578	4
1.32	6.116	.06205	466	511	25
869.60	15.535	.06437	.35777	1.3448	26
7.93	14.994	669	.36084	386	7
6.31	.490	901	380	327	8
4.74	.020	.07133	666	270	9
3.22	13.582	363	943	214	30
861.72	13.170	.07593	.37212	1.3160	31
0.27	12.783	823	473	108	2
858.86	.419	.08052	727	058	3
7.48	.075	282	975	009	4
6.13	11.751	510	.38217	1.2962	35

TABLE IV. PROPERTIES OF

p	t	q	H	r	APu
36	260.84	230.52	1161.49	930.97	76.16
7	2.46	2.18	1.99	929.81	.28
8	4.05	3.80	2.47	8.67	.40
9	5.60	5.39	2.95	7.56	.52
40	7.12	6.94	3.41	6.47	.64
41	268.61	238.46	1163.87	925.41	76.75
2	270.07	9.95	4.31	4.36	.86
3	1.51	241.42	4.75	3.33	.97
4	2.92	2.86	5.18	2.32	77.07
45	4.30	4.27	5.60	1.33	.18
46	275.65	245.65	1166.01	920.36	77.29
7	6.99	7.01	6.42	919.41	.39
8	8.30	8.35	6.82	8.47	.49
9	9.59	9.67	7.21	7.54	.58
50	280.85	250.97	7.60	6.63	.67
51	282.10	252.24	1167.98	915.74	77.76
2	3.33	3.49	8.35	4.86	.85
3	4.53	4.73	8.72	3.99	.94
4	5.72	5.96	9.09	3.13	78.03
55	6.90	7.15	9.44	2.29	.12
56	288.06	258.34	1169.80	911.46	78.21
7	9.20	9.50	1170.14	0.64	.29
8	290.32	260.66	0.49	909.83	.37
9	1.43	1.79	0.82	9.03	.45
60	2.52	2.91	1.16	8.25	.53
61	293.60	264.02	1171.49	907.47	78.61
2	4.66	5.11	1.81	6.70	.68
3	5.71	6.18	2.13	5.95	.76
4	6.75	7.25	2.45	5.20	.83
65	7.78	8.30	2.76	4.46	.90
66	298.79	269.34	1173.07	903.73	78.97
7	9.79	270.37	3.38	3.01	79.04
8	300.78	1.38	3.68	2.30	.11
9	1.75	2.38	3.97	1.59	.18
70	2.72	3.37	4.27	0.90	.25
71	303.67	274.35	1174.56	900.21	79.32
2	4.62	5.32	4.85	899.53	.39
3	5.55	6.28	5.13	8.85	.46
4	6.47	7.22	5.41	8.19	.53
75	7.39	8.16	5.69	7.53	.59
76	308.29	279.09	1175.97	896.88	79.65
7	9.18	280.01	6.24	6.23	.71
8	310.07	0.92	6.51	5.59	.77
9	0.95	1.82	6.78	4.96	.83
80	1.81	[illegible]	7.04	4.33	.89

SATURATED STEAM—*Continued.*

t	s	d	a	b	p
854.81	11.446	.08738	.38553	1.2916	36
3.53	.155	965	683	870	7
2.27	10.879	.09193	907	826	8
1.04	.616	420	.39124	784	9
849.84	.367	646	335	743	40
848.66	10.130	.09872	.39543	1.2702	41
7.50	9.902	.10099	748	661	2
6.36	.687	324	949	622	3
5.25	.480	549	.40146	584	4
4.15	.283	773	338	547	45
843.07	9.093	.10998	.40527	1.2511	46
2.02	8.911	.11222	712	475	7
0.98	.737	446	894	440	8
839.96	.570	669	.41071	406	9
8.96	.409	892	244	373	50
837.98	8.254	.12116	.41416	1.2340	51
7.01	.105	338	587	308	2
6.05	7.961	561	754	276	3
5.10	.823	783	918	245	4
4.18	.690	.13004	.42078	214	55
833.25	7.561	.13225	.42236	1.2184	56
2.35	.437	447	392	155	7
1.46	.316	668	546	126	8
0.58	.200	889	697	097	9
829.72	.087	.14109	845	069	60
828.86	6.979	.14330	.42992	1.2042	61
8.02	.873	550	.43138	015	2
7.19	.770	770	281	1.1988	3
6.37	.671	991	421	962	4
5.56	.574	.15211	560	936	65
824.76	6.481	.15431	.43697	1.1910	66
3.97	.390	650	832	885	7
3.19	.302	869	966	860	8
2.41	.216	.16087	.44099	836	9
1.65	.133	305	230	812	70
820.89	6.052	.16524	.44359	1.1788	71
0.14	5.973	741	485	764	2
819.40	.897	958	610	741	3
8.66	.822	.17175	733	718	4
7.94	.750	393	855	696	75
817.23	5.679	.17610	.44976	1.1674	76
6.52	.609	827	.45096	652	7
5.82	.542	.18045	214	630	8
5.13	.476	262	331	608	9
4.45	.411	479	446	587	80

TABLE IV. PROPERTIES OF

p	t	q	H	r	APu
81	312.67	283.59	1177.30	893.71	79.95
2	3.52	4.46	7.56	3.10	80.01
3	4.36	5.33	7.82	2.49	.07
4	5.20	6.19	8.07	1.88	.13
85	6.02	7.04	8.33	1.29	.19
86	316.84	287.89	1178.58	890.69	80.25
7	7.65	8.71	8.82	0.11	.30
8	8.45	9.54	9.07	889.53	.35
9	9.25	290.36	9.31	8.95	.40
90	320.04	1.17	9.55	8.38	.45
91	320.82	291.98	1179.79	887.81	80.50
2	1.60	2.78	1180.03	7.25	.56
3	2.37	3.57	0.26	6.69	.61
4	3.13	4.35	0.49	6.14	.66
95	3.88	5.13	0.72	5.59	.71
96	324.63	295.91	1180.95	885.04	80.76
7	5.38	6.67	1.18	4.51	.81
8	6.11	7.44	1.41	3.97	.86
9	6.85	8.19	1.63	3.44	.91
100	7.57	8.94	1.85	2.91	.95
101	328.29	299.68	1182.07	882.39	81.00
2	9.01	300.42	2.29	1.87	.05
3	9.71	1.14	2.50	1.36	.10
4	330.42	1.87	2.72	0.85	.14
105	1.11	2.59	2.93	0.34	.18
106	331.81	303.30	1183.14	879.84	81.23
7	2.49	4.01	3.35	9.34	.27
8	3.17	4.72	3.56	8.84	.31
9	3.85	5.42	3.77	8.35	.36
110	4.52	6.10	3.97	7.87	.41
111	335.19	306.79	1184.17	877.38	81.45
2	5.85	7.48	4.38	6.90	.50
3	6.51	8.16	4.58	6.42	.54
4	7.17	8.84	4.78	5.94	.58
115	7.81	9.50	4.97	5.47	.62
116	338.46	310.17	1185.17	875.00	81.66
7	9.10	0.83	5.37	4.54	.70
8	9.74	1.49	5.56	4.07	.74
9	340.37	2.14	5.75	3.61	.78
120	1.00	2.78	5.94	3.16	.82
121	341.62	313.43	1186.13	872.70	81.86
2	2.24	4.07	6.32	2.25	.90
3	2.85	4.71	6.51	1.80	.94
4	3.47	5.34	6.70	1.36	.98
125	4.07	5.97	6.88	0.91	82.02

SATURATED STEAM—*Continued.*

t	s	d	a	b	p
813.76	5.349	.18696	.45560	1.1566	81
3.09	.288	912	673	545	2
2.42	.228	.19128	784	525	3
1.75	.169	345	894	505	4
1.10	.112	561	.46003	485	85
810.44	5.056	.19777	.46111	1.1465	86
809.81	.002	993	218	446	7
9.18	4.949	.20208	324	427	8
8.55	.896	423	430	408	9
7.93	.845	639	535	389	90
807.31	4.795	.20854	.46639	1.1370	91
6.69	.747	.21068	741	352	2
6.08	.699	282	842	334	3
5.48	.652	497	942	316	4
4.88	.606	711	.47042	298	95
804.28	4.561	.21925	.47140	1.1280	96
3.70	.517	.22138	238	262	7
3.11	.474	351	335	245	8
2.53	.432	565	431	228	9
1.96	.390	778	526	211	100
801.39	4.349	.2299	.47620	1.1194	101
0.82	.309	.2321	714	177	2
0.26	.270	42	807	160	3
799.71	.232	63	899	144	4
9.16	.194	84	990	128	105
798.61	4.157	.2406	.48080	1.1112	106
8.07	.120	27	170	096	7
7.53	.084	48	259	080	8
6.99	.049	69	347	064	9
6.46	.015	91	434	049	110
795.93	3.981	.2512	.48521	1.1033	111
5.40	.948	33	607	018	2
4.88	.915	54	692	003	3
4.36	.883	76	776	1.0988	4
3.85	.851	97	860	973	115
793.34	3.820	.2618	.48944	1.0958	116
2.84	.789	39	.49027	943	7
2.33	.759	61	110	929	8
1.83	.729	82	192	915	9
1.34	.700	.2703	274	901	120
790.84	3.671	.2724	4.9354	1.0887	121
0.35	.643	45	433	873	2
789.86	.615	66	512	859	3
9.38	.588	87	590	845	4
8.89	.561	.2808	668	831	125

TABLE IV. PROPERTIES OF

p	t	q	H	r	APu
126	344.68	316.60	1187.07	870.47	82.06
7	5.28	7.22	7.25	0.03	.09
8	5.87	7.83	7.43	869.60	.13
9	6.46	8.44	7.61	9.17	.17
130	7.06	9.05	7.79	8.74	.21
131	347.64	319.66	1187.97	868.31	82.25
2	8.23	320.27	8.15	7.88	.28
3	8.81	0.87	8.33	7.46	.32
4	9.38	1.46	8.50	7.04	.35
135	9.95	2.06	8.68	6.62	.38
136	350.52	322.64	1188.85	866.21	82.42
7	1.09	3.23	9.02	5.79	.45
8	1.65	3.82	9.20	5.38	.49
9	2.21	4.40	9.37	4.97	.52
140	2.77	4.97	9.54	4.57	.56
141	353.32	325.54	1189.70	864.16	82.59
2	3.87	6.11	9.87	3.76	.63
3	4.42	6.68	1190.04	3.36	.66
4	4.96	7.24	0.20	2.96	.69
145	5.50	7.80	0.37	2.57	.72
146	356.04	328.36	1190.53	862.17	82.75
7	6.57	8.92	0.70	1.78	.79
8	7.11	9.47	0.86	1.39	.82
9	7.64	330.01	1.02	1.01	.86
150	8.16	0.56	1.18	0.62	.89
151	358.68	331.10	1191.34	860.24	82.92
2	9.20	1.64	1.50	859.86	.95
3	9.72	2.18	1.66	9.48	.98
4	360.24	2.71	1.81	9.10	83.01
155	0.75	3.24	1.97	8.73	.04
156	361.26	333.78	1192.13	858.35	83.07
7	1.77	4.30	2.28	7.98	.10
8	2.27	4.83	2.44	7.61	.13
9	2.78	5.35	2.59	7.24	.16
160	3.28	5.87	2.74	6.87	.19
161	363.77	336.38	1192.89	856.51	83.22
2	4.27	6.89	3.04	6.15	.25
3	4.76	7.41	3.19	5.78	.28
4	5.26	7.92	3.34	5.42	.31
165	5.74	8.42	3.49	5.07	.34
166	366.23	338.93	1193.64	[illegible]	83.37
7	6.72	9.44	3.79	[illegible]	[illegible]
8	7.20	9.94	3.94	[illegible]	[illegible]
9	7.68	340.43	4.08	[illegible]	[illegible]
170	8.16	0.94	4.23	[illegible]	[illegible]

SATURATED STEAM—*Continued.*

l	*s*	*d*	*a*	*b*	*p*
788.41	3.535	.2829	.49745	1.0817	126
7.94	.509	50	822	803	7
7.47	.483	71	899	790	8
7.00	.458	92	975	777	9
6.53	.433	.2913	.50051	764	130
786.06	3.408	.2934	.50126	1.0751	131
5.60	.384	55	201	738	2
5.14	.360	76	275	725	3
4.69	.336	97	348	712	4
4.24	.313	.3018	421	700	135
783.79	3.290	.3039	.50494	1.0687	136
3.34	.267	60	566	674	7
2.89	.245	81	638	662	8
2.45	.223	.3102	710	650	9
2.01	.202	23	781	637	140
781.57	3.181	.3144	.50852	1.0625	141
1.13	.160	65	922	613	2
0.70	.139	86	992	601	3
0.27	.118	.3207	.51061	589	4
779.85	.098	28	129	577	145
779.42	3.078	.3249	.51197	1.0565	146
8.99	.058	70	265	553	7
8.57	.039	90	332	541	8
8.15	.020	.3311	399	530	9
7.73	.001	32	466	519	150
777.32	2.983	.3353	.51532	1.0507	151
6.91	.964	74	598	496	2
6.50	.946	94	664	485	3
6.09	.928	.3415	729	474	4
5.69	.911	36	794	463	155
775.28	2.893	.3457	.51859	1.0452	156
4.88	.876	77	923	441	7
4.48	.859	98	987	430	8
4.08	.842	.3519	.52051	419	9
3.68	.825	40	114	408	160
773.29	2.809	.3560	.52177	1.0398	161
2.90	.793	81	239	387	2
2.50	.777	.3602	301	376	3
2.11	.761	22	363	365	4
1.73	.745	43	424	355	165
771.34	2.730	.3663	.52485	1.0345	166
0.96	.714	84	546	334	7
0.58	.699	.3705	607	324	8
0.20	.684	26	667	314	9
769.82	.669	47	727	303	170

TABLE IV. PROPERTIES OF

p	t	q	H	r	APu
171	368.63	341.42	1194.37	852.95	83.51
2	9.11	1.92	4.52	2.60	.54
3	9.58	2.41	4.66	2.25	.56
4	370.05	2.89	4.80	1.91	.59
175	0.51	3.38	4.95	1.57	.62
176	370.98	343.87	1195.09	851.22	83.64
7	1.44	4.35	5.23	0.88	.67
8	1.90	4.83	5.37	0.54	.70
9	2.36	5.30	5.51	0.21	.73
180	2.82	5.78	5.65	849.87	.75
181	373.28	346.26	1195.79	849.53	83.77
2	3.73	6.73	5.93	9.20	.80
3	4.18	7.20	6.07	8.87	.83
4	4.63	7.66	6.20	8.54	.86
185	5.08	8.13	6.34	8.21	.88
186	375.53	348.60	1196.48	847.88	83.90
7	5.97	9.06	6.61	7.55	.92
8	6.41	9.52	6.75	7.23	.95
9	6.85	9.97	6.88	6.91	.97
190	7.29	350.43	7.01	6.58	.99
191	377.73	350.88	1197.15	846.27	84.02
2	8.16	1.34	7.28	5.94	.04
3	8.59	1.79	7.41	5.62	.06
4	9.02	2.24	7.54	5.30	.08
195	9.45	2.69	7.67	4.98	.10
196	379.88	353.13	1197.80	844.67	84.12
7	380.31	3.57	7.93	4.36	.14
8	0.73	4.01	8.06	4.05	.16
9	1.15	4.45	8.19	3.74	.18
200	1.57	4.89	8.32	3.43	.20
201	381.99	355.33	1198.45	843.12	84.22
2	2.41	5.77	8.58	2.81	.24
3	2.83	6.20	8.70	2.50	.26
4	3.25	6.63	8.83	2.20	.28
205	3.66	7.06	8.96	1.90	.30
206	384.07	357.49	1199.08	841.59	84.32
7	4.48	7.92	9.21	[illegible]	.34
8	4.89	8.34	9.33	[illegible]	.36
9	5.29	8.76	9.45	[illegible]	.38
210	5.69	9.18	9.58	[illegible]	.40
211	386.09	359.60	1199.70	[illegible]	84.[illegible]
2	6.50	360.02	9.82	[illegible]	[illegible]
3	6.90	0.44	9.94	[illegible]	[illegible]
4	7.30	[illegible]	1200.07	[illegible]	[illegible]
215	7.69		0.19	[illegible]	[illegible]

SATURATED STEAM—*Continued.*

t	s	d	a	b	p
769.44	2.655	.3767	.52787	1.0293	171
9.06	.640	87	846	283	2
768.69	.626	.3808	905	273	3
8.32	.612	29	964	264	4
7.95	.598	49	.53022	254	175
767.58	2.584	.3870	.53080	1.0244	176
7.21	.570	91	138	234	7
6.84	.557	.3911	195	224	8
6.48	.543	32	252	215	9
6.12	.530	53	309	205	180
765.76	2.517	.3973	.53366	1.0195	181
5.40	.504	94	423	186	2
5.04	.491	.4014	479	176	3
4.68	.478	35	535	167	4
4.33	.466	55	591	157	185
763.98	2.453	.4076	.53647	1.0148	186
3.63	.441	97	702	138	7
3.28	.429	.4117	757	129	8
2.94	.417	38	812	120	9
2.59	.405	59	867	111	190
762.25	2.393	.4180	.53921	1.0102	191
1.90	.381	.4201	975	1.0093	2
1.56	.369	21	.54029	84	3
1.22	.357	42	083	75	4
0.88	.346	63	136	66	195
760.55	2.335	.4283	.54189	1.0057	196
0.22	.324	.4304	242	48	7
759.89	.312	25	295	40	8
9.56	.301	45	348	31	9
9.23	.291	66	400	22	200
758.90	2.280	.4386	.54452	1.0014	201
8.57	.269	.4407	504	05	2
8.24	.258	28	556	.9996	3
7.92	.248	48	607	88	4
7.60	.237	69	658	79	205
757.27	2.227	.4490	.54709	.9971	206
6.95	.216	.4510	759	62	7
6.63	.206	31	809	54	8
6.31	.196	52	859	46	9
6.00	.186	72	908	37	210
755.68	2.177	.4593	.54958	.9929	211
5.34	.167	.4614	.55007	21	2
5.04	.158	34	056	13	3
4.73	.148	55	105	04	4
4.42	.139	75	154	.9896	215

TABLE IV.—PROPERTIES OF

p	t	q	H	r	APu
216	387 09	361 95	1200 31	838 63	84.52
7	8 45	2 00	0 43	8.34	.54
8	8 87	2 50	0 55	8.05	.56
9	9 28	2 91	0 66	7 75	.58
220	9 66	3 32	0 78	7 46	.60
230	393 6	367 4	1202 0	834 6	84.6
40	7 3	371 2	3 1	1 9	.8
50	400 9	5 0	4 2	829 2	.9
60	4 4	8.7	5 3	6 6	85.1
270	407 8	382 3	1206 3	824 0	85 2
80	411 0	5 7	7 3	1 6	.3
90	4 2	9 1	8 3	819 2	.4
300	7.4	392.4	9 2	6 8	.5
350	432 0	407.8	1213.7	806 0	86.0
400	44 9	21.4	17 7	796 3	.2
450	56 6	33.8	21.3	87 5	.3
500	67.4	45.2	24 5	79.3	.3
550	477 5	455.8	1227 6	771.8	86.4
600	86 9	66.0	30 5	64 5	.4
650	95 7	75.4	33.2	57 8	.4
700	504.1	84.3	35.7	51.4	.3
750	512 1	492.9	1238 0	745 1	86.3
800	19.6	501.0	40 3	39 3	.3
850	26.8	08.8	42 5	33 8	.2
900	33.7	16.1	44.7	28.6	.1
950	540.3	523.3	1246.7	723.4	85.9
1000	46.8	30.5	48.7	15.2	.7

SATURATED STEAM—*Continued.*

l	s	d	a	b	p
754.11	2.130	.4696	.55203	.9888	216
3.80	.120	.4716	252	80	7
3.49	.111	37	300	72	8
3.17	.102	57	348	64	9
2.86	.093	78	396	56	220
750.0	2.006	.4984	.5588	.9777	230
747.1	1.927	.5190	633	703	40
4.3	.853	396	677	632	50
1.5	.786	601	719	563	60
738.8	1.724	.5807	.5760	.9497	270
6.3	.664	.6012	799	433	80
3.8	.609	217	837	371	90
1.3	.557	423	876	310	300
720.0	1.343	.7446	.6049	.9036	350
10.1	1.181	.8466	201	.8800	400
01.2	1.054	.9485	336	592	450
693.0	.952	1.0504	461	403	500
685.4	.868	1.1523	.6576	.8231	550
78.1	.797	1.2541	683	074	600
71.4	.738	1.3559	783	.7929	650
65.2	.686	1.4577	876	794	700
658.8	.641	1.5596	.6963	.7665	750
53.0	.602	1.6616	.7047	547	800
47.5	.567	1.7634	126	437	850
42.5	.536	1.8652	201	332	900
637.3	.508	1.9670	.7272	.7232	950
32.5	.483	2.0688	342	135	1000

TABLE V.

CONSTANTS FOR THE STEAM-CYCLE WITH ADIABATIC EXPANSION.

See § 24 (c) for illustration and explanation.

For a cycle under the conditions of the ideal heat-engine, or for adiabatic expansion in a steam-jet, from the pressure p_1 at the top of any column to any pressure p_2 at the side, the quantities given in this table are as follows:

E is the total heat-energy transformed into the mechanical form in Cycle B, Fig. 26 or Fig. 45, by one pound of steam saturated and dry at the beginning of the adiabatic expansion: this corresponds with AU' in Tables 16 A and 16 B.

E_1 is the similar effective performance of the Carnot cycle, Fig. 25, like AU in Tables 16 A and 16 B.

E_0 is the energy of the adiabatic expansion of hot water initially at p_1, equal to $E - E_1$.

x_2 is the fraction of steam in the mixture at the end of adiabatic expansion from p_1 to p_2, when the initial steam-quality was 1.00; $(1 - x_2)$ measuring the condensation.

x_{20} is a similar measure of the effect of expanding hot water adiabatically, and shows the amount of evaporation resulting from this process.

v_2 and v_{20} are the specific steam-volumes at the end of expansion, determined by x_2 and x_{20}.

TABLE V. ADIABATIC TABLE FOR SATURATED STEAM.

p_1 =		250	220	195	170	150	135	120
p_2	E	10.90						
	E_1	10.78						
	E_0	.12						
220	x_2	.9912						
	x_{20}	.0139						
	v_2	2.075						
	v_{20}	.047						
	E	21.06	10.22					
	E_1	20.66	10.08					
	E_0	.40	.14					
195	x_2	.9830	.9917					
	x_{20}	.0262	.0125					
	v_2	2.306	2.326					
	v_{20}	.079	.047					
	E	32.34	21.63	11.50				
	E_1	31.53	21.19	11.36				
	E_0	.81	.44	.14				
170	x_2	.9741	.9825	.9907				
	x_{20}	.0392	.0259	.0137				
	v_2	2.600	2.622	2.644				
	v_{20}	.122	.087	.054				
	E	42.55	31.93	21.86	10.45			
	E_1	41.17	31.05	21.43	10.32			
	E_0	1.38	.88	.43	.13			
150	x_2	.9661	.9743	.9823	.9915			
	x_{20}	.0514	.0374	.0254	.0120			
	v_2	2.899	2.924	2.948	2.975			
	v_{20}	.171	.129	.094	.054			
	E	50.96	40.42	30.43	19.13	8.75		
	E_1	49.07	39.14	29.69	18.76	8.63		
	E_0	1.89	1.28	.74	.37	.12		
135	x_2	.9595	.9676	.9755	.9845	.9929		
	x_{20}	.0593	.0456	.0347	.0212	.0098		
	v_2	3.179	3.206	3.232	3.262	3.289		
	v_{20}	.210	.168	.132	.088	.050		

TABLE V—*Continued.*

p_1	$p_2 =$	250	220	195	170	150	135	120
p_1	E	60.29	49.85	39.94	28.72	18.40	9.70	
	E_1	57.70	47.96	38.70	27.98	18.06	9.58	
	E_0	2.59	1.89	1.24	.74	.34	.12	
120	x_2	.9524	.9603	.9680	.9768	.9851	.9921	
	x_{20}	.0688	.0562	.0446	.0317	.0201	.0105	
	v_2	3.524	3.553	3.582	3.614	3.645	3.671	
	v_{20}	.269	.225	.182	.135	.092	.057	
	E	70.65	60.27	50.40	39.29	29.09	20.50	10.89
	E_1	67.22	57.71	48.54	38.17	28.45	20.15	10.77
	E_0	3.43	2.56	1.66	1.12	.64	.35	.12
105	x_2	.9444	.9523	.9598	.9684	.9765	.9834	.9911
	x_{20}	.0789	.0665	.0552	.0426	.0312	.0218	.0115
	v_2	3.961	3.994	4.025	4.062	4.095	4.124	4.156
	v_{20}	.347	.296	.253	.196	.148	.109	.066
	E	82.39	72.10	62.37	51.34	41.20	32.69	23.21
	E_1	77.97	68.65	59.80	49.58	40.09	32.00	22.84
	E_0	4.42	3.45	2.57	1.76	1.11	.69	.37
90	x_2	.9356	.9432	.9506	.9590	.9669	.9736	.9812
	x_{20}	.0899	.0778	.0667	.0544	.0433	.0341	.0241
	v_2	4.533	4.570	4.606	4.646	4.685	4.717	4.754
	v_{20}	.451	.393	.340	.280	.227	.182	.134
	E	96.04	85.78	76.25	65.34	55.28	46.89	37.48
	E_1	90.16	81.09	72.53	62.61	53.41	45.54	36.72
	E_0	5.88	4.69	3.72	2.73	1.87	1.35	.76
75	x_2	.9254	.9328	.9400	.9482	.9559	.9624	.9698
	x_{20}	.1019	.0901	.0794	.0673	.0565	.0476	.0378
	v_2	5.321	5.364	5.405	5.452	5.496	5.534	5.577
	v_{20}	.591	.525	.464	.397	.336	.286	.230
	E	112.25	102.23	92.71	81.96	72.08	63.77	54.48
	E_1	104.37	95.74	87.50	77.93	69.05	61.45	52.85
	E_0	7.88	6.49	5.21	4.03	3.03	2.32	1.63
60	x_2	.9135	.9206	.9276	.9356	.9430	.9493	.9565
	x_{20}	.1151	.1040	.0936	.0811	.0714	.0628	.0533
	v_2	6.474	6.524	6.574	6.630	6.683	6.727	6.779
	v_{20}	.833	.753	.679	.590	.522	.461	.394
	E	132.68	122.78	113.42	102.95	93.13	84.99	75.85
	E_1	121.91	113.94	105.84	96.71	88.21	80.94	72.71
	E_0	10.74	8.84	7.58	6.24	4.92	4.05	3.14
45	x_2	.8986	.9055	.9123	.9199	.9271	.9332	.9400
	x_{20}	.1302	.1200	.1100	.0987	.0887	.0804	.0712
	v_2	8.342	8.406	8.469	8.539	8.606	8.662	8.726
	v_{20}	1.223	1.129	1.036	.932	.839	.762	.677

TABLE V—*Continued.*

105	90	75	60	45	30	20	$=p_1$	
							E	p_2
							E_1	
							E_0	
							x_2	120
							x_{20}	
							v_2	
							v_{20}	
							E	
							E_1	
							E_0	
							x_2	105
							x_{20}	
							v_2	
							v_{20}	
12.44							E	
12.31							E_1	
.13							E_0	
.9899							x_2	90
.0128							x_{20}	
4.796							v_2	
.079							v_{20}	
26.85	14.57						E	
26.40	14.41						E_1	
.45	.16						E_0	
.9782	.9881						x_2	75
.0268	.0144						x_{20}	
5.625	5.681						v_2	
.168	.098						v_{20}	
44.00	31.88	17.61					E	
42.94	31.33	17.39					E_1	
1.06	.55	.22					E_0	
.9647	.9742	.9858					x_2	60
.0426	.0306	.0158					x_{20}	
6.836	6.905	6.986					v_2	
.319	.234	.129					v_{20}	
65.53	53.62	39.50	22.32				E	
63.22	52.09	38.70	21.99				E_1	
2.31	1.53	.80	.33				E_0	
.9479	.9571	.9682	.9819				x_2	45
.0608	.0494	.0361	.0200				x_{20}	
8.799	8.885	8.988	9.115				v_2	
.581	.475	.352	.202				v_{20}	

TABLE V—*Continued.*

p_2	$p_1 =$	250	220	195	170	150	135	120
	E	160.25	150.59	141.45	131.08	121.58	113.59	104.68
	E_1	145.10	137.40	130.05	121.48	113.51	106.68	98.93
	E_0	15.15	13.19	11.40	9.60	8.07	6.91	5.75
30	x_2	.8790	.8855	.8919	.8992	.9060	.9118	.9183
	x_{20}	.1500	.1396	.1301	.1195	.1099	.1020	.0933
	r_2	11.94	12.03	12.11	12.21	12.31	12.38	12.47
	r_{20}	2.05	1.91	1.78	1.64	1.51	1.40	1.28
	E	186.62	177.10	168.16	157.99	148.71	140.88	132.15
	E_1	166.62	159.41	152.53	144.49	137.00	130.54	123.83
	E_0	20.00	17.69	15.63	13.50	11.71	10.34	8.32
20	x_2	.8605	.8668	.8729	.8798	.8863	.8918	.8980
	x_{20}	.1663	.1564	.1473	.1372	.1281	.1206	.1123
	r_2	17.11	17.23	17.35	17.49	17.62	17.73	17.85
	r_{20}	3.32	3.12	2.94	2.74	2.56	2.41	2.25
	E	204.63	195.26	186.44	176.45	167.30	159.58	150.97
	E_1	180.95	174.09	167.52	159.83	152.66	146.50	139.50
	E_0	23.68	21.17	18.92	16.62	14.64	13.08	11.47
15	x_2	.8481	.8542	.8600	.8667	.8730	.8783	.8843
	x_{20}	.1764	.1668	.1580	.1482	.1394	.1321	.1241
	r_2	22.11	22.27	22.42	22.60	22.76	22.90	23.06
	r_{20}	4.61	4.36	4.13	3.88	3.65	3.46	3.25
	E	228.88	219.72	211.04	201.23	192.25	184.70	176.27
	E_1	200.02	193.59	187.43	180.22	173.48	167.68	161.03
	E_0	28.86	26.13	23.61	21.01	18.77	17.02	15.24
10	x_2	.8317	.8375	.8431	.8495	.8555	.8606	.8663
	x_{20}	.1889	.1798	.1714	.1620	.1535	.1466	.1389
	r_2	31.72	31.94	32.14	32.40	32.63	32.82	33.04
	r_{20}	7.22	6.87	6.54	6.19	5.87	5.60	5.31
	E	268.20	259.32	250.90	241.39	232.67	225.35	217.18
	E_1	229.79	224.06	218.55	212.07	205.99	200.75	194.76
	E_0	38.41	35.26	32.35	29.32	25.68	24.60	22.42
5	x_2	.8055	.8109	.8161	.8221	.8277	.8324	.8378
	x_{20}	.2065	.1980	.1901	.1814	.1735	.1670	.1599
	r_2	59.08	59.47	59.85	60.29	60.70	61.05	61.44
	r_{20}	15.16	14.53	13.96	13.31	12.74	12.26	11.74
	E	315.97	307.39	299.29	290.12	281.72	274.69	266.83
	E_1	264.52	259.61	254.85	249.22	243.98	239.33	234.07
	E_0	51.45	47.78	44.44	40.90	37.74	35.36	32.76
2	x_2	.7745	.7794	.7842	.7897	.7948	.7992	.8041
	x_{20}	.2240	.2162	.2090	.2010	.1937	.1878	.1812
	r_2	135.2	136.1	136.9	137.9	138.8	139.6	140.4
	r_{20}	39.1	37.8	36.5	35.1	33.8	32.8	31.7

TABLE V. *Continued.*

105	90	75	60	45	30	20	$=p_1$	p_2
94.62	82.97	69.20	52.43	30.64			E	
89.98	79.48	66.83	51.02	30.16			E_1	
4.64	3.49	2.37	1.41	.48			E_0	
.9257	.9345	.9450	.9580	.9752			x_2	30
.0836	.0726	.0599	.0447	.0257			x_{20}	
12.57	12.69	12.84	13.01	13.25			v_2	
1.15	1.00	.83	.62	.37			v_{20}	
122.31	110.94	97.46	81.08	59.83	29.92		E	
114.83	104.91	92.94	77.97	58.19	29.51		E_1	
7.48	6.03	4.52	3.11	1.64	.41		E_0	
.9051	.9134	.9235	.9358	.9522	.9758		x_2	20
.1030	.0925	.0804	.0659	.0479	.0234		x_{20}	
17.99	18.16	18.36	18.61	18.93	19.40		v_2	
2.06	1.85	1.61	1.33	.97	.48		v_{20}	
141.27	130.09	116.82	100.02	79.75	50.35	20.97	E	
131.40	121.87	110.36	95.94	76.88	49.18	20.66	E_1	
9.87	8.12	6.46	4.08	2.87	1.17	.31	E_0	
.8912	.8993	.9090	.9216	.9368	.9597	.9830	x_2	15
.1152	.1050	.0933	.0793	.0618	.0381	.0155	x_{20}	
23.23	23.44	23.70	24.03	24.42	25.02	25.63	v_2	
3.02	2.75	2.45	2.08	1.63	1.01	.42	v_{20}	
166.77	155.81	142.82	127.00	106.55	77.77	49.01	E	
153.46	144.41	133.47	119.89	101.71	75.33	48.11	E_1	
13.31	11.40	9.35	7.11	4.84	2.44	.90	E_0	
.8729	.8806	.8899	.9014	.9165	.9384	.9608	x_2	10
.1303	.1206	.1094	.0960	.0793	.0566	.0350	x_{20}	
33.29	33.59	33.94	34.38	34.96	35.79	36.64	v_2	
4.99	4.62	4.19	3.68	3.04	2.18	1.35	v_{20}	
207.99	197.38	184.81	169.53	149.79	122.04	94.35	E	
187.82	179.61	169.66	157.19	140.49	116.18	91.00	E_1	
20.17	17.77	15.15	12.34	9.30	5.86	3.35	E_0	
.8440	.8511	.8598	.8705	.8846	.9050	.9258	x_2	5
.1519	.1429	.1324	.1199	.1043	.0832	.0630	x_{20}	
61.89	62.42	63.06	63.84	64.88	66.37	67.90	v_2	
11.15	10.49	9.72	8.81	7.66	6.12	4.64	v_{20}	
258.01	247.81	235.74	221.04	202.14	175.92	149.08	E	
227.95	220.68	211.83	200.72	185.74	163.83	141.00	E_1	
30.06	27.13	23.91	20.32	16.40	12.09	8.08	E_0	
.8098	.8164	.8243	.8341	.8471	.8655	.8850	x_2	2
.1739	.1655	.1560	.1445	.1302	.1108	.0922	x_{20}	
141.4	142.5	143.9	145.6	147.9	151.0	154.5	v_2	
30.4	28.9	27.2	25.2	22.7	19.4	16.1	v_{20}	

TABLE VI.

CYLINDER CONSTANTS.

For a cylinder of the diameter d in inches, A is the area of cross-section, or of the piston, in square inches; V is the volume in cubic feet for each ten inches of length; and E.C. is the engine-constant $AS/396{,}000$, likewise for a length of ten inches, or it is $A/39{,}600$: see Eq. (97), page 102.

The small values in the first part of the table are for the piston-rod, and the E.C. for the latter can be subtracted from that for the full piston, and the result multiplied by $S/10$, where S is the stroke in inches. The displacement-volume of any engine-cylinder can be found in the same way, multiplying the net V from the table by $S/10$.

TABLE VI. CYLINDER CONSTANTS.

d	A	V	E.C.	d	A	V	E.C.
1	.79	.0045	.000020	4	12.57	.0727	.000317
1⅛	.99	58	25	4¼	14.19	.0821	358
1¼	1.23	71	31	4½	15.90	.0920	402
1⅜	1.49	86	38	4¾	17.72	.1026	448
1½	1.77	.0102	.000045	5	19.64	.1136	.000496
1⅝	2.07	120	52	5¼	21.65	.1253	547
1¾	2.41	139	61	5½	23.76	.1375	600
1⅞	2.76	160	70	5¾	26.00	.1503	656
2	3.14	.0182	.000079	6	28.27	.1636	.000714
2⅛	3.55	205	90	6½	33.18	.1920	838
2¼	4.00	230	.000100	7	38.49	.2227	972
2⅜	4.43	256	112	7½	44.18	.2557	.001116
2½	4.91	.0284	.000124	8	50.27	.2909	1269
2⅝	5.41	313	137	8½	56.75	.3284	1433
2¾	5.94	344	150	9	63.64	.3683	1606
2⅞	6.49	376	164	9½	70.88	.4102	1790
3	7.07	.0409	.000179	10	78.54	.4545	.001983
3⅛	7.67	444	194	10½	86.59	.5011	2187
3¼	8.30	480	210	11	95.03	.5500	2400
3⅜	8.95	518	226	11½	103.87	.6011	2623
3½	9.21	.0557	.000243	12	113.10	.6545	.002856
3⅝	10.32	600	261	12½	122.72	.7102	3099
3¾	11.05	639	279	13	132.73	.7681	3352
3⅞	11.79	683	298	13½	143.14	.8284	3615

TABLE VI—*Continued.*

d	A	V	E.C.	d	A	V	E.C.
14	153.94	.8908	.003887	57	2551.8	14.767	.06444
14½	165.13	.9556	4170	58	2642.1	15.290	6672
15	176.72	1.0227	4463	59	2734.0	15.822	6904
15½	188.69	1.0920	4762	60	2827.4	16.363	7140
16	201.06	1.1636	.005077				
17	226.98	1.3135	5732	61	2922.5	16.912	.07380
18	254.47	1.4726	6426	62	3019.1	17.472	7624
19	283.53	1.6408	7160	63	3117.2	18.040	7872
20	314.16	1.8181	7933	64	3217.0	18.617	8124
				65	3318.3	19.203	8380
21	364.4	2.004	.00875	66	3421.2	19.799	.08639
22	380.1	2.200	960	67	3525.7	20.403	8903
23	415.5	2.404	.01049	68	3631.7	21.017	9171
24	452.4	2.618	1142	69	3739.3	21.639	9443
25	490.9	2.841	1240	70	3848.5	22.271	9718
26	530.9	3.073	.01341				
27	572.6	3.313	1446	71	3959.2	22.912	.09998
28	615.8	3.563	1555	72	4071.5	23.562	.10282
29	660.5	3.823	1668	73	4185.4	24.221	.10569
30	706.9	4.091	1785	74	4300.8	24.889	.10861
				75	4417.9	25.566	.11156
31	754.8	4.368	.01906	76	4536.5	26.253	.11456
32	804.2	4.654	2031	77	4656.6	26.948	.11759
33	855.3	4.950	2160	78	4778.4	27.653	.12067
34	907.9	5.254	2293	79	4901.7	28.366	.12378
35	962.1	5.568	2430	80	5026.5	29.089	.12693
36	1017.9	5.891	.02570				
37	1075.2	6.222	2715	81	5153.0	29.821	.13013
38	1134.1	6.563	2864	82	5281.0	30.561	.13336
39	1194.6	6.913	3017	83	5410.6	31.311	.13663
40	1256.6	7.272	3173	84	5541.8	32.070	.13994
				85	5674.5	32.839	.14330
41	1320.3	7.640	.03334	86	5808.8	33.616	.14669
42	1385.4	8.018	3499	87	5944.7	34.402	.15012
43	1452.2	8.404	3667	88	6082.1	35.198	.15359
44	1520.5	8.799	3840	89	6221.1	36.002	.15710
45	1590.4	9.204	4016	90	6361.7	36.816	.16065
46	1661.9	9.618	.04197				
47	1734.9	10.040	4381	91	6503.9	37.638	.16424
48	1809.6	10.472	4570	92	6647.6	38.470	.16787
49	1885.7	10.913	4762	93	6792.9	39.311	.17154
50	1963.5	11.363	4958	94	6939.8	40.171	.17525
				95	7088.2	41.020	.17900
51	2042.8	11.822	.05159	96	7238.2	41.888	.18278
52	2123.7	12.290	5363	97	7389.8	42.765	.18661
53	2206.2	12.767	5571	98	7543.0	43.651	.19048
54	2290.2	13.254	5783	99	7697.7	44.547	.19439
55	2375.8	13.749	6000	100	7854.0	45.451	.19833
56	2463.0	[illegible]	.06220				

TABLE VII.

CONSTANTS FOR CIRCULAR MOTION

See § 35 (b) page 263

1 N	2 θ	3 C_1	4 C_2	1 N	2 θ	3 C_1	4 C_2
5	.524	.0114	.00035	205	21.468	19.212	.5971
10	1.047	.0457	.00142	210	21.991	20.151	.6266
15	1.571	.1028	.00320	215	22.515	21.122	.6568
20	2.094	.1825	.00568	220	23.038	22.115	.6892
25	2.618	.2855	.00888	225	23.562	23.132	.7193
30	3.142	.4112	.01306	230	24.086	24.172	.7516
35	3.665	.5597	.01740	235	24.609	25.234	.7846
40	4.189	.7311	.02273	240	25.133	26.319	.8184
45	4.712	.9252	.02877	245	25.656	27.427	.8528
50	5.236	1.1423	.03552	250	26.180	28.558	.8880
55	5.760	1.382	.04293	255	26.704	29.712	.9239
60	6.283	1.644	.05115	260	27.227	30.888	.9605
65	6.807	1.931	.06003	265	27.751	32.088	.9978
70	7.330	2.234	.06962	270	28.274	33.310	1.0358
75	7.854	2.570	.07992	275	28.798	34.555	1.0745
80	8.378	2.924	.09093	280	29.322	35.823	1.1139
85	8.901	3.301	.10265	285	29.845	37.114	1.1540
90	9.425	3.701	.11509	290	30.369	38.428	1.1949
95	9.948	4.124	.12823	295	30.892	39.764	1.2364
100	10.472	4.569	.14208	300	31.416	41.124	1.2787
105	10.996	5.038	.1566	310	32.463	43.91	1.3654
110	11.519	5.529	.1718	320	33.510	46.79	1.4549
115	12.043	6.043	.1878	330	34.558	49.76	1.5472
120	12.566	6.580	.2046	340	35.605	52.82	1.6424
125	13.090	7.140	.2221	350	36.652	55.97	1.7405
130	13.614	7.722	.2401	360	37.699	59.22	1.8414
135	14.137	8.328	.2589	370	38.746	62.55	1.9456
140	14.661	8.955	.2785	380	39.794	65.98	2.0516
145	15.184	9.607	.2987	390	40.841	69.50	2.1610
150	15.708	10.281	.3197	400	41.888	73.11	2.2733
155	16.232	10.979	.3414	410	42.935	76.81	2.3884
160	16.755	11.692	.3637	420	43.982	80.60	2.5063
165	17.279	12.440	.3868	430	45.030	84.49	2.6271
170	17.802	13.205	.4106	440	46.077	88.46	2.7506
175	18.326	13.993	.4351	450	47.124	92.58	2.8771
180	18.850	14.804	.4603	460	48.171	96.69	3.0064
185	19.373	15.638	.4863	470	49.218	100.94	3.1385
190	19.897	16.495	.5129	480	50.267	105.28	3.2735
195	20.420	17.375	.5403	490	51.313	109.71	3.4113
200	20.944	18.277	.5683	500	52.360	114.23	3.5520

TABLE VIII.

PISTON TRAVEL.

Values of the distance s of the piston from the head end of the stroke, for any crank-angle α, in terms of the stroke-length S as unity, and for different values of the ratio n of the connecting-rod to the crank: calculated from equation (210), put into the form

$$s = S\frac{(1-\cos\alpha)+n(1-\cos\beta)}{2}.$$

$\alpha°$		Values of n.				
		4	5	6	8	∞
5	355	.0023	.0023	.0022	.0022	.0019
10	350	.0095	.0091	.0089	.0086	.0076
15	345	.0212	.0204	.0198	.0191	.0170
20	340	.0375	.0360	.0350	.0338	.0302
25	335	.0581	.0558	.0543	.0525	.0468
30	330	.0827	.0795	.0774	.0748	.0670
35	325	.1111	.1069	.1042	.1007	.0904
40	320	.1430	.1377	.1343	.1299	.1170
45	315	.1780	.1716	.1674	.1621	.1464
50	310	.2156	.2081	.2032	.1970	.1786
55	305	.2556	.2470	.2413	.2342	.2132
60	300	.2974	.2878	.2814	.2735	.2500
65	295	.3407	.3301	.3231	.3145	.2887
70	290	.3850	.3735	.3660	.3567	.3290
75	285	.4298	.4177	.4097	.3998	.3706
80	280	.4747	.4622	.4539	.4436	.4132
85	275	.5194	.5066	.4981	.4876	.4564
90	270	.5635	.5505	.5420	.5314	.5000
95	265	.6066	.5987	.5852	.5747	.5436
100	260	.6484	.6358	.6275	.6173	.5868
105	255	.6886	.6765	.6685	.6587	.6294
110	250	.7270	.7157	.7080	.6987	.6710
115	245	.7633	.7527	.7457	.7371	.7113
120	240	.7974	.7878	.7814	.7735	.7500
125	235	.8292	.8206	.8149	.8078	.7868
130	230	.8584	.8509	.8459	.8398	.8214
135	225	.8851	.8787	.8745	.8692	.8536
140	220	.9090	.9038	.9003	.8959	.8830
145	215	.9303	.9261	.9233	.9199	.9096
150	210	.9487	.9455	.9435	.9409	.9330
155	205	.9644	.9621	.9606	.9588	.9532
160	200	.9772	.9757	.9747	.9735	.9698
165	195	.9872	.9863	.9858	.9851	.9830
170	190	.9943	[illegible]	.9937	.9934	.9924
175	185	.9986		.9984	.9983	.9981

TABLE IX.

FACTORS FOR ACCELERATION AND INERTIA OF THE PISTON

Values of $m = \left(\cos\alpha + \frac{R}{L}\cos 2\alpha\right)$.

See Equations (213) and (224), § 33 (e) and § 35 (a).

1		2	3	4	5	6	7
$\alpha°$		cos α	cos 2α	Values of $n = \frac{L}{R}$.			
				4	5	6	8
0	360	1.0000	1.0000	1.2500	1.2000	1.1667	1.1250
5	355	.9962	.9848	1.2424	1.1932	1.1603	1.1193
10	350	.9848	.9397	1.2197	1.1728	1.1414	1.1023
15	345	.9659	.8660	1.1824	1.1391	1.1103	1.0742
20	340	.9397	.7660	1.1312	1.0929	1.0674	1.0355
25	335	.9063	.6428	1.0670	1.0349	1.0134	.9867
30	330	.8660	.5000	.9910	.9660	.9494	.9285
35	325	.8192	.3420	.9047	.8876	.8762	.8619
40	320	.7660	.1737	.8095	.8008	.7950	.7878
45	315	.7071	.0000	.7071	.7071	.7071	.7071
50	310	.6428	−.1737	.5994	.6081	.6139	.6211
55	305	.5736	−.3420	.4881	.5052	.5166	.5308
60	300	.5000	−.5000	.3750	.4000	.4167	.4375
65	295	.4226	−.6428	.2619	.2941	.3155	.3423
70	290	.3420	−.7660	.1505	.1888	.2144	.2463
75	285	.2588	−.8660	.0423	.0856	.1145	.1506
80	280	.1737	−.9397	−.0613	−.0143	.0170	.0562
85	275	.0872	−.9848	−.1590	−.1098	−.0770	−.0359
90	270	.0000	−1.000	−.2500	−.2000	−.1667	−.1250
95	265	−.0872	−.9848	−.3334	−.2841	−.2513	−.2103
100	260	−.1737	−.9397	−.4086	−.3616	−.3303	−.2911
105	255	−.2588	−.8660	−.4753	−.4320	−.4032	−.3671
110	250	−.3420	−.7660	−.5335	−.4952	−.4697	−.4378
115	245	−.4226	−.6428	−.5833	−.5512	−.5298	−.5030
120	240	−.5000	−.5000	−.6250	−.6000	−.5830	−.5625
125	235	−.5736	−.3420	−.6591	−.6420	−.6306	−.6163
130	230	−.6428	−.1737	−.6862	−.6775	−.6717	−.6645
135	225	−.7071	.0000	−.7071	−.7071	−.7071	−.7071
140	220	−.7660	.1737	−.7226	−.7313	−.7371	−.7443
145	215	−.8192	.3420	−.7336	−.7508	−.7622	−.7764
150	210	−.8660	.5000	−.7410	−.7660	−.7827	−.8035
155	205	−.9063	.6428	−.7456	−.7805	−.7992	−.8260
160	200	−.9397	.7660	−.7482	−.7865	−.8120	−.8439
165	195	−.9659	.8660	−.7494	−.7927	−.8216	−.8577
170	190	−.9848	.9397	−.7499	−.7969	−.8282	−.8674
175	185	−.9962	.9848	−.7500	−.7992	−.8321	−.8731
180		−1.000	1.0000	−.7500	.8000	−.8333	−.8750

TABLE X.

TURNING-FORCE RATIOS.

Values of $m = \frac{\sin(\alpha+\beta)}{\cos\beta} = \frac{T}{P} = \frac{v}{v_0}$, for different values of $n = \frac{L}{R}$: see § 35 (d).

$\alpha°$		Values of n.				
		4	5	6	8	∞
5	355	.1089	.1045	.1016	.0980	.0872
10	350	.2164	.2079	.2022	.1950	.1737
15	345	.3215	.3089	.3005	.2901	.2588
20	340	.4227	.4065	.3957	.3822	.3420
25	335	.5189	.4995	.4866	.4706	.4226
30	330	.6091	.5870	.5724	.5542	.5000
35	325	.6923	.6682	.6523	.6325	.5736
40	320	.7675	.7421	.7253	.7054	.6428
45	315	.8341	.8081	.7910	.7699	.7071
50	310	.8915	.8657	.8488	.8279	.7660
55	305	.9392	.9144	.8982	.8782	.8192
60	300	.9769	.9540	.9390	.9205	.8660
65	295	1.0046	.9842	.9709	.9545	.9063
70	290	1.0224	1.0051	.9939	.9801	.9397
75	285	1.0303	1.0169	1.0082	.9974	.9659
80	280	1.0289	1.0197	1.0137	1.0064	.9848
85	275	1.0186	1.0139	1.0109	1.0072	.9962
90	270	1.0000	1.0000	1.0000	1.0000	1.0000
95	265	.9738	.9785	.9815	.9853	.9962
100	260	.9407	.9499	.9559	.9633	.9848
105	255	.9015	.9150	.9237	.9344	.9659
110	250	.8570	.8742	.8855	.8992	.9397
115	245	.8080	.8284	.8417	.8581	.9063
120	240	.7552	.7781	.7931	.8116	.8660
125	235	.6991	.7239	.7401	.7601	.8192
130	230	.6406	.6664	.6833	.7042	.7660
135	225	.5801	.6061	.6232	.6444	.7071
140	220	.5181	.5435	.5603	.5810	.6428
145	215	.4549	.4790	.4949	.5147	.5736
150	210	.3909	.4130	.4276	.4458	.5000
155	205	.3263	.3458	.3586	.3747	.4226
160	200	.2614	.2776	.2884	.3018	.3420
165	195	.1962	.2088	.2171	.2276	.2588
170	190	.1309	.1394	.1451	.1523	.1737
175	185	.0654	.0698	.0727	.0763	.0872

NOTES ON SUPERHEATED STEAM.

REGNAULT made four sets of experiments, upon steam under atmospheric pressure, superheated to from 252° to 448° F., or with 40° to 236° of superheat. His method was to condense a small portion of steam in an accurate water-calorimeter, so as to measure its total heat, and then subtract from this the total heat found in the same way for steam superheated about 15°. The results agreed very well; there was no apparent variation of the specific heat with the temperature; and the mean value which he found was 0.4805. Published in 1862, this was for a long time the only definite information in regard to the specific heat of steam under constant pressure; and was generally used, in spite of doubt as to its applicability under all conditions. It is as a consequence of the extensive adoption of superheating in the steam-plant, especially in Europe, that further investigations have been undertaken.

This determination by measurement of total heat is essentially a difficult method, because the heat-quantity sought is but a small fraction of the heat measured, so that extreme precision is necessary to reliable results. Later experiments have generally followed three lines: first, for small ranges above saturation, superheating by wire-drawing; second, for technical ranges of temperature and pressure, cooling in a water-calorimeter of the surface-condenser type, from a higher to a lower degree of superheat; third, for very high temperatures, exploding a mixture of hydrogen and oxygen and deducing specific heat from temperature and heat of combustion.

The last-named method was earliest used, and will first be briefly described. The purpose of the experiments was, primarily, to find the temperature of combustion, under the assumption that all the heat generated remains in the combined gases. Calorific power, or heat of complete combustion, was already known: if the specific heat determined for ordinary temperatures could be used for high ranges, the desired temperature was very simply calculable; but it was soon apparent that this was far from the truth, and that special experiments were necessary. These were made by enclosing a small body of the mixed gases in a strong chamber or bomb, igniting it with an electric spark, and measuring, by means of a very accurate manometer, the maximum pressure reached before cooling had time to occur. From observed pressure, temperature was calculated by the general law $pv = CT$; the heat of combustion Q being known, the specific heat at constant volume, c_v, could be found, and c_p deduced from it—the value of $k = c_p/c_v$ being involved.

From the results of numerous experiments, beginning with those of Mallard and Le Chatelier published in 1883, the metallurgists—who most use high ranges of temperature—have settled upon the formula

$$c_p = \underset{\text{Fahrenheit.}}{0.413 + .000192t} \quad \text{or} \quad \underset{\text{Centigrade.}}{0.419 + .000345t}. \quad . \quad . \qquad (290)$$

This gives the actual specific heat at any temperature t; to get the quantity of heat for a rise from t_1 to t, we must use a mean value; which will be, putting the formula into the general shape $c_p = a + bt$,

$$c_{pm} = a + b\frac{t_1 + t}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (291)$$

For the imaginary operation of heating from 0° to t (equivalent to assuming that the steam remains a perfect gas down to zero), when t_1 equals zero in (291), this gives, from (290),

$$c_{pm} = 0.413 + 0.000096t \quad \text{or} \quad 0.419 + 0.000173t. \quad . \qquad (292)$$

Very obviously, we have no assurance that these formulas will apply near saturation and for low degrees of superheat; for the properties of a perfect gas enter into their deduction.

Of experiments by wire-drawing, two first-class sets have been made: by Grindley, published in Philosophical Transactions of the Royal Society of London, 1900, Vol. 194; and by Griessmann, found in Zeitschrift des Vereins Deutscher Ingenieure, 1903, page 1850. In both cases, the apparatus was essentially a throt-

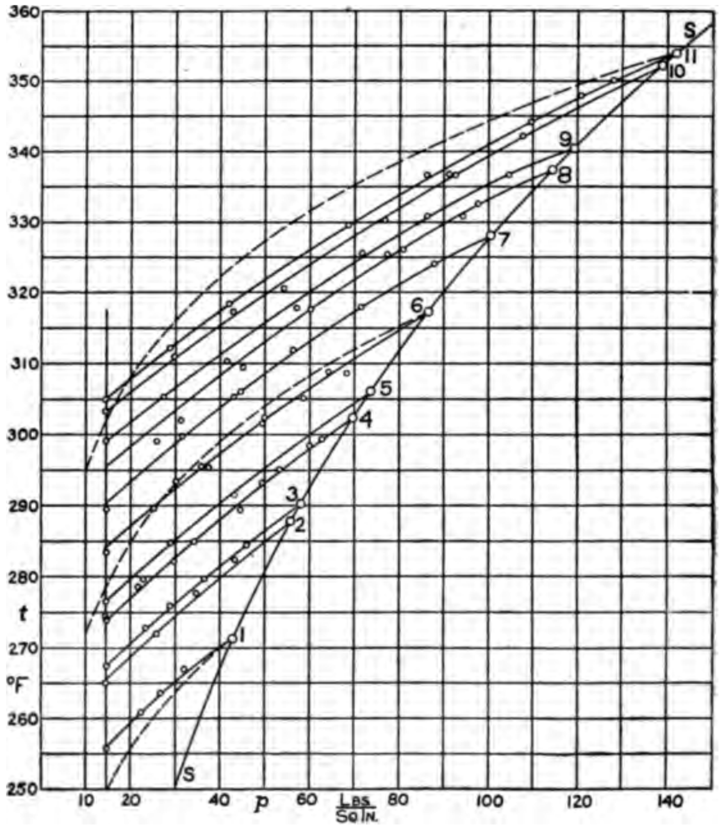


FIG. 188.—Curves of Throttling, according to Griessmann.

tling calorimeter which drew dry saturated steam from a specially arranged separator, great precautions being taken to avoid sources of error. The observations were, the pressure p_1 and temperature t_1 (checking each other) on the high side; and beyond the throttling-plate, the pressure p_0 and temperature t_2. Griessmann's results,

as the later and more accurate, are laid out in Fig. 188, on a diagram similar to that given by the author, but changed to English measures. For each initial pressure, a number of different discharge pressures were produced and maintained; and the observed lower temperatures are plotted on a pressure-base. SS is the curve of saturation, the same thing as curve I. on Fig. 22, but in a different position. Through the t_2 points belonging to each t_1 a curve is drawn; and the good agreement of the points with their respective curves speaks well for the accuracy of the experiments.

To each of these curves corresponds a certain value of the total heat in the steam, which value remains constant as the lower pressure changes—see § 28 (*b*), page 183. The several numbers are:

Curve No.	p_1 Lbs. Abs.	t_1 ° F.	H_1 B.T.U.	Curve No.	p_1 Lbs. Abs.	t_1 ° F.	H_1 B.T.U.
1	42.8	271.3	1164.65	7	100.8	328.1	1182.02
2	55.6	287.6	1169.66	8	114.4	337.4	1184.86
3	58.0	290.3	1170.49	9	119.4	340.5	1185.82
4	69.7	302.4	1174.15	10	138.7	352.0	1189.31
5	73.8	306.2	1175.32	11	141.9	353.8	1189.85
6	86.5	317.2	1178.70				

Having these values of the total heat and the corresponding temperatures of superheated steam as shown by the curves themselves, it is an easy matter to calculate the specific heat involved in a change of temperature along any one of the vertical lines of constant pressure. Thus at 40 lbs. pressure, curve 2 shows 279.8°; curve 10, 315.5°; the steam is raised through 35.7° by 1189.31 − 1169.66 = 19.65 B.T.U.; and the mean specific heat for the operation is 19.65 ÷ 35.7 = 0.550.

Turning to Eq. (154), page 184, we see that if the steam is originally dry-saturated, so that $x_1 = 1.00$, and if $c_p = 0.48$, the relation holds

$$0.305(t_1 - t_0) = 0.48\ (t_2 - t_0);$$

whence

$$t_2 - t_0 = 0.635(t_1 - t_0),$$

$$t_1 - t_2 = 0.365(t_1 - t_0). \quad . \quad . \quad . \quad . \quad . \quad (293)$$

This formula would give to t_2 the manner of variation shown by the dotted curves drawn from points 1, 6, and 11. Comparing these with the full-line curves of observed temperature, we can make the following deductions as to the true specific heat:

First, c_p is much greater than 0.48 for the higher pressures, because the actual temperature is so much less than that calculated from (293).

Second, as the saturation temperature is lower, c_p is less: for curve 1, it is less than 0.48 from the beginning at 1.

Third, along the lines of low pressure, as at 15 lbs., c_p increases steadily with the temperature: thus at 1, the true curve is well above the dotted, at 11 they are nearly together.

Dr. Griessmann worked out, from his curves, values of c_p for a number of different pressures. In Fig. 189 are plotted results

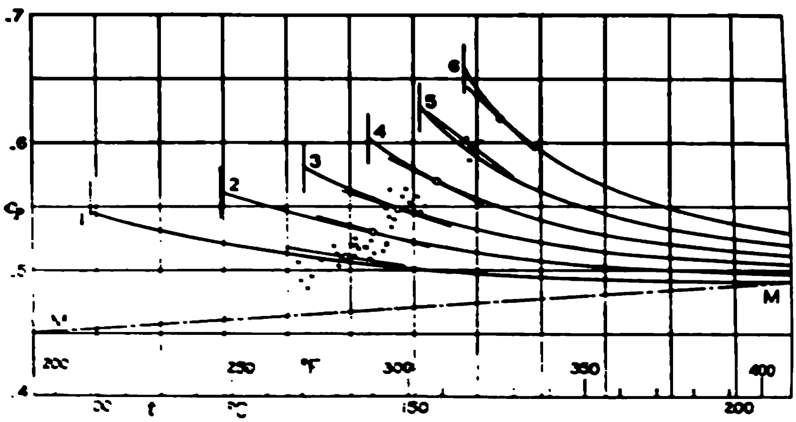


FIG. 189.—Curves through Griessmann's Results.

for pressures of 1, 2, 3, 4, 5, and 6 kg. per sq. cm. (1 kg. per sq. cm. = 14.224 lbs. per sq. in.) The mean of a number of determinations is given by each circle, while separate values are shown by the spots for 1, 3 and 5 kg. pressure. The average value of [illegible]

The most noticeable thing about these results is the rapid increase of c_p with t—far greater, for instance, than that in (290), which formula is represented by the line MM. To explain this apparent discrepancy, we note first that c_p, as found for 1 kg., is much farther from saturation than for 5 or 6: then it seems entirely reasonable to suppose that the specific heat would be greater near saturation, where there is likely to be disgregation work; and further, that it may increase with the pressure, that is, with the density, of the steam.

These ideas are given graphical expression in the curves sketched on Fig. 189, each beginning at the saturation temperature marked by a short vertical line. They constitute an hypothesis which, although it is not confirmed in detail by the primary results in Fig. 188, and is even contradicted by the third statement above Fig. 189, nevertheless offers the best way of reconciling what are otherwise conflicting data.

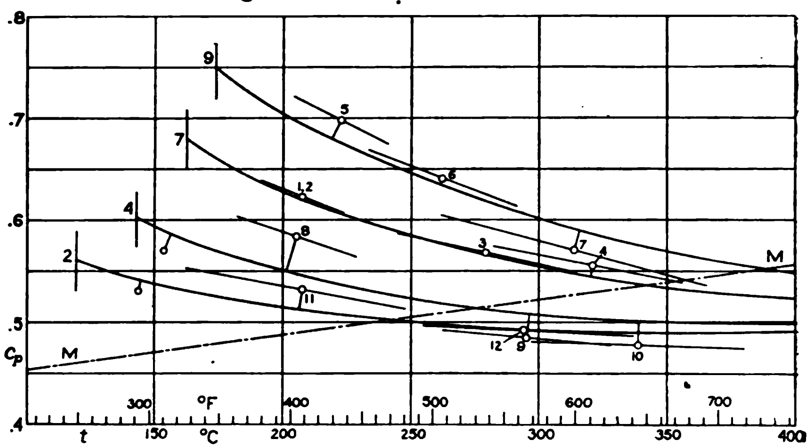


FIG. 190.—Results of Lorenz, by the Method of Cooling.

Strong confirmation of this idea as to the heat-properties of superheated steam is given by the experiments of H. Lorenz, published in Zeitschrift V. D. I., 1904, page 698. These were made by the method of cooling, the steam passing through a surface-calorimeter, escaping well above saturation, and then flowing into a surface-condenser. The results are represented

graphically on Fig. 190, where each numbered circle shows the mean value of c_p at a particular pressure and temperature, and the range through which the steam was cooled—the last by inclined straight lines, as in the preceding figure. The results are grouped according to pressure by drawing curves like those on Fig. 189; two points from that figure being also marked, for pressures of 2 and 4 kg. Here it is very evident that the specific heat is greater toward saturation and for higher densities.

The amount of data is too small, and its agreement not cordial enough, for any more definite expression of the apparent law than is given by these curves. They are made of fair form, and spaced with regard to the fact that there must be some smoothly-acting law of variation. The starting-points, on the saturation-temperature lines, lie on a fair curve. The curves evidently tend to merge into the line MM, or one of similar trend, for high temperatures. While not closely enough determined to give exact results, they are nevertheless likely to be of great practical utility for technical computations.

In regard to the throttling calorimeter, the statement made at the top of page 195 must be modified in view of this later interpretation of experimental results. It appears, however, from Fig. 188, that a mean value a little below 0.48, say 0.45, would be proper for use in Eqs. (154), (158), etc., when the discharge is at atmospheric pressure. In this we accept the results on Fig. 188 rather than the curve traced on Fig. 189; and it would be easy to derive from Fig. 188 a curve showing the true "normal temperature" for different initial temperatures.

The entropy-temperature curve for superheated steam, as drawn in Fig. 86, page 229, will be considerably modified by these later results. A different curve would have to be drawn for each starting-point on the line RS; but as the temperature rose, the curves would approach similarity and constant-distance spacing.

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